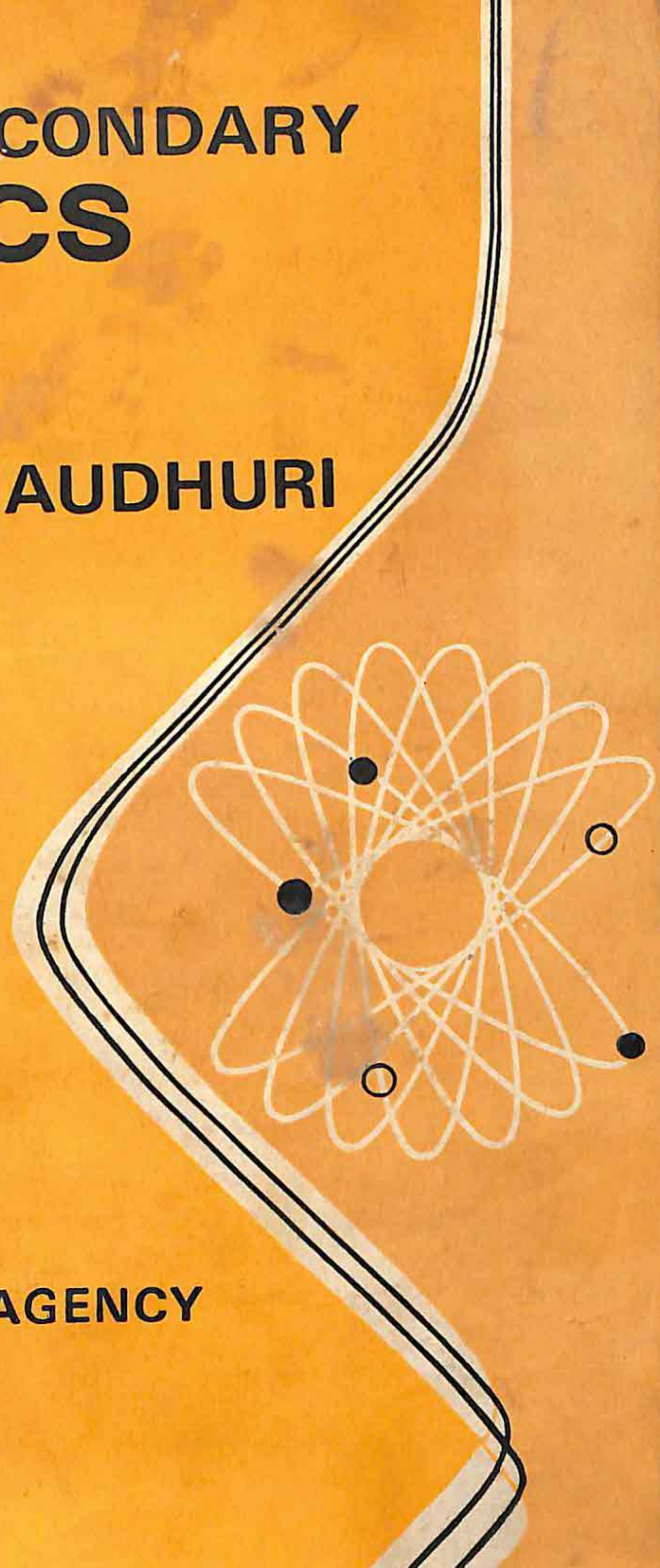
HIGHER SECONDARY
PHYSICS

D. P. RAYCHAUDHURI



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Written according to the New Syllabus introduced by the West Bengal Council of Higher Secondary Education for Classes XI *and* XII.

HIGHER SECONDARY PHYSICS

[For Classes XI & XII]

STATE INSTITUTE OF EDUCATION
Govt. of West Bengal

PART ONE

Group A : MECHANICS, VIBRATIONS AND WAVES

Group B : GENERAL PROPERTIES OF MATTER, HEAT

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PREFACE

I took up preparing the book on the basis of the syllabus forwarded by the Council of Higher Secondary Education to the West Bengal Publishers' and Book-sellers' Association. From time to time the Council forwarded modifications. These have been accommodated. Nevertheless, the printed syllabus differed in some minor respects from what was forwarded to the Association. Though they did not require any substantial modificaiton of the text, the author regrets deletion of reference to SI units. The topmost international scientific bodies strongly advocated the use of one system only (Syste'me Internationale) for all scientific measurements all over the world. West Germany accepted it even for domestic use from 1978. But in our country school children are still required to learn *systems* of units (meaning perhaps the cgs, fps and mks (?) systems). This is entirely unnecessary. It would have been good for all if the use of only the SI units were encouraged. Incidentally, it may be noted that the Council for the Indian School Certificate Examinations have issued clear instructions for the exclusive use of SI units for their examinations (Classes IX to XII). I wonder why our syllabus framers are so indifferent about international units and symbols, as well as the way how to use them. With the financial support of the UNESCO, a booklet entitled 'Symbols, Units and Nomenclature in Physics' was circulated around 1967 by the NCERT to all institutions of higher education in India. Authors, research workers, writers of popular scientific articles from the rest of India in general are following the international recommendations contained in the pamphlet. But in West Bengal we find very little evidence of it.

I have tried in this book to give the student some acquaintance with SI units and their handling. As there is a talk of syllabuses being soon revised and lightened, I fervently hope more attention will be paid to the SI units and obsolete topics dropped from the course. What matters primarily is to determine the purpose of the physics course. Should it be part of a 'General Science' course or a preparatory course for higher studies ? I apprehend the two purposes cannot be successfully combined. The present course seems to support my apprehension.

The syllabus might have advised use of modern nomenclature, such as 'Relative density' for 'Specific gravity', 'Specific heat capacity' for 'Specific heat', 'Specific latent heat' for 'latent heat per unit mass', etc. The word 'specific' is at present used in the sense 'per unit mass'. The usage should be encouraged. Since heat is energy, the 'calorie' is dropping out of use and the 'joule' is being used as the unit of heat. Such usage has the added advantage of doing away with the 'mechanical equivalent of heat'. Let us hope that the new syllabus will be more up-to-date in such matters.

When a page limit is prescribed for a book, it is necessary that the maximum number of pages that could be allotted to the main topics be at the same time specified. Otherwise, there will be too much variation from book to book, creating confusion among teachers and students alike. An approximate page limit to the text is desirable ; but wasting space by printers for profit may alone make a difference of 10% or more in the same text.

Let us hope that the relevant successor authorities will look into these matters.

D. P. R.

STATE INSTITUTE OF EDUCATION * Govt.

Syllabus in Physics (Elective)

(Ordinary Level)

Paper I—Mechanics ; General Properties of Matter ; Heat ; Vibrations and Waves. (80 marks)

Paper II—Optics ; Magnetism ; Electrostatics ; Current Electricity ; Modern Physics. (80 marks)

Practical—40 marks.

[The typed syllabuses that the West Bengal Council of Higher Secondary Education forwarded from time to time to the All Bengal Publishers' and Book-Sellers' Association to expedite writing of text-books, divided each paper into two Groups as follows :

Paper I : Group A : Mechanics ; Vibrations and Waves.
Group B : Heat ; General Properties of Matter.
Paper II : Group A : Electrostatics ; Magnetism ; Current Electricity.
Group B : Optics (Light) ; Modern Physics.
Each Group to carry 40 marks.

But nothing has been stated in the printed syllabus about such division.]

GROUP A

Mechanics

Particle Dynamics : Rest and motion, reference frame, displacement, velocity and acceleration, momentum, kinematical equations (in one dimension), elementary problems.

Scalars and Vectors. Composition and resolution of vectors. Representation of vector by coordinates. Addition of vectors by geometrical and analytical methods. Relative velocity and acceleration.

Newton's laws of motion, inertia, units of force, impulse and impulsive forces, conservation of linear momentum, elastic collisions of particles moving in the same line, jets and rockets. Friction, static and kinetic friction, coefficient of friction.

Statics : Centre of mass, centre of gravity. Conditions of equilibrium of a system of particles.

Dynamics of Rotational Motion : Rotational motion of a particle, angular velocity, angular acceleration, relation between angular velocity and linear velocity, angular momentum, moment of a force about a point and about an axis, torque, relation between angular momentum and torque (statement only), couples, centripetal force, centrifugal force (as a pseudo-force).

Work, Energy and Power : Definition of work, relevant units, work done by and against a force. Mechanical energy—kinetic and potential forms. Conservation of energy—with the case of a freely falling body as an example. Power—definition, units.

Vibrations and Waves

Vibrations: : Oscillation and its characteristics. Simple harmonic motion, examples. Relation with uniform circular motion. Graphical and mathematical representations. Energy in simple harmonic motion. Superposition of two simple harmonic motions in the same direction (graphical) (i) in phase, (ii) in opposite phases.

Nature of vibrations—(transverse and longitudinal). Free and forced vibrations, resonance, damped oscillations (qualitative discussions with examples).

Waves : Types of waves, characteristic features of propagating waves, preliminary definitions and relations. Reflection and refraction of waves.

Superposition of waves ; stationary waves ; vibrations of strings and air columns.

Interference, beats, Doppler effect, polarization (qualitative discussions).

Nature of waves : *(i)* Sound waves as elastic waves. Velocity of sound, Laplace's formula (Newton's formula $v=\sqrt{E/\rho}$ to be assumed.)

Sources of sound. Musical sound and noise. Principles of recording and reproduction of sound.

(ii) Light as a wave phenomenon. Finite velocity of light. Interference of light. Polarization (qualitative ideas). Validity of geometrical optics as an approximation.

GROUP B

General Properties of Matter

Gravitation : Newton's law of universal gravitation. Constant of Gravitation (no experimental details on the determination of the Gravitational Constant). Gravitational attraction for extended bodies. Gravitational attraction of the earth. Laws of falling bodies. Variation of acceleration due to gravity. Simple pendulum. Motion of planets, satellites. Escape velocity (no deduction). Weightlessness in orbiting satellites.

Elastic properties of matter : Stress, strain, elastic limit, Hooke's law, elastic moduli, Young's modulus, bulk modulus, rigidity modulus, Poisson's ratio.

Hydrostatics : Density, Specific Gravity, (methods of determination of Sp. Gr. not required), Archimedes' principle (demonstrations), flotation, pressure in fluids, transmission of fluid pressure, Pascal's law and its applications. Air pressure and its measurement. Siphon, principles of lift pump, compression pump, vacuum pump.

Surface Tension and Viscosity : Simple surface tension phenomena (illustrated with demonstrations). Motion in fluids—viscosity—streamline and turbulent flow (qualitative ideas).

Heat

Recapitulation of the basic concepts of heat and temperature.

Thermal expansions of solids and liquids. Simple demonstrations, Coefficients of expansion for solids, relation between them. Applications of expansion of solids.

Real and apparent expansions for liquids, relation between expansion coefficients. Anomalous expansion of water. Effect on marine life.

Thermal expansion of gases.

Boyle's law, Charles' law. Equation of state of an ideal gas; volume and pressure coefficients. Absolute scale of temperature.

Calorimetry : Preliminary definitions, principle of calorimetry (no questions on measurement to be set). Calorimetric problems.

Change of State : Latent heat (brief discussions of determination), evaporation and boiling. Effects of pressure on melting point and boiling point.

falling body as an example. Power—definition, units.

Vibrations and Waves

Vibrations: : Oscillation and its characteristics. Simple harmonic motion, examples. Relation with uniform circular motion. Graphical and mathematical representations. Energy in simple harmonic motion. Superposition of two simple harmonic motions in the same direction (graphical) (i) in phase, (ii) in opposite phases.

Nature of vibrations—(transverse and longitudinal). Free and forced vibrations, resonance, damped oscillations (qualitative discussions with examples).

Waves : Types of waves, characteristic features of propagating waves, preliminary definitions and relations. Reflection and refraction of waves.

Superposition of waves ; stationary waves ; vibrations of strings and air columns.

Interference, beats, Doppler effect, polarization (qualitative discussions).

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Surface Tension and Viscosity : Simple surface tension phenomena (illustrated with demonstrations). Motion in fluids—viscosity—streamline and turbulent flow (qualitative ideas).

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Thermal expansion of gases.

Boyle's law, Charles' law. Equation of state of an ideal gas; volume and pressure coefficients. Absolute scale of temperature.

Calorimetry : Preliminary definitions, principle of calorimetry (no questions on measurement to be set). Calorimetric problems.

Change of State : Latent heat (brief discussions of determination), evaporation and boiling. Effects of pressure on melting point and boiling point.

Vapour pressure. Relative humidity. Dew, fog and cloud. Hygrometry, Regnault's hygrometer.

Mechanical equivalent of heat : Heat as a form of energy. Relation between the calorie and the erg. Determination of mechanical equivalent of heat (paddle method). First law of thermodynamics. Isothermal and adiabatic expansions of gases. Specific heats of gases, definitions of C_p, C_v.

Kinetic Theory of Gases : Evidence of molecular structure of matter and of random molecular motion. Brownian movement (qualitative description). Basic assumptions of the kinetic theory of ideal gases. Pressure of an ideal gas (mention of the formula ; derivation not required). Concept of temperature from kinetic theory. Qualitative discussions of limitations of ideal gas laws.

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PHYSICS | INTRODUCTORY REMARKS

I-1. Matter and energy. The objects that we see all around us, e.g., books, tables, trees, stars etc. are called *bodies. Matter* is the material of which these bodies are made. Anything which has weight and occupies space is supposed to be made of matter.

The changes that we see in Nature may be divided into three classes, such as, change in (i) the position, (ii) the condition or (iii) the type of matter. A moving train shows a change of position. A boiling kettle shows a change of condition of water, from liquid to vapour. In burning coal, matter changes type—from coal to the gas carbon dioxide.

Matter cannot by itself undergo any change. Change of any kind in matter must be brought about by some agency. The invisible agency which does it is called *energy*. All natural phenomena are effects of the action of energy on matter. Matter and energy are all that exist in the physical world.

It will be of help to accept the following definitions :

Matter is anything which offers resistance when attempt is made to change its condition of rest or motion.

Energy is that which causes a change in the position, the condition or the type of matter.

A **body** is any piece of matter.

I-2. What is physics. Matter has been found to consist of about 90 elementary types called *elements*. Water is made up of two elementary types of matter, viz., hydrogen and oxygen. Air is a mixture, chiefly of two elements—nitrogen and oxygen.

We may say that *physics deals with those phenomena of the inanimate world in which the type of matter does not change.* Chemistry is essentially concerned with phenomena in which the type changes. This distinction does not, however, always hold.

Branches of physics. Physics has several main divisions. Of these we shall be mostly concerned with *mechanics, vibration and waves, heat, light, magnetism, electricity* and *modern physics*. Of all these, mechanics is the most fundamental because it is necessary in practically all the branches of physics. It deals with properties of matter, and treats of such ideas as motion, force, energy and their relations to one another and to matter. Heat, sound and light include those phenomena which we meet through our senses of touch, hearing and sight. We have no special senses for detecting electricity and magnetism. They are studied through the effects they produce.

Physics is built on accurate measurement. In the language of physics the term '*physical quantity*' (or sometimes simply 'quantity') is used to mean anything that can be measured. Length, weight, volume, time, speed, temperature etc. are common examples. The *laws of physics* are expressed as relations between different physical quantities. Measurement is the foundation on which physics is built.

To measure a quantity we compare it with a quantity of the same kind, which we call the *unit*. To find the length of a bench we may use a metre stick and see how many times the length known as the metre is contained in the length of the bench. If by laying the metre stick alongside the bench we find that the latter is three times as long as the former, we say that the length of the bench is 'three metres'. A measurement involves two things, a 'number' and a 'unit'. Mere mention of the number carries no sense ; the unit must be stated. *A physical quantity is made up of a number and a unit.*

I-3. Fundamental quantities and their units. Among the quantities which physics deals with, three are regarded as fundamental. They are *length*, *mass* and *time*. Length and time are well known concepts and cannot be explained in simpler terms. **Mass** *is the amount of matter contained in a body*. The units of these three quantities are called the *fundamental units*.

All quantities with which we are concerned in *mechanics* can be defined in terms of length, mass and time. Their units can also be derived from the units of length, mass and time. For this reason, these three quantities and their units are called *fundamental* (or *basic*). Units of the other quantities are called *derived* units,

I-3.1. Absolute (or coherent) units of a system. Derived units of a system which involve the fundamental units of the system only in unit measure, are known as *absolute units* (or coherent units). The absolute unit of work in the cgs system is 1 erg, which is $1\ \text{dyn} \times 1\ \text{cm} = 1\ \text{g} \times 1\ \text{cm/s}^2 \times 1\ \text{cm}$. In the erg, all the fundamental units occur in unit measure. The joule (symbol J), which is equal to 10^7 ergs, is not an absolute unit in the cgs system, as it involves a measure 10^7. It is the *practical unit* of work in the cgs system, the erg being too small a work for practical use. In the mks system, however, the joule comes out to be an absolute unit as 1 joule=1 newton×1 metre=1kg×1 m/s^2×1 m.

If in a numerical problem you find that the quantities are in different systems of units, or are not in absolute units of the same system, convert all of them to absolute units of the same system before you apply any formula to calculate the result.

I-4. SI Units (Syste'me Internationale Unite's) or the International System of Units. For the coherent* system based on the six basic units

Name	Symbol	Name	Symbol
metre	m	ampere	A
kilogram	kg	degree Kelvin	°K
second	s	candela	cd

the name *International System of Units* has been recommended by the General Conference on Weights and Measures in 1960. The units of this system are called SI units. They have been accepted by important international bodies as the only system for use in all scientific measurements.**

In developing any system of units, certain quantities are taken as *fundamental* (or *basic*). A very well defined amount of each such quantity is taken as a *fundamental unit*. In the International System of Units, the fundamental quantities are (1) length, (2) mass, (3) time, (4) electric current, (5) temperature and (6) luminous intensity. The International System defines the units of these quantities as accurately as present day technology permits. You are already familiar with the metre, the kilogram and the second. The old definitions of the metre and the second have been altered to make the amounts more precise. The definitions of the ampere, degree kelvin and candela will come up respectively under current electricity, heat and light (photometry).

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* A *coherent* system of units is a system based on a certain set of 'basic units' from which all 'derived units' are obtained by multiplication or division without introducing any numerical factors.

** You may be interested to learn that West Germany passed a law in 1970 to the effect that from 1st January, 1978, SI units would also be used there for all domestic purposes. Of late, the India Government have instructed that clinical (doctor's) thermometers should be graduated in celsius degrees instead of in fahrenheit degrees, as at present. When this comes into effect, a fever of temperature 104°F will read 40°C on our thermometer and 313°K on a West German thermometer.

Of late, the use of the degree sign (°) on the absolute scale has been dropped. So we write 273K instead of 273°K. But this method of writing has not yet become universal.

The MKS system of units. This system is based on the metre, the kilogram and the second as fundamental units. It is closely related to the CGS system, and a value in one system can be changed into that in another by merely shifting the position of the decimal point. All mechanical quantities can be measured in MKS units. All important international bodies have adopted this system for all scientific measurements. It has the advantage that the derived units in this system are not as small as those in the CGS system. Hence there is no necessity of having a separate set of practical units. In the MKS system

unit of force $=1$ newton (symbol N) $=1\ \text{kg}\times 1\ \text{m/s}^2=10^5$ dyn.
unit of work $=1$ joule (symbol J) $=1\ \text{N}\times 1\ \text{m}=10^7$ erg.
unit of power $=1$ watt (symbol W) $=1\ \text{J}/1\ \text{s}=10^7$ erg/s.

To give you a little acquaintance with these quantities we have occasionally used them. *All derived units of the MKS system are SI units.*

I-4.1. Unit for angle. For use in scientific measurements, the unit for *plane angle* is the *radian* (symbol, rad). A **radian** is the angle subtended at the centre of a circle by an arc equal in length to the radius. To express an angle in radian measure, also known as *circular measure*, we may describe an arc of a circle with the vertex of the angle as centre. The length of the arc limited by the angle divided by the radius (*i.e.*, arc $AB \div$ radius OA ; Fig. I.1) is the value of the angle in radians.

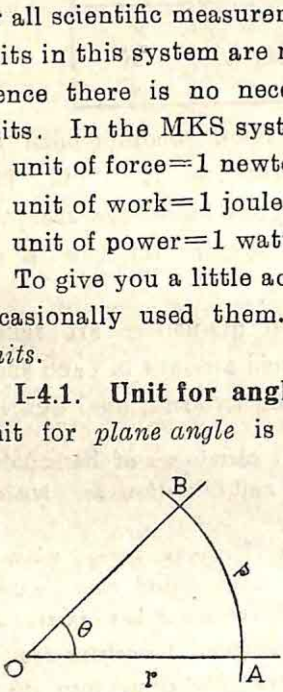

Fig. I.1

$$\text{Angle } (\theta) \text{ in radians} = \frac{\text{arc } (s)}{\text{radius } (r)}.$$

Hence length of arc=radius × angle subtended by arc in radians.

$$\text{or } s=r\theta.$$

The ratio of the circumference of a circle to its diameter is a constant. It is represented by the Greek letter π (read 'pie'). The total angle around a point is 2π radians ($\pi=3.1416...$). Hence 2π rad$=360^\circ$ or 1 rad$=360^\circ/2\pi=57^\circ 17' 45''=57.3^\circ$. The unit for *solid angle* is the **steradian** (symbol, sr).

I-5. International unit-symbols. In writing and using symbols of units of physical quantities, we shall follow the present international recommendations. Some of the symbols and prefixes are given below. The symbols are always printed in *roman (upright) type*. The symbol of a unit, derived from a proper name, should start with a capital roman letter (such as N for newton, J for joule, Hz for hertz).

CGS		MKS	
Name	Symbol	Name	Symbol
centimetre	cm	metre	m
gram	g	kilogram	kg
second	s	second	s
dyne	dyn	newton	N
erg	erg	joule	J
hertz	Hz	hertz	Hz

Prefixes to indicate decimal fractions or multiples of a unit

deci $(=10^{-1})$	d	kilo $(=10^{3})$	K
centi $(=10^{-2})$	c	mega $(=10^{6})$	M
milli $(=10^{-3})$	m	giga $(=10^{9})$	G
micro $(=10^{-6})$	μ	tera $(=10^{12})$	T

I-6. International recommendations for the use of unit-symbols.

The symbols for units (given after the names of the units) represent the respective units. They are to be treated as *algebraical quantities*, and not as abbreviations. They should not be used in the plural form. So, do not write 100 cms ; write it as 100 cm. This is the international practice. It applies to derived units also. Note that the international symbol for the 'gram' is 'g', not 'gm'. For the kilogram, it is 'kg', not 'kgm'. The symbol for the second is 's', not 'sec'. All symbols are printed in Roman characters (as we do in this book) and not in italics. No symbol shall be followed by a stop (·) as a sign of abbreviation. Write 'g', 'cm', 'kg', etc. and not 'g.', 'cm.', 'kg.'

We illustrate below the practices stated above :

$1\ \text{cm} \times 2\ \text{cm} \times 3\ \text{cm} = 6\ \text{cm}^3$ (just as in algebra you write $a \times 2a \times 3a = 6a^3$).

$5\ \text{g} \times 4\ \text{cm} \div 2\ \text{s} = 10\ \text{g cm/s}$ (or $10\ \text{g.cm/s}) = 10\ \text{g cm s}^{-1}$.

A velocity of 2 metres per second is written as 2 m/s or $2\ \text{ms}^{-1}$. An acceleration of 50 centimetres per second per second is written as $50\ \text{cm/s}^2$ or $50\ \text{cm s}^{-2}$.

In this book we shall follow the international recommendations*.

Remember that a physical quantity consists of a number and a unit together. The unit-symbol is an essential part of it and behaves like a proper algebraic quantity. *Symbols for units cancel or multiply like algebraical quantities.* For example, $25\ \text{g cm s}^{-2} \div 5\text{g} = 5\ \text{cm s}^{-2}$.

You will get further examples in the worked out problems in the book. Keep a watch.

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* In spite of all that has been said above, you may still find in many books and question papers the symbols written variously as 'gm', 'gm.', 'cm.', 'cms'. 'ft.', 'sec', 'sec.', etc. Though in the main text we shall follow the internationally recommended way of writing, we shall leave the questions compiled from various sources just as we found them. So you may not be at a loss when you see them.

MECHANICS

1

PARTICLE DYNAMICS

1-1. Introductory remarks. The science of **mechanics** is the most fundamental of all the branches of physics. It is used in practically all the other branches. It deals with the basic ideas of motion, force and energy, and their relations to one another. A knowledge of mechanics is essential for the proper understanding of physical phenomena.

Mechanics has two broad divisions, namely, *statics* and *dynamics*. Statics deals with bodies at rest and dynamics, with bodies in motion. *Kinematics* is the study of motion without regard to its cause ; *kinetics* is concerned with the causes and laws of motion. In our treatment of mechanics, we shall not follow these traditional subdivisions as we require for our purpose only an intelligent application of the results of mechanics. So, we shall not generally go into the proofs. (You will find them in your books on Statics and Dynamics prescribed for the Mathematics course.) Our purpose will be only to understand the basic ideas and results, and to learn to apply them intelligently to the problems.

In this chapter we shall deal with the elementary ideas concerned in the study of mechanics, and derive some basic results.

1-2. Some elementary concepts. In mechanics, the term **particle** means a body of negligible size. It is represented by a point. We have to consider its *position* and *mass* only. A **rigid body** is a body of significant dimensions. It is considered to be made up of an infinite number of particles. The distance between any pair of particles in a rigid body is supposed to remain unchanged. No perfectly rigid body exists, but all bodies will be considered rigid unless stated otherwise.

A **body** is any piece of matter. It may be treated as a particle if we are interested only in its position (and not in its rotation about any axis). Its entire mass may be treated as being concentrated at a point of the body called its *centre of gravity* (See Sec. 3-3) or more properly, *centre of mass* (Sec. 3-4). If we are not interested in the diurnal rotation of the earth about its own axis, we may represent the earth moving in its orbit round the sun as a point particle with the mass concentrated at the centre of the earth (which is its centre of mass).

Rest and motion. When we say that a body is at rest we ordinarily mean that it is at rest with respect to the surface of the earth or with respect to bodies around it. A body in motion means a body which is moving relative to the surface of the earth, or with respect to bodies around it, In general, a body is said to be at rest when it does not move relative to its surroundings. But when it moves relative to its surroundings, it is said to be in motion.

Displacement. A body is said to be displaced when its position changes relative to its surroundings. Let a body, here represented by a point, which was initially at O (Fig. 1.1), move to A. Whatever path the body follows in moving from O to A, its *displacement* is measured by the length and direction of OA. At some later time the body may move to B. Then it will be said to have a further displacement AB. The total displacement from the initial position will be OB. If it comes back to O the displacement is zero.

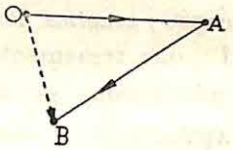

Fig. 1.1

Displacement always carries a sense of *direction* as well as of *magnitude*. A quantity which has both magnitude and direction is called a *vector* quantity.

Speed and velocity. Speed. Speed of a body is the distance it moves in unit time. If it covers a distance s in time t its speed is s/t.

If, while moving, a body covers equal distances in equal times, however small, its speed is said to be *uniform*. If it does not do so, the speed is *non-uniform* or *variable*. When speed is variable, s/t represents the *average speed* over the distance s or the time t. If t is extremely short s/t is called the *instantaneous speed*.*

Units. Speed is expressed in *units of length per unit of time*, such as 1 centimetre per second (1 cm/s or 1 cm s^{-1}), 1 metre per second (1 m/s or 1 ms^{-1}). These are absolute units of speed in the cgs and mks systems. We may use other units of length and time and write a speed as 1 km/hr or 1 mi/hr (or m. p. h.); but these are not absolute units.

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* If s_1 is the position of a particle at time t_1, and s_2 the position at time t_2, then $(s_2-s_1)/(t_2-t_1)$ is the *average speed* for the interval t_2-t_1. If t_2-t_1 is extremely short, then the value is the *instantaneous* value at time t_1.

The *knot* is a nautical unit of speed and implies a speed of one sea-mile (i.e., 6020 ft) per hour. It is correct to say 'a speed of 20 knots'.

Velocity. When a body moves in a definite direction, the speed of the body in that direction is called its *velocity*. Remember that *velocity always implies speed in a fixed direction.* While speed has magnitude only, velocity has both magnitude and direction.

Like speed, velocity may be uniform or variable and is expressed in the same units as speed, but with the direction mentioned, such as 'a velocity of 50 m/s due north'. Velocity will change when the speed changes, or the direction, or both.

Velocity may also be defined as *displacement per unit time* or as *the rate of displacement.*

Time-velocity graph. The velocities of a moving body at successive moments may be plotted graphically against the time and the points connected together by a smooth curve. Such a line is called a *time-velocity graph.* Let the velocities at different times be as follows:

Time in seconds	0	1	2	3	4	5
Velocity in m/s	2	2.4	3	3.5	3.8	4

Plot the time along the x-axis and the velocity along the y-axis. Join the points so plotted by a smooth line (Fig. 1.2). This line is the time-velocity graph.

If the velocity is uniform, the graph will be a straight line parallel to the x-axis. *The distance traversed between any two moments will be given by the area bounded by the ordinates for the initial and final moments, the graph and the x-axis.* This statement holds even when the velocity changes with time.

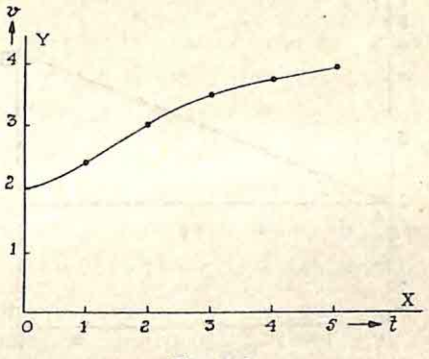

Fig. 1.2

Acceleration. When a train leaves a station it gradually gains in speed. Starting from rest it moves faster and faster till it acquires a practically constant speed. At the initial stages, when it is gaining in speed, it is said to be *accelerated*. When brakes are applied to bring it to a stop, it gradually loses speed and finally stops. It is then said to be *retarded* or *decelerated*. In both cases the speed changes with time.

A velocity will change if the speed or the direction of motion changes. *The rate of change of velocity with time is called* **acceleration**. Acceleration may involve a change only of speed or of direction of motion or both. We shall here consider only changes of speed, the direction of motion remaining the same. In rotational motion (Sec. 2-6) we shall come across acceleration due to change of direction only.

An acceleration is *constant* or *uniform* when the velocity changes by equal amounts in equal intervals of time, however small those intervals of time may be.

The following table illustrates a case of *uniform acceleration*.

Time in seconds	0	1	2	3	4	5	6
Velocity in m/s	2	2.5	3	3.5	4	4.5	5

The velocity changes by 0.5 m/s in every second. This is expressed by saying that the acceleration is 0.5 *metre per second per second* or 0.5 m/s^2, or ms^{-2}. In all expressions of acceleration the term 'per second' occurs *twice*. This is due to the fact that the unit of time is involved twice in such expressions,—once in the velocity, and then again in the rate of change of velocity.

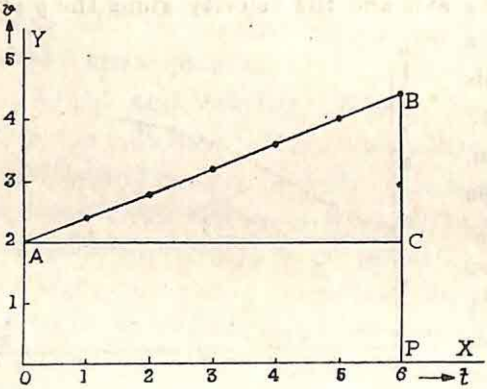

Fig. 1.3

The time-velocity graph of the values in the above table is shown in Fig. 1.3. It is the straight line *AB*. When the acceleration is constant, the time-velocity graph will be a straight line.

Deceleration or retardation is acceleration with a negative value. Here the velocity diminishes with time. If the initial velocity is 8 m/s, and after 2 seconds it comes down to 2 m/s, the deceleration is $(8 \text{m/s} - 2 \text{ m/s})/2 \text{ s}$, i.e., 3 m/s^2 or three metres per second per second. The acceleration is -3 m/s^2.

A useful concept in the relation between force and motion is **momentum**. *It is measured by the product of the mass of a body and its velocity*. (To distinguish it from angular momentum (Sec 2-2) it is proper to call it the *linear momentum*.) Consider a cricket ball thrown softly ; it will be easy to stop it. But when hit hard, it is much more difficult to stop it. In the latter case it has a larger momentum.

Think of a heavy iron roller lying on the ground or a four-wheeled carriage standing on a road. Because of its large mass you find it difficult to move. A continued push applied for some time gives it a certain velocity. A harder push over a shorter period will give rise to the same velocity. So, to give the body a momentum you had to *apply a force on it for a certain period*. (See 'Impulse of a force', Sec. 1-10.3.) To bring the moving roller or the carriage to rest, i.e., to destroy the momentum the body possesses, you must oppose the motion with a force and continue to exert the force for some time till the body stops. Production or destruction of a larger momentum requires application of a larger force for the same time or of the same force for a longer time.

The concept of momentum is of particular importance in connection with problems on blows, and action and reaction between moving bodies (see Sec.s 1-10.3 and 1-11).

Reference frame. A *frame of reference* is a set of lines or surfaces with the help of which we can describe the position of a particle or a straight line or a surface. If the motion of a particle is in a plane, we can use a frame of reference consisting of two straight lines at right angles in the plane. These are called respectively the X and the Y-axes. The point where the two axes cut each other is called the *origin* of the reference frame. A point in the plane represents a position of the particle.

Drop perpendiculars from the point on the two axes. The distances x and y of the feet of the perpendiculars from the origin are called the *co-ordinates* of the point. If x and y both represent distances, the combination (x, y), that is, the co-ordinates of the point, tells us where the point lies on the plane of reference. Co-ordinates indicating the position of a point are called its *space co-ordinates*.

We may extend the reference frame to *three dimensions*. In that case we imagine a third axis, the Z-axis, perpendicular to both the X and Y-axes and passing through the origin. A point in space is then defined by *three co-ordinates* (x,y,z). In this frame we are using three axes or three mutually perpendicular planes. They are the X-Y plane, Y-Z plane and Z-X plane.

Reference frames may be of very many kinds. The lines need not be perpendicular to one another nor the surfaces plane. But at the beginning we shall use only rectangular frames of reference with two or, if need be, three rectangular axes.

Any given frame of reference is generally called a *co-ordinate system*. In a broader sense, any system with reference to which we describe an event, is a reference system. (An *event* is anything that happens, such as the motion of a particle, a rotation, a collision between two particles, etc.)

1-3. Algebraic formulæ for uniformly accelerated motion (Kinematical equations in one dimension).

Let u = initial velocity of a particle,
v = its final velocity,
t = the time in which this change occurs, and
f = the acceleration,

all quantities being expressed in the same system of units.

Since f is the increase in velocity per second, the velocity after one second is $u+f$; after two seconds, $u+2f$ etc. Hence at the end of the time t seconds, the velocity will increase by ft. The final velocity, which we have called v, will then be given by

$$\mathbf{v=u+ft} \tag{1-3.1}$$

Distance traversed in a given time. Referring to Fig. 1.3, let OA represent the initial velocity u and PB, the final velocity. OP is the time interval t. CB is the increase ft in the velocity. As it is a case of constant acceleration, the time-velocity graph AB is a straight line.

The distance traversed in time $t=OP$ is given by the area bounded by the ordinates OA and PB at the initial and final moments, the graph AB and the x-axis, i.e , the area $OABP$.

Now area $OABP$ = rectangle $OACP$ + triangle ABC

$$=OA.OP+\tfrac{1}{2}.AC.BC=ut+\tfrac{1}{2}ft^2.$$

This is the distance s traversed.

$$\therefore\ \mathbf{s=ut+\tfrac{1}{2}ft^2} \tag{1-3.2}$$

Algebraic proof. A simple algebraic proof of the above formula may be considered as follows :

Since the velocity increases uniformly from u to $u+ft$, the body may be taken to have moved with a constant velocity equal to the average of the initial and final values.

The average velocity is, therefore,

$$=\tfrac{1}{2}\{u+(u+ft)\}=u+\tfrac{1}{2}ft.$$

The distance s traversed in time t with this velocity will be given by

$$s=t(u+\tfrac{1}{2}ft)=ut+\tfrac{1}{2}ft^2.$$

The two equations 1-3.1 and 1-3.2 may be combined to form a new one by eliminating t from the equations. Thus

$$v^2=(u+ft)^2=u^2+2f(ut+\tfrac{1}{2}ft^2)=u^2+2fs$$

$$\text{or } \mathbf{v^2-u^2=2fs}. \qquad (1\text{-}3.3)$$

Distance traversed in a particular second. Eq. 1-3.2 gives us the total distance s which a body travels in time t. From it we can find how far the body travels in a particular second, say, the nth. (The nth second may be the first, second, third or any other second.)

Let s_n denote the total distance which the body moves in n seconds, and s_{n-1} the distance it travels in $n-1$ seconds. (n may have any value, integral or fractional.)

Writing n for t in Eq. 1-3.2, we have $s_n=un+\frac{1}{2}fn^2$.

Similarly, writing $n-1$ for t we have $s_{n-1}=u(n\text{-}1)+\frac{1}{2}f(n-1)^2$.

∴ The distance a body travels in the nth second is

$$s_n-s_{n-1}=(un+\tfrac{1}{2}fn^2)-\{u(n-1)+\tfrac{1}{2}f(n-1)^2\}$$

$$=u+\tfrac{1}{2}f(2n-1) \qquad (1\text{-}3.4)$$

The four equations 1-3.1 to 1-3.4 describe fully the motion of a particle in a straight line with constant acceleration. These are the beginning equations of kinematics. *The first and the second equations are fundamental.* The other two can be derived from them.

Examples. (1) A particle starts from rest and acquires a velocity of 42 cm/s in 3·5s. Find the acceleration and the distance it travels in 3 seconds.

Solution : Since the particle starts from rest, its initial velocity $u=0$.

From Eq. 1-3.1, $f=\dfrac{v}{t}=\dfrac{42\text{cm/s}}{3{\cdot}5\text{s}}=12\text{ cm/s}^2$

From Eq. 1-3.2, the distance traversed in 3 seconds

$$=0+\tfrac{1}{2}\times 12\text{ cm/s}^2\times(3\text{s})^2=54\text{ cm}.$$

(2) The acceleration of a particle is 4 m/s². What will be its velocity after it has travelled 20,000 cm from rest ?

(*Note*—In solving probleme in which the units belong to different systems, they must all be reduced to the same system. Let us use the mks system here.)

Solution : Here f=4m/s², $u=0$, s=20,000 cm=200 m. To find v. From Eq. 1-3.3, that is, $v^2-u^2=2fs$, we get $v^2=u^2+2fs$.

Or $v^2=2\times 4\text{m/s}^2\times 200\text{m}=1600(\text{m/s})^2$. $\therefore$ v=40m/s. (Try working it out using the cgs system.)

(3) A particle moves 25 cm in the 3rd second and 55 cm in the 6th second. Find how far it will travel in 8 seconds.

Solution : (The quantities are all in cgs units. So no change of unit is necessary.) Let u=initial velocity and f=the acceleration of the particle in cgs units. The 3rd second is the interval between the completion of 2 seconds and 3 seconds. If s_2 and s_3 represent respectively the distances traversed in 2 and 3 seconds, then s_3-s_2 is the distance traversed in the third second.

$$\therefore\ 25=(3u+\tfrac{1}{2}f.3^2)-(2u+\tfrac{1}{2}f.2^2)=u+\tfrac{5}{2}f.$$

Similarly, $55=s_6-s_5=(6u+\frac{1}{2}f.6^2)-(5u+\frac{1}{2}f.5^2)=u+\frac{11}{2}f.$

Solving for u and f from these two relations, we get $u=0$ and $f=10$ in cgs units, that is, f=10 cm/s².

Distance traversed in 8 seconds is

$$=8u+\tfrac{1}{2}f.8^2=0+\tfrac{1}{2}\ .\ 10\ \text{cm/s}^2.(8\text{s})^2=320\ \text{cm}.$$

1-4. Acceleration due to gravity. By *gravity* we mean the pull of the earth on a body. A body is said to fall 'freely' when its fall under gravity is not opposed in any appreciable way. We now know that *for a body falling freely under gravity, the motion is one of* **constant acceleration**.

The acceleration with which a freely falling body moves towards the earth is called the acceleration due to gravity. It is denoted by the symbol g. (Do not confuse it with the symbol 'g' for gram. The former is printed in italics (or inclined) type. The latter is printed in erect type.)

g may be taken as 9.8 m/s², 980 cm/s², or 32 ft/s² in numerical problems unless some other value is given.

1-4.1. Equations of vertical motion under gravity. When a body moves under the action of gravity, its motion is determined by the equations

$$v=u\pm gt \tag{1-4.1}$$

$$h=ut\pm\tfrac{1}{2}\,gt^2 \tag{1-4.2}$$

$$v^2-u^2=\pm 2gh \tag{1-4.3}$$

$$h_n-h_{n-1}=u\pm\tfrac{1}{2}g(2n-1) \tag{1-4.4}$$

h is the vertical distance travelled from the starting point in the direction of u. We have put h for s and g for f in Eq.s 1-3.1 to 1-3.4.

The *plus sign* is used for *falling bodies.* The *minus sign* is used for bodies which are *thrown vertically upwards.* This is so because during upward motion, the velocity and the acceleration are in opposite directions.

A. Free fall under gravity. When bodies fall from rest, $u=0$. The equation of free fall *from rest* under gravity then reduces to

$$h=\tfrac{1}{2}gt^2. \tag{1-4.5}$$

h is positive downward.

If projected vertically downward with an initial velocity u, the equation will be

$$h=ut+\tfrac{1}{2}\,gt^2. \tag{1-4.6}$$

B. Maximum height of ascent. A body projected vertically upward gradually loses speed as it rises. This is so because the acceleration which it has due to the action of gravity, is directed downward, i.e., against the motion. For upward projection h and u are positive while g is negative. The speed gradually falls to zero ; it is then at the topmost point of its rise. v, *the velocity at the topmost point, is zero.* Thereafter it starts moving downward.

If u is the initial velocity of projection, and H the maximum height of ascent, then from the relation $v^2-u^2=-2gh$, we get

$$0-u^2=-2gH \quad \text{or} \quad H=u^2/2g. \tag{1-4.7}$$

C. Time for ascent. How long does the body take to rise to its maximum height ? If T is this time, then from the relation $v=u-gt$, we get

$$0=u-gT \text{ or } T=u/g. \tag{1-4.8}$$

It is easy to see that the *time for ascent=time for descent.* The time T_1 for descent is the time required by the body to fall from rest through a height $H=u^2/2g$. Then $u^2/2g=\frac{1}{2}gT_1{}^2$ or $T_1=u/g$. This is also the time for ascent.

D. Two values of time to reach the same height. When a body is projected vertically upward, it will be at a given point of its path at two moments—once during rise and again during fall. Since for upward projection $h=ut-\frac{1}{2}gt^2$, we have $\frac{1}{2}gt^2-ut+h=0$. This is a quadratic equation in t. Solving for t, we get

$$t=\frac{u\pm\sqrt{u^2-2gh}}{g} \tag{1-4.9}$$

h must be less than the maximum height of ascent $H=u^2/2g$.

E. **Same speed at the same height during ascent and descent.** A vertically upward projected body will have the same speed at the same point of its path while going up and coming down. This follows from the relation $v^2-u^2=-2gh$. It gives $v^2=u^2-2gh$

$$\text{or} \quad v=\pm\sqrt{u^2-2gh} \tag{1-4.10}$$

The plus sign relates to the upward velocity and the minus sign, to the downward velocity.

Examples. (1) How far will a body fall from rest in 5 seconds ? Take $g=9{\cdot}8$ m/s^2. [Ans : 122·5 m]

(2) If the same body were projected downward with a velocity of 10 m/s from rest, how far would it fall in 5 seconds ? [Ans : 172·5 m]

(3) A body is projected vertically upward with a velocity of 20 m/s. Find (a) how far it will rise, (b) how long it will take to reach the topmost point. Find also when it will be at a height which is half of the maximum. What will be its velocity then ?

[*Hints* : (a) $v=0$ at the top. Put $v=0$ in $v^2-u^2=-2gh$ and find h. $h=(20\times20)/(2\times9{\cdot}8)$ m. (b) At the top $u-gt=v=0$. This will give t in seconds. Half the max. height in about 10·2 m. Find t from the relation $10{\cdot}2=20t-\frac{1}{2}\times9{\cdot}8\times t^2$. You will get two values of t. The shorter one is for ascent, and the longer, for descent.]

1-5. Vector and scalar quantities.

The term *vector* is applied to quantities which have both direction and magnitude. Displacement, velocity, acceleration, momentum and force are examples of vector quantities. The term *scalar* refers to quantities which have magnitude only. Mass, volume, density, speed etc. are examples.

1-5.1. Representation of vectors.

This may be done in three ways, namely, (i) by *line segments*, (ii) by *components* and (iii) by *co-ordinates*.

(i) In representing a vector by a line segment, a straight line is drawn parallel to the direction of the vector. A segment of the line is chosen to represent the vector. The length of the segment is made proportional to the magnitude of the vector to some convenient scale. The length of the line segment which represents unit magnitude of the vector is called the *unit vector* in the given direction. An arrow-head is put on the line segment to indicate the *sense* in which (that is, the side toward which) the vector acts along the line.

This is illustrated in Fig. 1.4. Suppose we have to represent two vectors. One of them is a velocity of 30 cm/s due north and the other is a velocity of 40 cm/s due east. In the figure, the line segment OE represents the velocity to the east. The line segment ON represents the velocity to the north. The lines are at right angles (as east and north are). Their lengths are in the ratio 3 : 4. The unit vectors are portions of the segments between two successive dots. They are all equal in length.

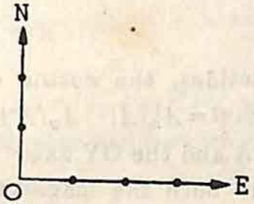

Fig. 1.4

To represent a force by a line segment, we have to consider the point at which it is applied, and also the magnitude and direction of the force. The point of application of a force is represented by the point from which the line segment is drawn. The line segment is parallel to the direction of the force, and its length is proportional to the magnitude of the force. In the cases of vectors like velocity, acceleration, momentum, etc., we represent them by line segments of appropriate lengths drawn parallel to the directions of the actual quantities.

When a line segment AB represents a vector and the vector acts from A towards B, we write the vector as $\overrightarrow{AB}$ or $\overline{AB}$, putting an arrow-head or bar above AB. $\overrightarrow{BA}$ or $\overline{BA}$ then represents a vector equal and opposite to $\overline{AB}$. In $\overline{AB}$, A is the *initial point* of the vector and B the *final point*. In $\overline{BA}$, B is the initial point and A the final point.

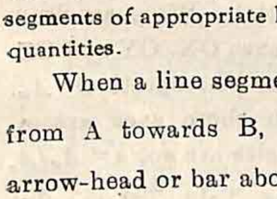

Fig. 1.5

(ii) Representation of a vector *by components* can be understood from Fig. 1.5. Suppose a vector **A**[*] acts along the line DE in the sense D to E. Suppose further that the line segment $\overline{OA}$ represents this vector. With O as origin draw a line OX as axis at any angle θ to OA. Draw another line OY as axis perpendicular to OX in the same plane as OA and OX.

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[*] Vectors are printed in thick, capital letters, such as **A**, **B**, **P**, **Q**, etc. To represent its *magnitude only* we write *A*, *B*, *P*, *Q*, etc. [that is, we use *italic* (slant) type]. Another way to write the magnitude is $|\mathbf{A}|$, $|\mathbf{B}|$, etc.

Let OB be the *projection* of OA on the OX axis. Let us write this projection as A_x. Then $A_x = \text{OB} = \text{OA} \cos\theta$. Similarly, the projection A_y on the OY axis is $A_y = \text{OC} = \text{OA} \sin\theta$. We thus get that the magnitude A of the vector **A** is given by

$$A = \sqrt{A_x^2 + A_y^2} \qquad (1\text{-}5.1)$$

Besides, the cosine of the angle between OA and the OX axis is $\cos\theta = A_x/A$. A_y/A is the value of the cosine of the angle between OA and the OY axis, So, if we know the values of A_x and A_y, we get both the magnitude and the direction of the vector **A** in the reference frame (or coordinate system) which we have taken. (The reference frame consists of the axes OX and OY at right angles. Clearly, the reference frame can be chosen in an infinite number of ways, for θ may have any value.)

A_x and A_y are called the **components** of **A** in the reference frame chosen.

In the above case, we have so chosen the plane of the axes that it contains the line OA. But this is not essential. We may draw through O any three mutually perpendicular axes OX, OY and OZ. Let the projections of OA on these three axes be respectively A_x, A_y and A_z. If the angles which OA makes with these axes are α, β and γ respectively, then the cosines of these angles are $\cos\alpha = A_x/A$, $\cos\beta = A_y/A$ and $\cos\gamma = A_z/A$. Also $A^2 = A_x^2 + A_y^2 + A_z^2$. A_x, A_y and A_z are called the components of **A** in the reference frame chosen. We thus find that when we know the values of the components of a vector (in a tri-rectangular frame of reference), we know both the magnitude and the direction of the vector. The *triplet of numbers* A_x, A_y, A_z completely defines the vector **A**. (Leaving aside the unit, we find that while a scalar requires only *one* number to represent it, a vector requires *three*.)

(iii) *Representation of a vector by coordinates* follows from what we have already said. For this purpose we want the coordinates of the initial and final points of the line segment which represents the vector. Let the coordinates of the initial point (in a tri-rectangular coordinate system) be x_1, y_1, z_1, and those of the final point x_2, y_2, z_2. Then the components of the vector **A** will be as follows :

$$A_x=x_2-x_1\ ;\ A_y=y_2-y_1\ ;\ A_z=z_2-z_1$$

The magnitude of the vector will be

$$A=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

If α, β, γ be the angles which the vector makes with the x-axis, y-axis and z-axis respectively, then

$$\cos\ \alpha=A_x/A=(x_2-x_1)/\{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2\}^{\frac{1}{2}}\quad \text{etc.}$$

If the x-y plane contains the vector, the z-coordinates will be zero.

1·6. Composition and resolution of vectors. By the term 'composition' of vectors we mean the *addition* of two or more vectors. For simplicity we shall confine ourselves to *coplanar* vectors, that is vectors which lie in the same plane. Scalar quantities are added *algebraically* : but not so the vectors. Vectors are added *geometrically* as indicated in section 1-6.1 below.

'Resolution' of vectors means splitting (=dividing) a vector into two components, the components and the vector lying in the same plane. Resolution of a vector can be easily understood when we have learned how to add vectors.

1-6.1. The geometrical law of addition of vectors. In Fig. 1.6, let the vectors **P** and **Q** to be added be represented by the line segments OA and OB drawn from the same point. (The vectors may be a pair of velocities which a particle has at the same moment. Or they may be a pair of forces acting on the same particle at the same time.) To get their sum, called the **resultant**, complete the parallelogram OACB. Then OC, the diagonal, will represent in magnitude and direction the vector sum **R** of $\overline{\text{OA}}$ and $\overline{\text{OB}}$, that is, of **P** and **Q**. In symbols, **P**+**Q**=**R** or $\overline{\text{OA}}+\overline{\text{OB}}=\overline{\text{OC}}$.

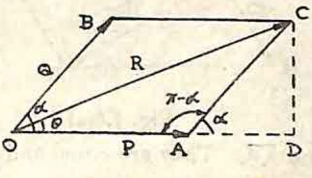

Fig. 1.6

This law of addition, which applies to any quantity which has both magnitude and direction, is known as the **parallelogram law of addition.** Let us accept the law without asking for proof at this stage. In words, the parallelogram law of addition may be stated as follows :

If two vectors are represented in magnitude and direction by the two sides of a parallelogram drawn from a point, they are equivalent to a single vector represented in magnitude and direction by the diagonal of the parallelogram passing through the point.

The single vector to which the other vectors are equivalent is called the **resultant** of the vectors.

A. Principle of independence of vectors. The law of addition of vectors is based on a principle known as the principle of independence of vectors. It states that *each vector produces its own effect as if the others did not exist.* When we add two or more forces, velocities, displacements, etc., this principle operates. To emphasize this independence in the cases of forces and velocities, we put down the following two statements.

B. Principle of independence of forces. If a system of forces acts on a particle at rest or in motion, the combined effect of the system is found by considering the effect of each force as if the other forces did not exist, and then adding up these effects.

C. Principle of independence of velocities. When a particle has two (or more) velocities at the same time, the velocity in one direction is not affected by that in the other direction. Each velocity produces its own motion as if the other (or others) did not exist.

D. Vector subtraction. If the sum of two vectors **P** and **Q** is **R**, that is, if $\mathbf{P}+\mathbf{Q}=\mathbf{R}$, then $\mathbf{P}-\mathbf{Q}=\mathbf{P}+(-\mathbf{Q})$. This means that to subtract a vector **Q** from a vector **P**, we should reverse **Q** and add it to **P**. If we do so in Fig. 1·6, we shall find that $\mathbf{P}-\mathbf{Q}=\mathbf{R}'$ (Fig. 1·6*a*) will be represented by the other diagonal of the parallelogram. $\mathbf{P}-\mathbf{Q}$ will be represented by $\overline{BA}$ and $\mathbf{Q}-\mathbf{P}$ by $\overline{AB}$. They are equal and opposite.

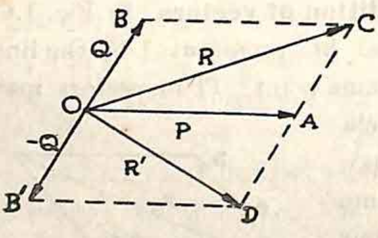

Fig. 1.6(a)

1-6.2. Magnitude and direction of the resultant. Fig 1.6 shows the resultant R of two vectors P and Q inclined to each other at an angle α.

$$R^2=OC^2=OD^2+DC^2=(OA+AD)^2+DC^2=OA^2+(AD^2+DC^2)$$
$$+2OA.AD=OA^2+AC^2+2OA.AC\cos\alpha=P^2+Q^2+2PQ\cos\alpha. \qquad (1\text{-}6.1)$$

R is inclined at an angle θ to P where

$$\tan\theta=\frac{CD}{OD}=\frac{AC\sin\alpha}{OA+AC\cos\alpha}=\frac{Q\sin\alpha}{P+Q\cos\alpha} \qquad (1\text{-}6.2)$$

Equations 1-6.1 and 1-6.2 give respectively the magnitude and direction of the resultant of two vectors. (Say, forces or velocities etc.)

There is a theorem in trigonometry which states that in a triangle the ratio of any two sides is equal to the ratio of the sines of their opposite angles. Applying this result to the triangle OAC (Fig. 1.6), we may write

$$\frac{R}{\sin(\pi-\alpha)}=\frac{Q}{\sin\theta}=\frac{P}{\sin\phi}$$

These relations may be used for solving problems involving the resultant of two vectors. P and Q stand for any two vectors of the same kind and ϕ for the angle opposite P.

Problems. (1) A boat is towed by two ropes, each inclined at 30° to the stream, but on opposite sides of it. If the pull along each rope is 50 kg-wt. find the magnitude and direction of the resultant force. (Draw the necessary diagram.) [Ans : $R=86{\cdot}6$ kg-wt ; $\theta=30°$].

(2) Forces 3 g-wt and 4 g-wt act at a point. Find the angle between their lines of action if they have a resultant of (i) 7 g-wt, (ii) 6 g-wt, (iii) 5 g-wt, (iv) 4 g-wt, (v) 1 g-wt.

[Ans : (i) 0°, (ii) 62°42′, (iii) 90°, (iv) 131°48′, (v) 180°.]

1·6.3. Alternative construction for addition of vectors : the polygon method. An alternative construction for adding two or more vectors is as follows. It is a graphical method.

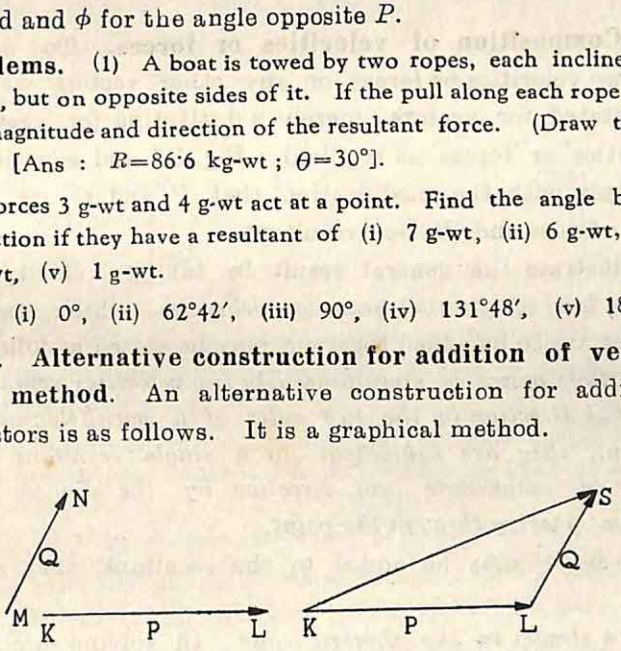

Fig. 1.7

Let P and Q (Fig. 1.7), represented respectively by KL and MN, be the vectors to be added. From the end point L (Fig. 1.7) of the straight line KL (representing the vector P) draw LS equal and parallel to MN (representing Q).

Then the straight line KS represents in magnitude and direction the sum of P and Q.

Any number of vectors may be added in this manner by *placing them head to tail* (tail of the second to the head of the first and so

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on) *in a continuous sequence* (Fig. 1.8). *The order in which they are taken is immaterial.* The line drawn from the origin of the first to the terminus of the last represents the resultant in magnitude and direction. Fig. 1.8 shows the addition of three vectors, P, Q and R, in this way. $KN = S$ is their resultant. This is known as the *polygon method* of addition. For two vectors the polygon reduces to a triangle.

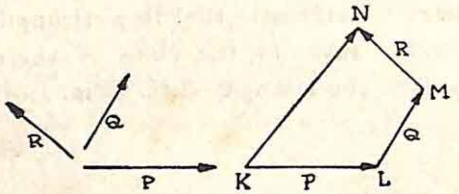

Fig. 1.8

1-6.4. Composition of velocities or forces. To compound (i.e., add) two velocitics or forces or any other vectors we procced exactly as stated for vectors, merely substituting for 'vectors' the word 'velocities' or 'forces' as required. Fig. 1.6 and equations 1-6.1 and 1-6.2 apply with the modification that P and Q are the two velocities, or forces and, R their resultant.

Let as illustrate the general result by taking 'velocity' as the vector. The *law of parallelogram of velocities*, which states how two velocities are to be added together, may be stated as follows :

If a particle possesses simultaneously two velocities represented in magnitude and direction by the two sides of a parallelogram drawn from a point, they are equivalent to a single resultant velocity represented in magnitude and direction by the diagonal of the parallelogram passing through the point.

A third velocity may be added to the resultant, then a fourth and so on.

To cross a stream in the shortest time. In solving problems in which a particle has more than one velocity at the same time, it is well to remember the principle of independence of velocities. When a man wants to cross a river the distance he needs to cover is the shortest if it is perpendicular to the bank. If he swims in this direction he will cross the river in the shortest time, though in this attempt he will be carried downstream by the current. According to the principle of independence of velocities, his velocity perpendicular to the stream will not be affected by the velocity of the stream. So, in his attempt to cross in the shortest time, he swims *as if there were no current* in the stream.

1-6.5. Special cases of composition. Equations 1-6.1 and 1-6.2 are perfectly general and hold for all values of α. We take up here a few special cases :

(1) The forces or velocities are in the *same direction, i.e,* $\alpha=0$. Since $\cos\theta=1$, equation 1-6.1 gives $R^2=u^2+v^2+2uv$ or $R=u+v$,

(2) The forces or velocities are in *opposite directions*, i.e., $\alpha=180^\circ$. Since $\cos 180^\circ=-1$, $R^2=u^2+v^2-2uv$ or $R=u-v$. Here the resultant is the difference of the two and acts in the direction of the larger.

(3) The forces or velocities are *at right angles, i.e.,* $\alpha=90^\circ$ (Fig. 1.9). Since $\cos 90^\circ=0$,

$$R^2=u^2+v^2 \quad \text{or} \quad R=\sqrt{u^2+v^2} \qquad (1\text{-}6.3)$$

$$\text{and } \tan\theta=v/u \qquad (1\text{-}6.4)$$

Note that in this case

$$\left.\begin{aligned} u&=R\cos\theta \\ \text{and} \quad v&=R\sin\theta \end{aligned}\right\} \qquad (1\text{-}6.5)$$

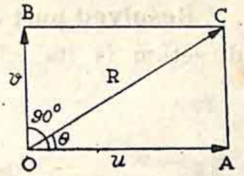

Fig 1.9

Examples. (*i*) Suppose a steamer moves downstream with a velocity of 16 km per hour, while a man walks across its deck with a velocity of 6 km per hour. What velocity will this man appear to have to a msn standing on the bank of the river ?

Solution : The man on the steamer has two simultaneous velocities at right angles to each other. One is that of the steamer at 16 km/hr and the other his own, at 6 km/hr,

From Eq. 1-6.3 the resultant velocity $R=\sqrt{16^2+6^2}=17{\cdot}1$ km/hr.

If θ is the angle between the direction of motion of the steamer and the direction of the resultant velocity, then, from equation 1-6·2, $\tan\theta=3/8$.

Thus to the man at rest on the bank of the river the other man will appear to move with a speed of about 17·1 km/hr in a direction inclined at an angle $\tan^{-1} 3/8$ with the downstream direction.

[*Note* : It will be convenient if the student draws a diagram representing the velocities and then proceeds to solve.]

(*ii*) A crane pulls a body vertically upward with a velocity of 4·8 m/min, while it itself moves along horizontal rails at 2 m/min. Find the velocity of the body (relative to an observer at rest on the surface of the earth).

Solution : Here also the two velocities are at right angles.

$\therefore$ The resultant $R=\sqrt{4{\cdot}8^2+2^2}=5{\cdot}2$ m/min

The angle between the vertical and the direction of the resultant R is $\tan^{-1} 5/12$.

1-7. Resolution of vectors. We have seen in sections 1-6 and 1-6.3 how two vectors may be compounded into one. By following the reverse process (*i.e.*, constructing a parallelogram on a given diagonal, the directions of the sides of the parallelogram being given) it is possible to split a vector into two parts in two given directions. The case in which these two directions are at right angles is of special importance. The parts into which a given vector is split are called the *components* of the vector, and the process is known as *resolution* of the vector. If the components are at *right angles*, the parallelogram reduces to a rectangle. Each component in such a case is called the *resolved part of the vector* in the direction concerned.

Resolved part of a vector. The resolved part of a vector in a direction is its effective part* in that direction. The resolved part of a force R (Fig. 1. 10) in the direction OX making an angle θ with R, is given by $P=R\cos\theta$. This together with Q given by $Q=R\sin\theta$, equals R, as may be seen by the vector addition of P and Q.

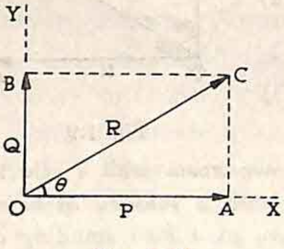

Fig. 1.10

The effect of R in the direction OX is the same as that of P where $P=R\cos\theta$. P and Q are two *components* of R.

Definition. The *resolved part* of a force (or any vector) in a given direction is that force (or vector) which, together with another force (or vector) in a perpendicular direction, produces the same effect as the given force (or vector).

If the vector is A and it makes an angle θ with some given direction OX the *resolved part* (or *component*) of A in the given direction is given by

$$A_x = A\cos\theta \qquad (1\text{-}7.1).$$

Problems. (1) A railway truck is pulled along level rails by a horse pulling a rope with a force of 500 kg-wt in a direction making an angle of 30° with the rails. What is the force pulling the truck along the rails? What is the force pulling it sideways against the rails? [Ans. : 433 kg-wt ; 250 kg-wt.]

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* This may be understood with the help of the following example. Suppose O represents a railway carriage standing on rails parallel to OX (Fig. 1.10), and is being pulled in the direction OC with a force R. R is equivalent to P along OX and Q along OY. The action of Q cannot displace the carriage along the rails. The motion of the carriage along the rails is therefore determined entirely by the resolved part P of R along the rails.

(2) A gale forces a mooring cable of a captive balloon to an angle of 30° with the vertical. The tension in the capable is found to be 50 kg-wt. Find (a) the horizontal force exerted by the wind and (b) the net up-thrust on the baloon. [Ans : (a) 25 kg-wt ; (b) $25\sqrt{3}$ kg-wt.]

1-8. Addition of vectors by geometrical and analytical methods. We have seen in Sec.s 1-6 and 1-6.3 how two or more vectors may be added together by the geometrical method. Here we shall see how they can be added in another way, called the *analytical method.* For convenience, we shall consider all the vectors to lie in the same plane. We shall illustrate the method by taking 'force' as the vector. For any other vector, replace the term 'force' by the name of that vector.

First, resolve all forces into rectangular components along any convenient pair of axes, and then combine these into a single resultant.

This is illustrated in Fig. 1.11. F_1, F_2, F_3 are the forces to be added. Let us take the x-axis along any suitable direction (it may be along F_1) and the y-axis perpendicular thereto. If θ_1, θ_2 and θ_3 be the angles the forces make with the x-axis, then the x-components of the forces are $F_1 \cos \theta_1$, $F_2 \cos \theta_2$ and $F_3 \cos \theta_3$. Adding these up we get the x-component R_x of the resultant R. The y-component R_y of the resultant is similarly $F_1 \sin \theta_1 + F_2 \sin \theta_2 + F_3 \sin \theta_3$. Then

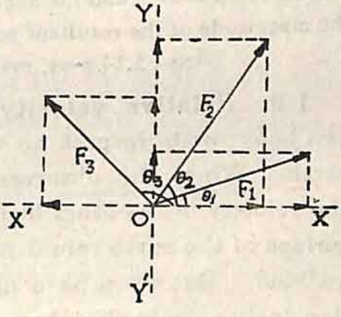

Fig. 1.11

$$R = \sqrt{R_x{}^2 + R_y{}^2}$$

$$= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2},$$

where ΣF_x is written for the *sum* of x-components of the forces and ΣF_y for the sum of the y-components.

If α is the angle which R makes with the x-axis, then

$$\tan \alpha = \frac{R_y}{R_x} = \frac{\Sigma F_y}{\Sigma F_x}. \qquad (1\text{-}8.2)$$

Example. In Fig. 1.11, let $F_1 = 60$ g-wt, $F_2 = 100$ g-wt, $F_3 = 75$ g-wt, $\theta_2 = 60°$, $\theta_3 = 135°$. Resolving along F_1 and perpendicular thereto, we shall have $\theta_1 = 0°$, or $F_1 \cos\theta_1 = F_1$. The calculations may be tabulated as follows :

Force (F) in g-wt	Angle (θ)	x-component $= F\cos\theta$ (g-wt)	y-component $= F\sin\theta$(g-wt)
$F_1 = 60$	0°	+60	0
$F_2 = 100$	60°	+50	+86·6
$F_3 = 75$	135°	−53	−53
		$\Sigma F_x = +57$	$\Sigma F_y = +33{\cdot}3$

$$R = \sqrt{(57)^2 + (33{\cdot}3)^2}\ \text{g-wt} = 66\ \text{g-wt}.$$

$$\tan\alpha = \frac{33{\cdot}3}{57} = 0{\cdot}588 \quad \text{or} \quad \alpha = 30{\cdot}4°.$$

Problems. (1) Two equal forces, inclined at 60°, act on a particle. Resolve the forces along the bisector of the included angle and perpendicular thereto. Hence find the magnitude and direction of the resultant. [Ans : $\sqrt{3}$ times the magnitude of either force ; along the bisector of the included angle.]

(2) Three coplanar forces of 1, 2 and 3 g-wt respectively act at a point. The angle between the first and the second is 60° ; the angle between the second and the third is 30° ; and the angle between the first and the third is 90°. Find the magnitude of the resultant and the angle it makes with the first force.

[Ans : 5.14 g-wt, making an angle of 67°5′ with the first force.]

1-9. Relative velocity. Ordinarily, we consider the motion of a body with respect to an observer at rest on the surface of the earth. When the observer himself is in motion it seems to him that the velocity of the other body has changed. To a man at rest on the surface of the earth rain-drops appear to fall vertically when there is no wind. But when he is moving in a train the drops seem to move slantingly even in the absence of wind. When a person looks at a moving body, the motion he sees is always a *relative motion*, *i.e.*, the motion of the body relative to the observer, whether the latter is at rest or in motion.

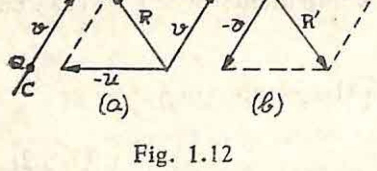

Fig. 1.12

The relative velocity between two bodies, both of which are in motion, may be obtained easily by the method of composition of velocities. Suppose two bodies P and Q move with velocities u and v along the lines AB and CD respectively (Fig. 1.12), u and v being values relative to an observer at rest. What will be the magnitude and the direction of the velocity which Q will appear to have to an observer situated on P? To picture it clearly, P and Q may be looked upon as two ships proceeding in different directions, whose velocities are u and v with respect to a person at rest on the seabeach.

If we superpose on both P and Q a velocity equal to $-u$, their relative velocities will not alter. This is the same as imagining that the sea has a current whose velocity is equal and opposite to that of the ship P. P will then appear to be at rest to the observer on the beach. Q will seem to him to have two simultaneous velocities, v along CD and $-u$ along AB. The velocity of Q to both the stationary observers is the same, and is the resultant of v and $-u$. This resultant R is obtained graphically as shown in Fig. 1.12 (*a*).

When we want the relative velocity of P with respect to Q (*i.e.*, if the observer instead of being in P were in Q), we bring Q to rest by superposing a velocity of $-v$ on both. Then the relative velocity will be the resultant of u and $-v$. Fig. 1.12(*b*) gives its value R', which is easily seen to be equal and opposite to R.

Generalising, we may say that to find the relative velocity of a body Q with respect to another body P when both are in motion, reverse the velocity of P and combine it with that of Q. The resultant gives the relative velocity. P may be called the *observer* and Q, the *observed*. The working rule then reduces to—'*Reverse the velocity of the observer and combine it with that of the observed*'. The magnitude and direction of the resultant are best obtained by geometrical construction.

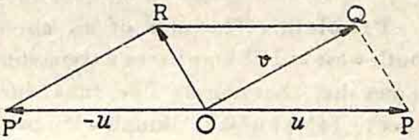

Fig. 1.13

Note. *Reversing* a vector u and adding it to a similar vector v means *subtracting* u from v, that is, getting $v-u$. So relative velocity is obtained by appropriate *vector subtraction*. If $\overline{OP}$ and $\overline{OQ}$ (Fig. 1.13) represent the two vectors u and v, then $v-u=$ the vector represented by $\overline{OR}=\overline{PQ}$. $u-v$ is the vector represented by $\overline{QP}$. So, we may say that the velocity of Q relative to P is equal to the velocity of Q *minus* the velocity of P. (Note that while $\overrightarrow{OC}$ in Fig. 1·6 represents $P+Q$, the other diagonal represents the difference of P and Q; $P-Q=\overrightarrow{BA}$ and $Q-P=\overrightarrow{AB}$).

Examples. (1) A ship steams due east at 18 km/hr and another due south at 24 km/hr. Find the velocity of the first ship with respect to the second.

Solution : The student is advised to draw the necessary diagram.

Here the observer is in the second ship. So the relative velocity is obtained by combining the velocity of 18 km/hr due east with 24 km/hr due north. This gives a velocity of 30 km/hr at an angle $\tan^{-1}\frac{3}{4}$ east of north.

(2) A cyclist rides horizontally at 10 km per hour. There is no wind and rain-drops appear to him to fall at an angle of 10° to the vertical. Find the speed of the rain-drops.

Solution : In Fig. 1.14, OA is the velocity of the cyclist and OC that of rain-drops. The relative velocity OR between them is obtained by reversing OA and combining it with OC. $\angle COR = 10°$, $CR = 10$ km/hr. To find OC.

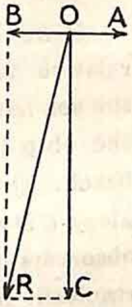

Fig. 1.14

$$\text{Now } \frac{CR}{OC} = \tan 10° = 0{\cdot}176 \text{ (from table of tangents).}$$

$$\therefore \quad OC = \frac{10}{0{\cdot}176} \text{ km/hr} = 56{\cdot}8 \text{ km/hr.}$$

Relative motion sometimes produces surprising effects. Suppose a passenger in a stationary train finds the wind blowing from due east. As the train moves south, the wind direction will appear to change and seem to come from the south. When two boats are moving at a small angle with the same speed, an observer in one will find the other boat moving in a direction about perpendicular to his own motion. (Explain these two cases by drawing diagrams.)

Problem. The pilot of an aircraft flying on a horizontal course due south-west at 200 kmph sees a train apparently moving due north, although he knows that the railway line runs due west. Find the speed of the train. [Ans : 141·4 kmph]. (kmph = km per hour.)

1-9.1. Relative motion in the same straight line with acceleration. Suppose two bodies, P and Q, are moving in the same direction with initial velocities u_1 and u_2, and accelerations f_1 and f_2, and that at the initial moment they are at a distance s apart (Fig. 1.15). Suppose P overtakes Q after travelling a distance x_1. Let the distance which Q travels in the same time t be x_2. Then

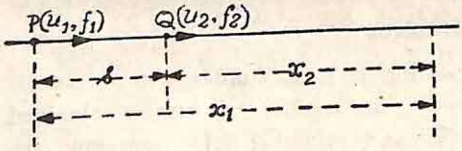

Fig. 1.15

$$x_1 = u_1 t + \tfrac{1}{2} f_1 t^2 \text{ and } x_2 = u_2 t + \tfrac{1}{2} f_2 t^2 .$$

$$\therefore \quad s = x_1 - x_2 = (u_1 - u_2)t + \tfrac{1}{2}(f_1 - f_2)t^2 . \qquad (1\text{-}9.1)$$

This is the same as if Q were stationary and P approached Q with a velocity $u_1 - u_2$ and acceleration $f_1 - f_2$. (Q's velocity and acceleration are reversed and added to those of P.) The time required by the faster body to overtake the slower one is that which it will take in travelling the distance s with a velocity $u_1 - u_2$ and acceleration $f_1 - f_2$.

Example. A stone is dropped from a high altitude and 3·00 s later another is projected downward with a speed of 150 ft/s. When and where will the second overtake the first ? ($g = 32$ ft/s^2).

Solution : At the moment the second stone is projected the first one, *i.e.*, the slower one has fallen $\frac{1}{2} \times 32 \times 3^2 = 144$ ft, and its speed $u_2 = 32 \times 3 = 96$ ft/s. The second stone has $u_1 = 150$ ft /s. The acceleration is $g = 32$ ft/s^2 for both.

We, therefore, have $s = 144 = (150 - 96)t + \frac{1}{2}(g - g)t^2 = 54t$, i.e., $t = 2\frac{2}{3}$ s.

The distance x_1 from the point of projection $= 150 \times 2\frac{2}{3} + 16 \times (2\frac{2}{3})^2 = 514$ ft nearly.

Problems. (1) A bullet is shot vertically upward with a speed of 320 ft/s, and 4·0 s later a second bullet is shot upward with a speed of 190 ft /s. Will they ever meet ? If so, where ?

(*Hint* : Calculate the speed of the first bullet after 4 sec. Note whether the fasterbullet is approaching the slower one or receding from it. They wiil not meet.)

(2) An elevator is ascending with an acceleration of 4·0 ft/s^2. At the instant when its upward speed is 8·0 ft/s, a bolt drops from the top of the cage, which is 9.0 ft from the floor. Find the time until the bolt strikes the floor and the distance it has fallen. [ANS : 0·71 s ; 2·3 ft.]

(*Hint* : For the elevator $u_1 = 8$ ft/s, $f_1 = 4$ ft/s^2. For the bolt $u_2 = 8$ ft/s, $f_2 = -32$ ft/s^2, $s = 9$ ft.)

1-10. Newton's laws of motion. The exact relation between force and motion was stated by Newton in the form of three laws known as Newton's laws of motion. These laws form the basis of mechanics. The laws may be stated as follows :

Law I. *Every body continues in its state of rest or of uniform motion in a straight line unless impelled by an external force to change that state.*

Law II. *The change of momentum per unit time is proportional to the impressed force, and takes place in the direction in which the force acts.*

Law III. *To every action there is an equal and opposite reaction.*

1-10.1. Discussion of the first law—Inertia and Force. The first law introduces two concepts—those of *inertia* and of *force*.

The tendency of a material body to continue in its state of rest or of uniform motion in a straight line is a fundamental property of matter and is called **inertia**. Due to inertia, matter resists any attempt to start it if it is at rest, or to stop it if it is in motion. It also resists any attempt to change its direction or speed of motion. It is due to inertia that a body on a moving train tends to move towards the forward end when the train stops, and towards the back end when the train starts. In both cases the body tends to continue in its previous state, whether that was one of rest or of motion. Other examples of inertia of motion are the particles of mud which fly off tangentially from the wheel of a moving motor car, or the sparks that fly off in straight lines from a grinding wheel when a metal is pressed on to it. A ball thrown upward in a moving train moves with the train because of its inertia of motion. It has the velocity of the train, and moves with the same velocity when thrown up. The *mass* of a body is a measure of its amount of inertia.

Force. The first law also defines force. The law means that an *unbalanced*[*] force must act on the body from outside to bring about a change in its state of rest or of motion.

If there are opposing forces, a motion may not be produced by a force. For example, we may push a heavy piece of furniture without moving it. Here forces of friction prevent motion. If these were absent, the applied force would have moved the piece of furniture. The push failed to produce motion as it was opposed.

We may, therefore, define force by saying that :

Force is the agency which can change the state of rest or of motion of a body.

1·10.2. The second law—the fundamental kinetic equation. While the first law defines force and inertia, the second law provides means of measuring them.

Let u = initial velocity of a body,
v = its final velocity,
t = the time in which this change occurs,
m = its mass, and
f = its acceleration.

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[*] An *unbalanced force* is one which is not opposed by an equal and opposite force.

The change of momentum in time $t = mv - mu$.

$$\therefore \text{ the rate of change of momentum} = \frac{m(v-u)}{t} = mf$$

$$= \text{mass} \times \text{acceleration}.$$

According to the second law the change of momentum per unit time, and hence the acceleration, will take place in the direction in which the force acts.

If P is the applied force, we have, from the second law

$$P \propto mf$$

$$\text{or} \quad P = kmf \tag{1-10.1}$$

where k is the factor of proportionality and is a constant. The value of this constant depends on our choice of units of P, m and f. If we agree that the force which produces unit acceleration on unit mass will be called the unit of force, then, in Eq 1-10.1, $P=1$, $m=1$ and $f=1$. This makes $k=1$. On this basis we write

$$P = mf$$

$$\text{or} \quad \textbf{Force} = \textbf{Mass} \times \textbf{Acceleration} \tag{1-10.2}$$

The is the *fundamental kinetic equation* and gives us a means of measuring force, as also mass.* For, according to Eq. 1-10.2 **mass** *of a body may be defined as the ratio of the force to the acceleration it gives to the body*. The mass, so defined, is a measure of the inertia of the body, and is called the *inertial mass*.

Inertial mass shows the following three properties :—

(*i*) The inertial mass of two bodies is the sum of the masses of the two.

(*ii*) Inertial mass of a body does not depend on the shape or composition of the body.

(*iii*) Chemical reaction does not change the total mass of the reactants.

To compare the masses m_1 and m_2 of two bodies, we may apply the same force P to them and find the accelerations they acquire. We shall then have $P = m_1 f_1 = m_2 f_2$ or $m_1/m_2 = f_2/f_1$. The accelerations produced will be inversely proportional to the masses. If $m_1 = 1$ kg, $m_2 = f_1/f_2$ kg.

1-10.3. Impulse of a force.

The product of a force (P) and the time (t) for which it acts on a body is known as the *impulse of the force*. An impulse produces a change of momentum. For

$$P \times t = m \times f \times t = m(v-u) \tag{1-10.3}$$

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* In Sec. I-1, of the chapter marked 'Physics : Introductory Remarks' we said matter offers resistance to any change in its state of rest or motion, and that mass is the amount of matter in a body. This implies that matter has inertia and mass is a measure of inertia.

or *Impulse of a force = Change of linear momentum of the body.*

A large force acting for a short time is called an **impulsive force.** Kicking a football, hitting a cricket or tennis ball, etc., are examples of impulsive forces. The effect of an impulsive force is to change the momentum of the body on which it acts. If the body was at rest, it starts with a velocity as a result of the impulse. The velocity will be greater the longer the force acts on the body. Hence a kick to a football will be more effective if the foot follows through, i.e., maintains a longer contact with the ball.

1-10.4. Units of force. A. Absolute units. A unit of force defined according to the equation $P=mf$ is called an absolute unit.

In *c.g.s. units*, the absolute unit of force is the dyne. The **dyne*** is that force which, acting on a mass of 1 g, gives it an acceleration of 1 cm/s^2.

$$1 \text{ dyne} = 1 \text{ gm} \times 1 \frac{\text{cm}}{\text{s}^2} = 1 \frac{\text{g cm}}{\text{s}^2}$$

The dyne (symbol dyn) can be replaced by its equivalent g cm/s^2 or g cm s^{-2}.

In *mks units*, the unit of force is called the **newton****. It is the force required to produce an acceleration of 1 m/s^2 in a mass of 1 kg. Thus

1 newton (symbol N) $= 1 \text{ kg} \times 1 \text{m/s}^2 = 1 \text{ kg m s}^{-2}$.

The newton (N) can be replaced by its equivalent kg m s^{-2} or kg m/s^2.

In the British system the absolute unit of force is called the poundal. The **poundal**† is that force which, acting on a mass of 1 lb, gives it an acceleration of 1 ft/s^2.

The poundal can be replaced by its equivalent lb ft/s^2 or lb ft s^{-2}.

Relation between the newton and the dyne. Let us write 1 newton $= 1 \text{kg} \times 1 \text{m/s}^2 = x$ dynes $= x$ g cm s^{-2}.

$$\therefore \quad x = 1 \frac{\text{kg}}{\text{g}} \cdot \frac{\text{m}}{\text{cm}} = 1000 \times 100 = 10^5$$

Thus 1 N $= 10^5$ dyn.

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* The dyne is a very small force, being about the weight of a mosquito.

** A newton is about equal to the weight of 100 g.

† The poundal is about equal to the weight of a 14-gram mass.

Relation between the poundal and the dyne :

$$\begin{aligned}1 \text{ Poundal} &= 1 \text{ lb} \times 1 \text{ ft/s}^2\\ &= 453.6 \text{ g} \times 30.48 \text{ cm/s}^2\\ &= 453.6 \times 30.48 \text{ g cm/s}^2\\ &= 1.382 \times 10^4 \text{ dyn} = 0{\cdot}1382 \text{ N}.\end{aligned}$$

B. Gravitational units of force. We use such terms as 'a kilogram of force or a kilogram-weight' and 'a gram of force or a gram-weight', etc. These are *gravitational units of force* and represent the attraction of the earth on a unit mass, *i.e.*, the weight of a kilogram or a gram or a pound. As the weight of a body varies slightly at different places it is necessary for the sake of accuracy to specify where this weight is measured.

A *kilogram of force* (written kg-wt or kgf) is the weight of a kilogram of mass at 45° latitude and mean sea-level.

A *gram of force* (or a gram-weight, written g-wt or gf) is the weight of a gram of mass at 45° latitude and mean sea-level.

A *pound of force* (or a pound-weight, written lb-wt or lbf) is the weight of a pound of mass at 45° latitude and mean sea-level.

For most practical purposes the slight variations in weight due to differences in location may be neglected.

Conversion from gravitational to absolute unit. A kilogram of force makes a kilogram of mass move with an acceleration g ($=9{\cdot}8$ m/s^2). Hence, to convert a force in gravitational unit into one in absolute unit, multiply by the appropriate value of g. Thus

1 kg-wt or kgf $=9{\cdot}8$ N (because $g=9{\cdot}8$ m/s^2 in mks units)

1 g-wt or gf $=980$ dyn (because $g=980$ cm/s^2 in cgs units)

1 lb-wt or lbf $=32$ poundals (because $g=32$ ft/s^2 in fps units)

Where extremely high precision is needed we use the *standard value of g*, which is $980{\cdot}665$ cm/s^2 or $32{\cdot}1741$ ft/s^2. This value closely represents the value of g at 45° latitude and mean sea-level.

Note from the above that if in the relation $P=kmf$ (Eq. 1-10.1), we take $k=g$ in the same system of units as m and f and put m and f equal to unity, then P becomes the gravitational unit of force.

Examples. (1) A force of 20 dynes acts on a body of mass 5 g initially at rest. What is the acceleration produced ? If the force ceases to act after 5 seconds how will the body move ?

Solution : From the relation that acceleration=force÷mass, we have $f = 20\text{ dyn}/5\text{ g} = 20\text{ g cm/s}^2 \div 5\text{ g} = 4\text{ cm/s}^2$.

As the force acts for 5 seconds, the increase in velocity in this time is $4\text{ cm/s}^2 \times 5\text{ s} = 20\text{ cm/s}$. Since the body was at rest initially, this is the velocity when the force ceases to act. By Newton's first law of motion, the body will continue to move with this constant velocity after the force ceases to act. The motion is in the direction in which the force acted.

(2) Under the action of a force a body of mass 10 g moved 40 cm from rest in 4 seconds. What is the magnitude of the force? If the force acts altogether for 5 seconds, how far will the body move in 3 seconds after the force has ceased to act?

Solution : Here $u=0$, $s=40$ cm, $t=4$ s. To find the force we must know the aceeleration. From the relation $s=ut+\frac{1}{2}ft^2$, we get

$$40\text{ cm} = \tfrac{1}{2}f \times (4\text{s})^2 \text{ or } f = 5\text{ cm/s}^2.$$

Since the force produces this acceleration in a mass of 10 g, the

$$\text{force} = 10\text{ g} \times 5\text{ cm/s}^2 = 50\text{ g cm/s}^2 = 50\text{ dynes}.$$

The velocity acquired can be had from the relation $v=u+ft$. Since $u=0$, we shall have $v = 5\text{ cm/s}^2 \times 5\text{ s} = 25\text{ cm/s}$. This is the velocity of the body when the force ceases to act. Hence the body travels with this constant velocity thereafter, and in the next 3 seconds covers $25\text{ cm/s} \times 3\text{ s} = 75\text{ cm}$.

(3) A football player takes a spot kick as a result of which the ball moves with a speed of 10 m/s. If the ball weighs 350 g and its time of contact with the foot during the kick is 1/10 second, what is the force applied to the ball?

Solution : In the relation $Pt=m(v-u)$ (Eq. 1-10·3), we have $u=0$, $m=350$ g, $v=1000$ cm/s.

$$\therefore \quad P = \frac{350\text{ g} \times 1000\text{ cm/s}}{1/10\text{s}} = 3{\cdot}5 \times 10^6 \text{ dynes}.$$

(4) You throw a stone weighing 60 g vertically upward. It reaches the top of a three-storeyed building which is 10 m high. If in throwing the stone you swung it for $\frac{1}{5}$ second, what force did you apply on it? ($g=9{\cdot}8\text{ m/s}^2$).

(Note that the values are in mixed units. Convert all of them into the same system, cgs or mks.)

Solution : It is a case of an impulsive force. Calculate the momentum you imparted to the stone. If it started upward with a velocity u, then from the relation $H=u^2/2g$, we get $u^2 = 2gH = 2 \times 9{\cdot}8\text{ m/s}^2 \times 10\text{ m}$ or $u=14$ m/s. Hence the momentum imparted was $0{\cdot}06\text{ kg} \times 14\text{ m/s} = 0{\cdot}84\text{ kg m/s}$. This momentum was imparted in 0·2 s. Hence force (or rate of change of momentum) $= 0{\cdot}84\text{ kg m/s} \div 0{\cdot}2\text{ s} = 4{\cdot}2\text{ kg m/s}^2 = 4{\cdot}2\text{ N}$. Try working out in cgs units.

(5) A pile driver weighing 400 lb falls from a height of 16 ft and comes to rest in $\frac{1}{5}$ sec after striking the pile. What is the force exerted on the pile?

Solution : The velocity acquired by the fall is obtainable from the relation $v^2=2\,gh$. With $g=32\text{ ft/s}^2$ and $h=16$ ft, we get $v=32$ ft/s. The momentum of the pile driver is 400×32 lb ft/s. The force = momentum ÷ time = $(400 \times 32 \div \frac{1}{5})$ poundals = 64,000 poundals = 2000 lb-wt.

(This is the force of the blow. The total force on the pile will be this plus the weight of the pile driver, i.e. 2400 lb-wt.)

[In the above problem read 400 kg for 400lb, 5 m for 16 ft, and work out the force. $g = 9{\cdot}8$ m/s^2.]

Solution of problems based on the equation P=mf.

In applying the equation $P = mf$, remember that

(i) When an unbalanced force P acts on body of mass m, the body moves with an acceleration $f = P/m$.

(ii) This acceleration takes place in the direction of the applied force.

(iii) The acceleration contiues so long as the force acts.

(iv) When the force ceases to act, the acceleration also ceases. The body then continues to move with the velocity it had at the moment when the force stopped.

Also remember

(v) **1 dyn = 1 g. cm/s^2. 1 N (newton) = 1 kg. m/s^2 = 10^5 dyn.**
1 poundal = 1 lb. ft/s^2

(vi) In equations, unit symbols cancel like algebraic quantities.

1-10.5. Third law : Action and reaction. The first and the second laws apply to the body on which the force acts. The third law introduces the body (or agency) which exerts the force. The terms *action* and *reaction* are simply other names for force. If a body A applies a force on another body B, which we call the *action*, then, according to the third law, the body B will apply an equal and opposite force on A, which is called the *reaction*. The law thus states that *in nature forces always occur in pairs*, and that *each force of the pair acts on a different body*. Whether the bodies are at rest or in motion, touch each other or are separated by a distance (as in the case of two magnets), the law is of universal application.

Consider a book lying on a table. The weight of the book exerts a downward force on the table. According to the third law, the table will exert an equal and opposite force on the book. If the former force is called the action, the latter force is the reaction. Note that *reaction lasts only so long as the action is there.* Besides, *action and* *reaction act on two different bodies, and cannot, therefore, produce equilibrium.* For equilibrium the forces on one and the *same* body must be balanced.

Other examples. (i) When a ladder leans against a wall it presses upon the wall. This pressure, here called the atcion, tends to overturn the wall. The counter-thrust exerted by the wall on the ladder is the reaction. It keeps the ladder in position.

(ii) Consider a rope tied to a tree and a man pulling at the other end of the rope. The force exerted by the man on the rope is the action. The reaction is the equal and opposite force exerted by the rope on the man.

The force exerted by the man on the rope is transmitted through the rope to the tree. The rope thus exerts an action on the tree. The tree exerts an equal and opposite reaction on the rope. The rope is in equilibrium under the action of the pulls exerted on it by the man and the tree.

[In this example the rope transmits two forces through it—one exerted by the man which acts through the rope on the tree, and the other, the reaction to this force which is exerted by the tree on the man. The rope is stretched and is said to be in 'a state of tension'. *Either force* transmitted through the rope is called a *tension*. This would show that when two men pull at two ends of a string, each with a force of 20 kg, the tension in the string is still 20 kg-wt.]

Problems. (1) Two boys pull at two ends of a string, each with a force of 20 kg-wt. What is the tension in the string ? [*Ans* : 20 kg-wt.]

(2) One end of a string is tied to a fixed post. Two boys pull the string at two places, each with a force of 20 kg-wt. What is the tension in the string (*a*) between the post and the first boy, (*b*) between the two boys ?

[*Ans* : (*a*) 40 kg-wt, (*b*) 20 kg.wt.]

(3) In a tug-of-war each team pulls with a force of 100 kg-wt. If a spring balance is inserted in the string between the teams what will it read ?

Reaction in a moving lift. Consider a person weighing m kg standing in a lift while it is moving *upward* with an acceleration f m/s^2. The forces on him are his weight mg acting downward, and R, the reaction of the floor, acting upward. Since the acceleration is upward, we must have

$$R-mg=mf \text{ or } R=m(g+f)$$

If the lift were moving *downward* the man moves downward too. Then mg must be greater than R. In this case, we shall have

$$mg-R=mf \text{ or } R=m(g\text{-}f)$$

When $f=g$, the reaction R reduces to zero.

(Our sense of weight arises from the reaction of the floor on which we stand. If this reaction is zero, we shall feel *weightless*. So when the lift descends with an acceleration equal to that due to gravity, a person in the lift will feel weightless.)

Example. A man weighing 70 kg stands in a lift which starts moving downward with an acceleration of 0·8 m/s². Calculate the thrust he exerts on the floor. If the lift moved upward with the same acceleration, what thrust will the man experience ? ($g=9{\cdot}8$ m/s²)

Solution : The thrust he exerts is equal to the reaction on him.

From the relation $R=m(g-f)$ for downward motion we get

$R=70$ kg $(9{\cdot}8-0{\cdot}8)$ m/s² $=630$ N.

For upward motion $R=m(g+f)=70(9{\cdot}8+0{\cdot}8)$ kg m/s² $=742$ N.

1·11. Principle of conservation of linear momentum. A very important principle follows from Newton's third law of motion and the action of impulse of a force (Sec. 1·10.3). Let a particle A act on another particle B with a force P for a time t. The impulse Pt of the force gives the change in linear momentum of B. Since B exerts an equal and opposite force on A, this reaction acts for the same time t on A, and producs a change of momentum Pt in A which is opposite in direction to that produced in B. The changes in momenta due to action and reaction are thus equal and opposite.

Hence the total momentum of the system* consisting of the particles A and B is not changed by the action and reaction between them. This is true whatever the number of particles in the system. We can therefore say that

The total linear momentum of an isolated system of particles is not altered by the actions and reactions between the particles comprising the system. This statement is known as the **principle of conservation of linear momentum.** For 'particles' we can read 'bodies', the bodies obeying Newton's third law.

To change the linear momentum of a system of bodies, or particles, a force must be applied from outside the system. Thus a cyclist cannot slow down his cycle by merely pulling at the handle. The change in momentum due to this pull on the cycle is balanced by that due to the forward reaction on the cyclist.

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* The term 'system of bodies' as used in physics should be understood in the following sense. We often select for our discussion two or more bodies which exert forces on one another. For convenience of analysis we imagine the bodies under discussion to be isolated from all other bodies, as if all that existed were the bodies in question and nothing else in the rest of the universe. Such bodies are referred to as *a system af bodies* or *an isolated system.*

A shell bursting in mid-air has a certain momentum before it explodes. When the fragments fly off in all directions after the explosion, each one of the fragments has a momentum of its own, According to the principle of conservation of momentum the sum total of the momenta of the fragments must be equal to the momentum of the entire shell.

The flight of **rockets** is another example of this principle. The rocket has solid fuel. One of the constituents of the fuel provides the oxygen necessary for the combustion of the other constituents. The products of combustion within the rocket come out with a very high velocity from the tail end of the rocket. *The main body of the rocket gets an equal and opposite momentum and moves forward.* To change the direction of a rocket ship in space, small jets are fired from the sides of the rocket. These may be oriented as desired.

Jet planes utilize the same principle as rockets for motion. In the jet engine, air blows in at the front, is compressed and led into a combustion chamber. Here it is mixed with fuel and burned. The high pressure of the resulting gases is made to work a turbine. The turbine not only works the compressor which compresses the incoming air, but also drives the gases through the tailpiece of the engine at a tremendous speed. *The plane gets a forward momentum equal to the backward momentum of the exhaust gases, and moves forward.*

Gun firing a shell. When a gun fires a shell, the shell acquires a momentum in its direction of motion. Before firing, the system consisting of the gun and the shell was at rest, and their combined momentum was zero. Since the explosion exerts equal and opposite forces on the gun and the shell, their combined momentum will not change due to the explosion. The gun therefore acquires a momentum equal and opposite to that of the shell. The backward velocity which the gun acquires on firing is called the *velocity of recoil*.

Let M and m be the masses of the gun and the shell respectively and $-V$ and v their velocities.

Then $-MV+mv=0$ or $MV=mv$.

Example. A shell weighing 8 kg is fired from a gun weighing 800 kg. If the speed of the shell is 300 m/s, find the velocity of recoil of the gun.

Solution : From the relation $MV=mv$,

we have $V=mv/M=8\text{ kg}\times 300\text{ m/s}\div 800\text{ kg}=3\text{ m/s}$.

1-12. Elastic collisions. You are familiar with collisions, such as between a man and a cyclist, two cyclists, two trains, etc. Mathematical analysis of any such real collision is about impossible. But in idealized simple cases, it is possible to do so.

Suppose two particles collide with each other while moving along the same straight line. A collision is said to be *elastic* when the sum of the kinetic energies of the particles remains the same before and after collision. The linear momentum is of course conserved. Atoms, molecules, nuclei, electrons, etc. may suffer elastic collisions.

Let two particles of masses m_1 and m_2 move *along the same line* with velocities u_1 and u_2 respectively (Fig. 1.16a). The velocities will be taken as positive if the motion is from left to right. Momenta are in the direction of velocities. If any of the particles moves from right to left its velocity and momentum will be taken as negative.

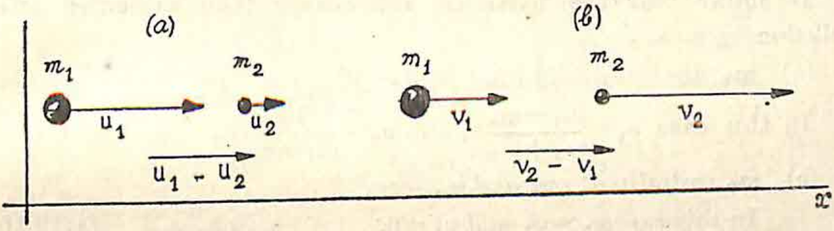


Fig. 1.16

Suppose after the collision the velocity of m_1 becomes v_1, and that of m_2 becomes v_2. From the principle of conservation of momentum, we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{1-12.1}$$

In elastic collisions, kinetic energy is also conserved.

Therefore $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ (1-12.2)

These two equations will give us the answer to all questions of elastic collision in a straight line. Let us consider a few cases. Note that we are taking $u_1 > u_2$ and $v_2 > v_1$.

(*i*) *Relative velocities before and after collision.*

From 1-12.1 $m_1(u_1 - v_1) = m_2(v_2 - u_2)$ (1-12.3)

From 1-12.2 $m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$ (1-12.4)

Dividing 1-12.4 by 1-12.3 we get

$$u_1 + v_1 = v_2 + u_2$$

$$\text{or} \quad u_1 - u_2 = v_2 - v_1 = -(v_1 - v_2) \tag{1-12.5}$$

This shows that the *relative velocity of approach before collision is equal to the relative velocity of separation after collision.*

(*ii*) *Dependence of final velocities on the colliding masses.*

From 1-12.5, we have $v_2 = u_1 - u_2 + v_1$. Putting this value in 1-12.3 we get $m_1(u_1 - v_1) = m_2(u_1 - u_2 + v_1 - u_2)$

or, $v_1(m_1 + m_2) = (m_1 - m_2)u_1 + 2m_2u_2$

$$\text{or, } v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2 \tag{1-12.6}$$

Similarly,

$$v_2 = \frac{2m_1}{m_1 + m_2}u_1 + \frac{m_2 - m_1}{m_1 + m_2}u_2 \tag{1-12.7}$$

(*a*) *If both masses are equal* $(m_1 = m_2)$

$$v_1 = u_2 \text{ and } v_2 = u_1 \tag{1-12.8}$$

It shows that the particles *interchange* their velocities after collision.

(*b*) m_2 *fixed intitially* (*that is*, $u_2 = 0$).

$$\text{In this case } v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 \text{ and } v_2 = \frac{2m_1}{m_1 + m_2}u_1 \tag{1-12.9}$$

(*c*) m_2 *initially at rest and* $m_1 = m_2$.

$$\text{In this case } v_2 = u_1 \text{ and } v_1 = 0. \tag{1-12.10}$$

This means that the first particle comes to rest after collision and the second starts with the velocity of the first.

(*d*) *The second particle is very heavy relative to the first* $(m_2 \gg m_1)$ *and is initially at rest* $(v_2 = 0)$. In this case, we get from Eq.s 1-12.6 and 1-12.7

$$v_1 = \simeq - u_1 \text{ and } u_2 \simeq 0. \tag{1-12.11}$$

(The sign $\simeq$ means 'about equal to')

This means that the lighter particle turns back practically with the same velocity after collision while the heavy particle remains practically at rest.

[It follows that on colliding with a rigid wall, a particle will turn back with the same speed.]

(*e*) m_2 *is very small compared with* m_1 *and is at rest.* We neglect m_2 compared with m_1 and take $u_2 = 0$ in Eq.s 1-12.6 and 1-12.7. This gives

$$v_1 \simeq u_1 \text{ and } v_2 \simeq 2u_1 \tag{1-12.12}$$

This means that the velocity of the heavier particle remains practically unchanged, while the lighter particle shoots off with a speed about twice that of the heavier particle.

1-13. General ideas on friction. When one surface moves over another, or tends to do so, a force comes into play which opposes the motion. This force is called the *force of friction*. The term 'friction' means the resistance to motion which a body meets with when it moves over another body.

Friction between solid bodies may be classified as *sliding* and *rolling*. We discuss below the case of sliding friction.

1-13.1. Sliding friction. Consider a heavy body resting on a horizontal table (Fig. 1.17a). Its weight W acts on it vertically downward. The reaction R of the table acts on the body vertically upward. Since the body is at rest, these two opposite forces are equal. (They act in the same line through the centre of gravity, but are shown separated in the figure for the sake of clarity.) If we now apply a small horizontal force P to the body it will not move (Fig. 1.17b). This force will be balanced by the force of friction F which comes into play at the surface separating the body and the table. P and F must be equal and opposite. Their lines of action are parallel. S is the resultant of F and R.

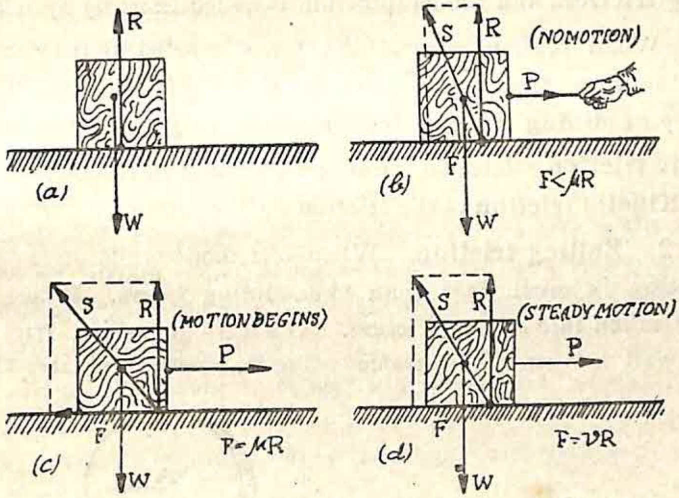


Fig. 1·17

As we increase P, F also increases and balances P. But when P exceeds a certain value, the body starts to move. After the motion has begun, the value of the force of friction slightly decreases (Fig. 1.17d). The force of friction reaches the maximum value just before the motion begins. This value is called the *limiting friction* (or the *limiting value of static friction*). The slightly smaller value when the body is moving is called the *force of kinetic friction.*

Summarising, we may say that

(*i*) Friction is a *self-adjusting force*, increasing from zero to a maximum. So long as there is no motion, just as much of it comes into play as is necessary to prevent motion.

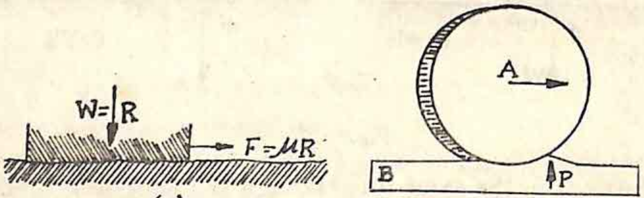
(*ii*) It always acts in the plane of sliding and its direction is always opposite to that in which the motion occurs or tends to occur.

(*iii*) When the surfaces are just on the point of relative motion, the force of friction is the greatest. This value is known as the **limiting friction**, and the equilibrium is called *limiting equilibrium.*

(*iv*) When motion occurs, the force exerted by friction against the motion is *less* than the value of the limiting friction and is called the **force of sliding friction**, (also **kinetic** or **dynamic** friction).

Static friction relates to the friction so long as the body does not move. **Kinetic friction** is the friction during motion.

1-13.2. Rolling friction. When one body rolls over another, the friction is much less than when sliding occurs. *Wheels change sliding friction into rolling friction.* The laws of rolling friction are not so well known. The static value is, however, greater than the kinetic value. The coefficients of rolling friction are much smaller than those of sliding friction. The force required to overcome rolling friction is inversely proportional to the radius of the roller.



(a) (b)

Fig. 1.18

Causes of friction. Surfaces of solids are generally uneven. Even on a plane or polished surface there are small irregularities (Fig. 1.18*a*). Sliding friction is caused by the interlocking of these irregularities. Besides, a harder body produces depressions in the softer body. This causes more interlocking. In rolling, the supporting surface is slightly depressed at the place of contact and a slight elevation forms ahead of it (Fig. 1.18*b*). During rolling, the rolling body has constantly to mount such a minute hill. There is also some slip at the place of contact.

Molecular forces across the surface of contact between two bodies also contribute to frictional resistance to motion.

1-13.4. Reduction of friction. *(i)* **Lubrication.** Friction between two surfaces can be reduced by lubrication. A substance used for the purpose is called a **lubricant.** Very often the lubricant is an oil. Properly lubricated surfaces are not in contact ; a layer of the lubricant, be it an oil, exists between them. *Sliding friction is therefore replaced by 'liquid friction'*, which is much smaller. A good lubricant must have the property of adhering to the surfaces. Mixtures of mineral and vegetable oils serve as good lubricants. Sometimes a little colloidal graphite is added to great advantage.

(ii) In rotating machinery friction is reduced by using *ball* or *roller bearings.* In such bearings the surfaces do not slide, but roll over one another. *Sliding friction is reduced by this means to rolling friction.*

1-13.5. Advantages and disadvantages of friction. Though friction always opposes motion, reduces the efficiency and life of our machines, and leads to wastage of energy, it is not an unmixed evil. Friction makes walking possible, holds the nails to the walls, drives machinery by means of belts coupled to engines, and starts and stops our cars and railroad trains. Many essential activities would have been impossible without friction.

1-14. Laws of sliding friction.

Some general results relating to *sliding friction between two dry solid surfaces*, known as the 'laws of friction', are as follows. They are approximately true within limits.

(i) For a given pair of surfaces the limiting friction F is proportional to the normal force R acting at right angles to the plane of contact between the surfaces. The ratio $F/R=\mu$ is called the **coefficient of static friction** between the two surfaces.

$$\text{Coeff. of static friction } (\mu)=\frac{F}{R}=\frac{\text{Limiting friction}}{\text{Normal reaction.}} \qquad (1\text{-}14.1)$$

(ii) The limiting friction between two bodies in contact depends upon

- *(a)* the nature of the substances (such as wood, iron, leather etc.) of which the bodies are made ;
- *(b)* the condition of the surfaces (moist or dry, plane or rough, etc.)

(*iii*) The limiting friction is independent of the area of contact *so long as the normal force pressing the bodies together remains unchanged.* Thus a brick lying flat on a floor, or standing on edge, will offer the same frictional resistance to its motion.

(*iv*) When motion* occurs, the force F' of sliding (kinetic) friction is proportional to the normal force R between the surfaces. The ratio $F'/R=\nu$ is called the **coefficient of sliding** or **kinetic friction.**

$$\text{Coeff. of kinetic friction } (\nu)=\frac{\text{Force of sliding friction}}{\text{Normal reaction}} \qquad (1\text{-}14.\ 2)$$

(*v*) Within wide limits the force of sliding (or kinetic) friction is independent of the relative velocity between the surfaces, and their areas of contact, provided the normal reaction is the same. Kinetic (or sliding) friction is by far the more important in practice. When a body is at rest, but a force of friction is operating, its value is rarely the limiting value. In practical cases we are more concerned with bodies in motion with friction opposing motion. Thus *it is the* (*kinetic* or *sliding*) *friction which is the one to be considered.*

Table. Coefficient of sliding friction

Wood on wood	0·2 to 0·5	Wood on stone	0·6 to 0·7
Leather on metal (dry)	0·56	Earth on earth	0·25 to 1·0
,, ,, (oily)	0·15	Smooth oiled surfaces	0·03 to 0·036
Metal on metal (dry)	0·15 to 0·20	Iron on stone	0·4

1-15. Angle of repose and angle of friction. Let a body A of weight W (Fig. 1.19) rest on an inclined plane B whose inclination θ to the horizontal can be changed at will. If the weight W be resolved

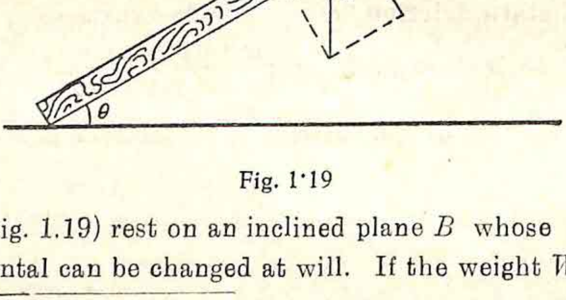


Fig. 1·19

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* This motion is *motion without acceleration* (of one surface with respect to the other.)

along and normal to the inclined plane then, for equilibrium of the body, we must have normal reaction $R=W \cos \theta$, and $F=W \sin \theta$, where F stands for the force of friction.

Let the inclination of the plane be gradually increased so that the body is just on the point of sliding. Let the inclination be now $\theta=\alpha$. In this limiting condition, we get the coefficient of static friction.

$$\mu=\frac{F}{R}=\frac{W \sin \alpha}{W \cos \alpha}=\tan \alpha. \qquad (1\text{-}15.1)$$

The angle α is called the **angle of repose.** It may be defined as the maximum angle of inclination that a plane may have before a body on it begins to slide under the action of its weight.

The *angle of friction* is defined as the angle between the resultant reaction S and the normal reaction R (Fig. 1.17*c*) when the friction is limiting. It may be shown that this is equal to the angle of repose.

1-16. Measurement of the coefficient of friction.

A. **Inclined plane method**. In this method a block of one material (A ; Fig. 1. 19) is placed on an inclined plane (B) of the other material. The inclination of the plane with the horizontal is gradually increased until A starts sliding down the plane. The angle of inclination α is measured. From Eq. 1-15.1, $\tan \alpha=\mu$, the coefficient of static friction between A and B.

At a slightly smaller angle than the above, A will slide down the plane *with a uniform speed* if started from rest by a gentle push. If this angle is θ, $\tan \theta=\nu$, the coefficient of sliding (or kinetic) friction. The component $W \sin \theta$ of the weight parallel to the plane just overcomes the frictional force during the downward motion without acceleration.

To find μ or ν several observations have to be made for each and the average value taken.

B. **Horizontal plane method.** The arrangement is as represented in Fig. 1.20. A string passing over a pulley connects the movable block A with a scale pan on which different weights can be placed. The load on the pan is adjusted until A starts moving. If F is the total weight of the scale pan and the load on it, and R is

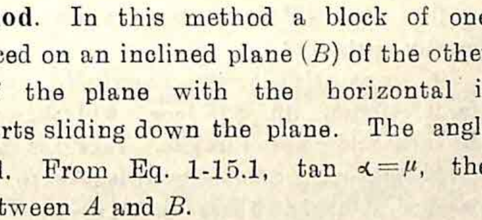


Fig. 1.20

the normal reaction on A, then $F/R=\mu$, the coefficient of static friction. R is obviously the weight of the block A together with that of any additional load which may be placed on it.

With a slightly smaller load than F, A will move *with a constant speed* if started by a light push. The ratio of this load F' to R is ν, the coefficient of kinetic friction ($\nu=F'/R$).

Exercises

1. (*a*) What do we mean when we say a body is at 'rest' or in 'motion'? Why is 'rest' or 'motion' considered *relative*?

(b) Explain the term 'acceleration' with suitable examples.

(c) Distinguish between 'average value' and 'instantaneous value' of a quantity. When are they the same?

2. What is meant by a 'reference frame'? What are 'space coordinates'?

3. (a) Derive the equations $s=ut+\frac{1}{2}ft^2$ and $v^2-u^2=2fs$, clearly explaining the meanings of the symbols.

(b) What will the equations reduce to when a particle moves freely under gravity vertically?

4. A particle is projected vertically upward with a velocity v. Find (i) how far it will rise, (ii) how long it will take to rise, (iii) what will its velocity be at the mid-point of its rise. Take g as the acceleration due to gravity.

Show that the time the particle takes to rise the last half-metre of its path is equal to the time it takes to fall the same distance from rest.

5. What is a vector quantity? In what way is it different from a scalar quantity? Give examples of each.

How can a vector be represenied by a line segment?

State the law of addition of vectors. How can one vector be subtracted from another? Draw diagrams.

What is meant by the *Principle of independence of vectors*?

6. (a) Vectors P and Q are inclined to each other at an angle θ. Find their resultant (Give both magnitude and direction). Find also the magnitude of their difference, and also its direction.

(b) A particle is acted on simultaneously by two forces F_1 and F_2 inclined to each other at an angle α. Find the magnitude and direction of their resultant.

(c) What is meant by the *principle of independence of velocities*? If a person wants to cross a river perpendicularly to the bank, in what direction will he swim, and why? Where will he reach on the other bank?

7. What is meant by the *resolved part* of a vector? How is a vector related to its components?

8. Three forces act on a particle in the same plane. One is 100 dynes due east, the second is 200 dynes due north and the third is 200 dynes acting

north-east. Resolve the forces into components and find their resultant. [Hint : See Sec. 1-8.]

9. What do you understand by *composition* and *resolution* of forces or velocities ? When is a resolved part the same as the component in that direction ?

10. A and B have velocities u_1 and u_2 respectively. Their directions are inclined at an angle θ. In what direction will B appear to move when seen from A ? Draw a diagram to explain and calculate. [See Sec. 1-9].

11. Two stones are dropped from the same height at an interval of 2 seconds. What is their initial distance ? Explain if these distances will increase or decrease as they continue to fall freely. Take $g=980$ cm/s². [*Hint* : See Sec. 1-9.1. Distance will increase with time at the rate of 1960 cm/s.].

12. (a) State and discuss Newton's laws of motion in about 40 lines.

(b) Give examples of inertia of rest and inertia of motion.

(c) Establish the equation $P=mf$, explaining the meanings of the symbols.

13. (a) 'Newton's first law introduces the concepts of *inertia* and *force*' —Explain the statement.

(b) Explain the statement that Newton's second law indicates how force and inertia can be measured. What is *inertial mass* ?

14. Explain the terms *dyne*, *newton*, *gram-weight*, *kilogram-weight*. What symbols do you use for them ? Express the last three quantities in dynes. Take $g=980$ cm/s².

15. What do you understand by *impulse of a force* and an *impulsive force* ? Find how impulse and momentum are related.

16. 'Newton's first and second laws relate to the same body ; but the third law introduces another body.' Explain this statement with examples.

If action and reaction are equal and opposite, why should a body move at all under the action of a force ?

17. State the principle of conservation of linear momentum, and explain how you can arrive at it from Newton's laws.

If we accept this principle as basic, we can get at Newton's first law from the third. Can you say how this could be done ?

Give three examples of conservation of linear momentum. Discuss the motion of a rocket on this basis.

18. What is an *elastic collection* ? Establish the momentum and energy equations for an elastic collision in a straight line.

A particle at rest is hit by another particle of equal mass moving with velocity v. Show that the second will impart its momentum fully to the first. [See Eq. 1-12·10].

19. (a) Friction is said to be a 'self-adjusting' force. What does this statement mean ? A force is said to have a direction. What is the direction of the force of friction ? Is the direction fixed ? Define the coefficients of

static and dynamic friction, Which of these two is the more important in everyday life ?

(b) State and explain rhe laws of friction.

20. A particle moves with an accleration of 5 cm/s^2. If its initial velocity is 100 cm/s, what will be its velocity after 20 s ? How far will the particle move in this time ? [Ans : 200 cm/s , 3000 cm].

21. A car has a velocity of 20 m/s. It is decelerated at 5 m/s^2. How far will it move before stopping, and how long will it take to come to a stop ? [Ans : 4 s ; 40 m].

What should be the retardatiod of the car if it is wanted to stop it in (a) one-fourth the distance, (b) one-fourth the time. [Ans : 20 m/s^2 in both cases].

22. A stone is thrown horizontally with a velocity of 10 m/s from the top of a tower 20 m high. How far from the tower will the stone reach the ground ? At what angle will it meet the ground ? Take $g=9{\cdot}8$ m/s^2.

[*Solution* : The stone has two simultaneous velocities $-u_1$ of horizontal projection, and u_2 of free fall under gravity. Each acts independently. First find the time to reach the ground due to vertical motion from $h=\frac{1}{2}\,g\,t^2$ or $20=\frac{1}{2}\times 9{\cdot}8\times t^2$. At the end of this time $u_2=gt$. In time t, the stone moves horizontally a distance u_1t. This is the distance sought. Combine u_1, u_2 (which are at right angles) to get the direction of the resultant with u_1. The stone strikes the ground at this angle.]

23. What is meant by the resolved part of a force ?

In a tug-of-war, should all members of a team pull in a straight line, or in a zig-zag fashion ? Explain your answer.

A body weighing 2 kg is placed on a smooth plane inclined to the horizontal at 30°. What is the magnitude of the force tending to slide it down the plane ? What acceleration does it give to the body ? ($g=9{\cdot}8$ m/s^2) [Ans : 9·8 newtons or $9{\cdot}8\times 10^5$ dynes ; 4·9 m/s^2].

24. A body is pulled simultaneously with a force of 50 kg acting due east and a force of 20 kg acting at an angle of 60° east of north. Find the magnitude and direction of the resultant. [Ans : Magnitude of resultant = 62·4 kg. If the resultant makes angle θ with east towards the north $\tan\theta=1/2\sqrt{3}$].

25. The resultant of two forcss makes angles 30° and 45° with the forces. If the resultant is 1000 dynes, what are the magnitudes of the forces ?

26. A particle of mass 2 g is acted on by a force of 26 dyne wt an angle θ north of east. $\tan\theta=5/12$. How far will this force move the particle towards the east in 5 s ? [*Hint* : The resolved part of the force towards the east is $26\cos\theta=24$ dyn. It gives the particle an acceleration of $f=12$ cm/s^2. In 5 s the particle moves through a distance $\frac{1}{2}ft^2$. Ans : 150 cm].

27. A ball weighing 150 g was moving with a speed of 10 m/s. It is stopped in 0·2 s. How much force was applied to stop it ? [Ans : $7{\cdot}5\times 10^5$ dyn].

28. A bullet weighing 10 g is fired from a suspended gun weighing 1 kg. The bullet lodges into a piece of wood weighing 990 g. If the speed of the bullet was 500 m/s, with what velocity will the gun recoil ? What will be the velocity of the piece of wood ? [Ans : Both 5 m/s]

29. A boy weighing 50 kg stand on a lift. What force will the floor of the lift exert on the boy when

(a) the lift is at rest ;

(b) the lift is ascending with an acceleration of $4{\cdot}9\ m/s^2$;

(c) the lift is ascending at a constant speed ;

(d) the lift is moving up, but has a retardation of $4{\cdot}9\ m/s^2$?

Take $g=9{\cdot}8\ m/s^2$.

[Ans : (a) 50 kg-wt ; (b) 75 kg-wt ; (c) 50 kg-wt ; (d) 25 kg-wt.]

30. A passenger sits in an aeroplane. State the conditions in which the action and reaction between the two will be

(a) equal to the weight of the passenger ;

(b) greater than the weight of the passenger ;

(c) less than the weight of the passenger ;

(d) zero.

[Ans : (a) The plane is at rest or moving with a constant velocity ; (b) Plane moving up with an acceleration ; (c) Plane descending with an acceleration ; (d) descending with an acceleration equal to that due to gravity.]

31. The barrel of a gun is 50 cm long. A bullet weighing 10 g is fired from the gun with a velocity of 400 m/s. What is the average acceleration of the bullet inside the barrel ? What is the average force on it ? [Ans : $1{\cdot}6\times10^7$ cm/s^2 ; $1{\cdot}6\times10^8$ dyn.]

32. A bullet weighing 50 g is fired with a speed of 300 m/s. It enters 2·5 cm inside a target before stopping. What is the retarding force ? How long did it take the bullet to come to a stop in the target ? [Ans : 9×10^9 dyn ; 1/6000 s.]

CHAPTER

2

DYNAMICS OF ROTATIONAL MOTION

2-1. Terms for describing rotational motion. When a body rotates about an axis, such as a wheel spinning about its axle, it is said to have *rotational motion*. To describe rotational motion, we have first to introduce a few new terms. For simplicity, we shall consider the motion of a particle in a circle.

Consider a particle P (Fig. 2.1) moving in a circle of radus r. As it moves, the line drawn from the centre to the particle sweeps out gradually increasing angles. (The line is called the *radius vector*.) Let this angle be measured from some standard position of the radius vector (say, from OA on the line OAx). The angle θ which the radius vector makes with the standard line is called the **angular displacement** of the particle. θ is generally measured in radians.

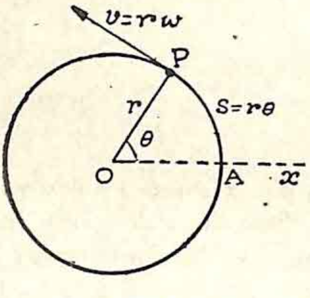


Fig. 2.1

The **angular velocity** of the particle is the time rate of change of its angular displacement, or the angle turned through by it in unit time. If θ_1 and θ_2 are the angular displacements at times t_1 and t_2 respectively, then angular velocity

$$\omega = \frac{\theta_2 - \theta_1}{t_2 - t_1} \qquad (2\text{-}1.1)$$

Angular velocity is expressed in **radians per second**. It is treated as a vector along the rotation axis. The magnitude of angular velocity is **angular speed**. So long as the direction of the axis remains fixed, it does not matter whether we call ω the angular velocity or the angular speed. Like linear velocity or speed, ω *may be constant or variable*. When $t_2 - t_1$ is very very small, ω is called the *instantaneous angular velocity*.

The number of revolutions executed in one second is called the **frequency** of rotation of the particle. The time required to execute one revolution is called the **period** of rotation. If the angular speed is ω rad/s, the period

$$T = 2\pi/\omega \text{ seconds.} \qquad (2\text{-}1.2)$$

The number of revolutions executed in one second, i.e., **frequency**
$n=1/T=\omega/2\pi$ per second. (2-1.3)

Example. A particle executes 200 revolutions in 5 seconds. Find (*i*) its frequency, (*ii*) angular speed, (*iii*) period and (*iv*) the time required to turn through 90°:

Solution : Frequency n=number of revolutions per second (rps in brief)

$$=\frac{200}{5\text{ s}}=40 \text{ per second or } 40\text{ s}^{-1} \text{ or } 40/\text{s}.$$

[The unit of frequency has been named the **hertz**, symbol Hz.]

Angular speed $\omega=2\pi n=80\pi$ radians/second (or 80π rad/s in modern symbols).

Period $T=\frac{1}{n}=\frac{1}{40\text{ per second}}=\frac{1}{40}$ second (0·025 s in modern language).

$$\text{Time to turn through } 90^\circ=\frac{\text{angle turned through}}{\text{angular speed}}$$

$$=\frac{90^\circ}{80\pi\text{ radians/second}}=\frac{\pi/2\text{ rad}}{80\pi\text{ rad/s}}=\frac{1}{160}\text{ s.}$$

The rate of change of angular velocity with time is called **angular acceleration** (α).

$$\alpha=(\omega_2-\omega_1)/(t_2-t_1) \qquad (2\text{-}1.4)$$

As in the case of linear acceleration (f), it may be constant or variable. When t_2-t_1 is very small, it is called instantaneous acceleration.

Relation between angular and linear speed. Suppose the particle (Fig. 2.1) turns through an angle θ radians in t seconds. Then its angular speed is $\omega=\theta/t$ rad/s. In this time the particle describes an arc AP (Fig. 2.1) of length s. Since angle in radians$=\frac{\text{length of arc}}{\text{radius}}$, we have $\theta=s/r$ or $s=r\theta$. Dividing by t, we get $s/t=r.\theta/t$

or *linear speed=radius × angular speed* (in radians per second)

In symbols, $v=r\omega$. (2-1.5)

Example. A stone is whirled at the end of a string 1 metre long and describes 6 revolutions in 4 seconds. Find its angular and linear speeds.

Solution : In 6 revolutions the stone describes $6\times2\pi=12\pi$ radians.

$$\therefore \text{ Angular speed}=\frac{\text{angle described}}{\text{time taken in doing so}}=\frac{12\pi\text{ rad}}{4\text{ s}}=3\pi\text{ rad/s.}$$

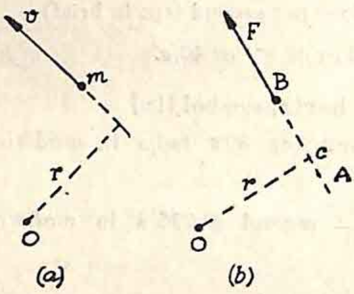
Linear speed=radius × angular speed in radians per sec.

$$=1\text{ metre}\times3\pi\text{ rad/s}=300\pi\text{cm/s}=942\text{ cm/s.}$$

2-2. Angular momentum. A particle of mass m moving with velocity v has a *linear momentum mv* in the direction of v. Let there be a point O whose *perpendicular distance* from the direction of v is r (Fig. 2.2a). Then the product $r \times mv$ is called the **angular momentum** (L) of the particle about the point O.

$$L=mvr \tag{2-2.1}$$

Angular momentum is a very important quantity in rotational motion. Like linear momentum, it is a conserved quantity ; hence its importance. In Fig. 2.1, if the particle P has a mass m, its angular momentum about O, the centre of the circle, will be mvr where v is the linear velocity of P. Since $v=\omega r$, we have, in this case



(a) (b)

Fig. 2.2

$$L=mvr=mr^2\omega \tag{2-2.2}$$

2-3. Moment of a force about a point.

Let AB (Fig. 2.2b) be the line of action of a force F. O is a point whose perpendicular distance OC from AB is r. Then the product $r \times F$ is called the moment N of the force F about O. We may define it by saying

The moment of a force about a point is the product of the force and the perpendicular distance of the point from the line of action of the force.

$$M=rF \tag{2-3.1}$$

M measures the turning effect of F about O. If O lies on the line of action of F, its moment vanishes. In such a case the force F cannot produce any rotation about O.

2-3. Moment of a force about an axis.

Consider a heavy door hinged about a vertical axis AB (Fig. 2.3). In our everyday experience we find that it requires a smaller force to swing the door when the force is applied to the door *perpendicularly* at a point close to the handle, than when applied closer to the axis AB. If we could use some device to measure the force, such as a spring balance, we should find that *to produce a slow uniform rotation of the door about the axis*

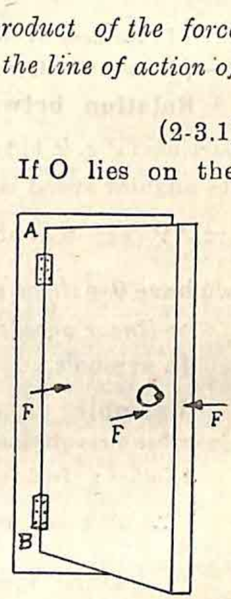


Fig 2.3

the force × its distance from the axis of rotation = constant.

The perpendicular distance l from the axis to the line of action of the force is called the *lever arm* or *moment arm* of the force, and the product of the force F and the lever arm (*i.e.*,Fl) is called the **moment** of the force (or **torque**) about the given axis. The term *torque* is also used to mean moment of a force about a point.

The above discussion shows that the *rotational effect of a force is measured* not by its own magnitude, but *by the magnitude of its moment about the given axis.* A force applied at a certain distance from the axis has the same turning effect as a force of half the magnitude applied at double the distance.

If the line of action of the force passes through the axis of rotation, the perpendicular distance between them is zero and so also is the torque. Such a force cannot produce rotation about the axis. This can easily be verified by pushing at a door in its own plane.

Inclined force. If F is not perpendicular to the axis (AB), we have to resolve F into two components F_x, F_y in a plane parallel to the axis. If the x-axis is perpendicular to the given axis AB and l is the perpendicular distance between the x-axis and the AB-axis, then the moment M of F about AB is $M=F_x l$. F_y being parallel to AB produces no turning effect.

Moment as a vector. Positive and negative moments. Moments which tend to produce rotation in an anticlockwise direction are conventionally taken as positive. Those producing a clockwise rotation are considered negative. A moment is a vector quantity, its direction being taken as that of the axis about which it causes rotation. In Fig. 2.4, the moment of the force F_1 about O is positive, while that of F_2 is negative. If the figure represents a lamina which can turn about O, F_1 turns it anticlockwise and B clockwise.

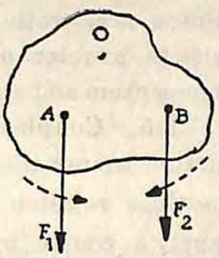


Fig. 2.4

Equilibrium of moments. Consider a body capable of rotation about an axis (Fig. 2.4). Let two forces, F_1 and F_2, act on it in a plane perpendicular to the axis, and let their distances from the axis be a and b respectively. The moments of the forces about the axis will be $F_1 a$ and $F_2 b$. If one produces clockwise motion and the other anticlockwise, there will be no rotation when $F_1 a=F_2 b$.

Two moments (or torques) balance each other when they are equal and opposite. The forces need not be equal.

2-4. Relation between angular momentum and torque.

For simplicity, let us suppose that a particle of mass m is moving with velocity v. The force on the particle is F. v and F have the same direction. O is a point at a perpendicular distance r from the line of action of F or v.

The angular momentum of the particle about O is $L=mvr$. The moment of the force F about O is $M=Fr$. Now $F=mf=mv/t$ or $F.t=mv$.

$$\therefore \quad L=mvr=Ft.r=Mt \text{ or } L/t=M \qquad (2\text{-}4.1)$$

In words, we may say

The rate of change of angular momentum of a particle about a point is equal to the moment of the force (that is, the torque) about the point.

In the case of rotational motion, we may get an important result from Eq. 2-4.1.

$$M=\frac{L}{t}=\frac{mvr}{t}=\frac{mr^2\omega}{t}=mr^2\left(\frac{\omega}{t}\right)=mr^2\alpha \qquad (2\text{-}4.2)$$

where $\alpha=\omega/t=$angular acceleration of m. So, we find that a torque (or the moment of a force) causes angular acceleration.

In linear motion, a force gives rise to linear momentum and linear acceleration. In rotational motion, a moment or torque gives rise to angular momentum and angular acceleration. Both linear momentum and angular momentum are conserved quantities.

2-5. Couples. Two equal, unlike, parallel forces whose lines of action are not the same, form a **couple**. The effect of a couple is to produce rotation. When we turn a door knob or wind a watch, we apply a couple to the knob or the winding key with our fingers. In using a screw-driver, turning a tap or spinning a top we do the same.

The accompanying figure (Fig. 2.5) represents a couple consisting of the forces F and $-F$ acting at the points A and B of a body. The negative sign merely implies that one of the forces is opposite in sense to the other.

The perpendicular distance between the lines of action of the forces is called the *lever arm* or the **arm** of the couple. By the term **moment** (or **torque**) or a couple is meant the algebraic sum of the moments of the component forces forming the couple about any point in the plane of the couple. If O be any such point and OC and OD its perpendicular distances from lines of action of the forces, $F.OC$ and $F.OD$ are the moments of the components about O. We find from the figure that both these moments tend to produce rotation in the same sense. Hence the algebraic sum of the moments is $F(OC+OD)=F.l$. Thus we can say that the **moment or torque of a couple is the product of either force and the perpendicular distance between the forces.** The moment of a couple (*i.e.*, its torque) measures its ability to produce rotation.

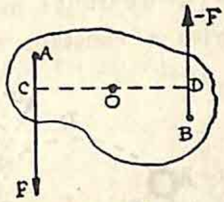


Fig. 2.5

A line perpendicular to the plane in which the forces forming the couple lie, is called the **axis** of the couple. When a couple acts on a freely movable body, its axis of rotation passes through the *centre of gravity* of the body (See Sec. 4-4). The torque is said to be positive when, looking towards the body along the rotation axis, the rotation appears to be anticlockwise. It is negative when the rotation appears clockwise. A torque is a vector whose direction is that of the axis.

We shall use the term **torque** to represent the moment either of a force or of a couple. Either of them can produce a rotation about an axis.

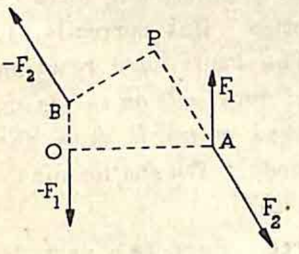


Fig. 2.6

Equilibrium of couples. A force acting singly cannot balance the effect of a couple. *Two couples impressed on a body will be in equilibrium if their moments are equal and opposite.* In Fig. 2.6, let $F_1, -F_1$ and $F_2, -F_2$ be two coplanar couples acting on the same rigid body. The forces F_1, F_2 meet at A, and $-F_1$, $-F_2$ at B. AO is the arm of the first couple and BP that of the second. If $F_1.AO=F_2.BP$, the torques will be equal. If, further, the senses of the torques be opposite, they will produce no net effect on the body.

2-6. Uniform motion in a circle : Centripetal force. The simplest case of rotation is the motion of a particle with uniform angular velocity ω in a circle. It corresponds to the motion of a particle in a straight line with uniform speed (that is, rectilinear motion with uniform velocity).

According to Newton's first law of motion, a moving particle will, due to inertia, continue to move in a straight line unless acted upon by some external force.

If we want a particle to describe a circle, such as $ABCD$ (Fig. 2.7), with a constant speed, we must apply some force f on it to keep it on the circle. At every point of its path the particle tends to move along the tangent to the path, *i.e.*, along AA' at A, along BB' at B, etc. Since the speed is to remain constant, the force to be applied cannot have any component in the direction of motion of the particle, as it will then produce an acceleration in the direction of motion and thus change the speed. The force is therefore everywhere perpendicular to the path, *i.e.*, *it must be directed towards the centre of the circle.*

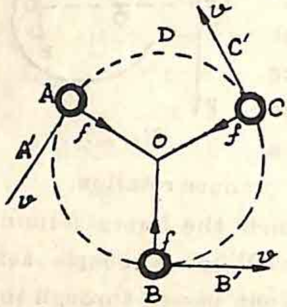


Fig. 2.7

When a body moves in a circle, *the inward force which, acting towards the centre of the circle, keeps the body moving in the circle, is called the* **centripetal force**. The agent which exerts the centripetal force is acted on by the reactional force. This reactional force is called *centrifugal force* in engineering practice. But correctly, it should be called the **centrifugal reaction**. The centrifugal reaction is thus *equal and opposite to the centripetal force, acts on the agent which restrains the moving body, and is directed outwards from the centre of the circle.* It does not act on the body. We shall come to the correct meaning of *centrifugal force* in Sec. 2-7.

2-6.1. Magnitude of the centripetal force. Suppose a particle of mass m moves round in a circle with a constant speed v and is at the point A (Fig. 2.8) at a given moment. Consider a *very short* interval of time t. If there were no force acting towards the centre, the particle would move a distance $AB=vt$ tangentially in time t, which is taken to be very short. But, because of the centripetal force P there will be an acceleration $f=F/m$ normal to the track (*i.e.*, to AB). Due to this

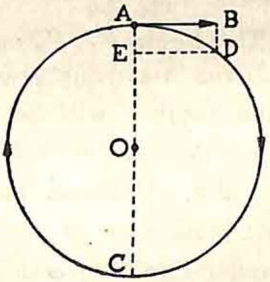


Fig. 2,8

acceleration the particle would move a distance $\frac{1}{2} ft^2$ perpendicular to AB in time t, and be brought to a point D on the circle where BD is parallel to AO. Drawing DE parallel to AB, we have from geometry

$DE^2 = AE.EC$, where AC is the diameter

$= BD.EC = \frac{1}{2} ft^2 \left(2r - \frac{1}{2} ft^2\right) = rft^2$, since, t being very small, $\frac{1}{2} ft^2$ may be neglected in comparison with $2r$.

Thus $DE^2 = AB^2 = v^2 t^2 = rft^2$ or $f = v^2/r$.

$\therefore$ **Centripetal (or central) acceleration = $\mathbf{v^2/r}$** (2-6.1)

and **centripetal force $\mathbf{F = mv^2/r}$.** (2-6.2)

Alternative proof. The moving particle has a velocity v along the local tangent to the circle. Its speed does not change with time, but the direction of motion changes continuously at the uniform rate of 2π radians in T seconds, that is, at the rate $2\pi/T = \omega = v/r$. Acceleration means rate of change of velocity, that is, of both speed and direction. Here speed does not change; only the direction changes at the rate v/r. Therefore the acceleration is $v.v/r = v^2/r$. We have seen earlier that the force, hence the acceleration, is directed towards the centre. Thus the centripetal acceleration is v^2/r and centripetal force is mv^2/r.

If ω is the angular velocity of the particle, T its time period and n its frequency, we have

$$v = \omega r = 2\pi r/T = 2\pi nr$$

When necessary we may substitute any of these values in Eq.s 2-6.1 or 2-6.2.

The above analysis shows that if we want to make a particle of mass m and speed v move round in a circle of radius r, we must apply to it a force $F = mv^2/r$ *perpendicular to* v.

Example. A small body of mass 200 g revolves in a circle on a horizontal frictionless surface, being attached by a string, 20 cm long, to a pin in the surface. Find the tension in the cord if the body executes 90 revolutions per minute.

Solution : Angular velocity $\omega = 2\pi n = 2\pi(90 \div 60)$ rad/s. The tension in the string supplies the centripetal force P.

$\therefore \quad F = m\omega^2 r = 200 \text{ g} \times (3\pi)^2 \text{ s}^{-2} \times 20 \text{ cm}$

$= 9\pi^2 \times 4000 \text{ g cm s}^{-2} = 3{\cdot}55 \times 10^5$ dyn.

Problem 1. A particle is on the top of a smooth sphere of radius r. What is the smallest horizontal velocity that should be given to it so that it may leave the sphere without sliding down it ? [Ans : $v = \sqrt{gr}$]

Problem 2. A body of mass m lies on a horizontal rotating table at a distance r from the rotation axis. The coefficient of friction between the body and the table is μ What is the magnitude of the force of friction if the table is rotating at n rps ? At what angular velocity will the body begin to slide ?

(*Hint* : The force of friction provides the centripetal force. ANS : $\omega=\sqrt{\mu g/r}$.)

2-7. Centrifugal force. The centrifugal reaction is called the centrifugal force by many modern authors, particularly in engineering practice. It is exerted by the body on any thing that makes the body follow a circular path. It is equal and opposite to the centripetal force. Application of the term 'centrifugal force' to mean the 'centrifugal reaction' is a misuse.

The true sense of the term centrifugal force may be represented in the following way. Consider a horizontal table rotating about a vertical axis at a constant angular speed. Imagine, further, an observer situated at the centre of the table and rotating along with it, but unconscious of the motion of the table. He holds in his hand one end of a string, to the other end of which is attached a stone. When the whole system rotates uniformly, the stone appears to him to be at rest. But he is conscious of the pull he exerts on the stone. How is it that the stone on which he exerts a pull does not come nearer to him, but maintains a constant distance ? It then *seems* to him that the stone is acted on by a force equal and opposite to the pull he exerts on it. This force is called the **centrifugal force**. It must not be confused with the centrifugal reaction which the stone exerts on the observer. *To an observer standing at rest outside the table the centrifugal force has no existence. To the observer who has the same motion as that of the table, this force is as real as any other force.* When the centripetal force ceases, the stationary observer finds that the stone flies off tangentially to its path ; but to the rotating observer it appears to move radially outwards because of the centrifugal force.

Definition. The centrifugal force may then be defined as a force equal and opposite to the centripetal force, acting radially outwards on a body rotating in a circle and existing only *relatively* to an observer who has the same rotational motion as that of the body. Centripetal force is due to interaction between two bodies. But centrifugal force is not due to any interaction. It may therefore be called a 'pseudo' (false) force or 'fictitious' force. The centrifugal force is due to the rotation of frame of reference from which observation is made.*

The use of the term centrifugal force has thus been made confusing. *A beginner will do well not to use the term, and treat problems of motion in a circle on the basis of the centripetal force acting on the body.*

2-8. Some examples of centripetal force and its reaction.

(*i*) A motor car cannot of itself change the direction of its motion.

When it takes a bend it moves in a circle. To make it so move, it requires an external force to be applied towards the centre of the circle. On a level road this force can be supplied only through friction between the wheels and the road. The required force is mv^2/r. But the friction which is to supply this force is small and of uncertain magnitude. So the car should be driven slowly; otherwise it may skid in its tendency to continue in a straight line. Skidding occurs when v is too large and r too small.

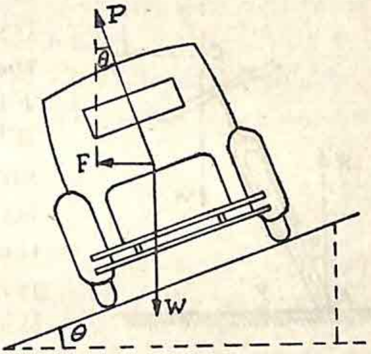


Fig. 2.9

To remove this difficulty the road bed is inclined to the horizontal at the bends, the slope being towards the centre of the bend.

Fig, 2.9 shows a moving car on such an inclined road bed. The weight W of the car acts vertically downward at its centre of gravity. The normal reaction P of the road on the car is perpendicular to the bed, friction being left out of account. The vertical component of P, i.e., $P \cos \theta$, balances the weight, while the horizontal component of P, i.e., $P \sin \theta$, supplies the necessary centripetal force. Thus we have

$$P \sin \theta = mv^2/r, \text{ and } P \cos \theta = W = mg, \quad \text{whence } \tan \theta = v^2/gr. \tag{2-8.1}$$

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* Newton's second law of motion $P = mf$ holds in a frame of reference which is either at rest or moving with a uniform velocity. It does not hold in the given form when the reference frame is rotating. To write the law of motion in the Newtonian form when the reference frame is rotating, it is necessary to add other forces to the applied force. *Centrifugal force* is such a force. *Coriolis force* is another. These added forces are called pseudo-forces or fictitious forces as they are not due to interaction between bodies. Ordinary forces with which we are familiar, such as a push or a pull or an attraction or repulsion, are due to interaction between bodies.

The above equation gives the proper bending angle for any assumed speed v and radius of curvature r of the road.

(ii) The case of a cycle or a railway train may be similarly treated. While rounding a curve each of them moves in a circle, so that necessary centripetal force must be supplied. The cyclist leans towards the centre of the circle (Fig. 2.10) in which he wants to move and derives his centripetal force from the horizontal component of the reaction P (=CS) of the ground. P is made up of a vertically upward component R (=CR=AR) equal and opposite to W, the weight, and a horizontal component F (=CD=AF) due to friction. $P^2 = W^2 + F^2$. The maximum value of F is μW, where μ is the coefficient of friction between the road and the tyres. Hence the maximum available centripetal force is μW. If mv^2/r exceeds μW the cycle skids. If θ is the inclination of the cyclist to the *vertical*, $P \cos \theta = W$ and $P \sin \theta = F = mv^2/r$. For larger speeds and smaller radii θ must increase. The maximum inclination α which the cyclist can have without skidding is given by $\tan \alpha = \mu W/W = \mu$.

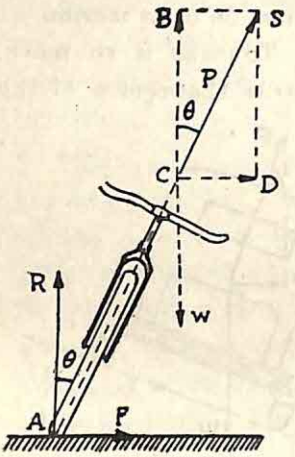

Fig. 2.10

Problems. (1) A cyclist takes a bend of 20 ft radius at 10 mph. At what angle must he lean, and what must be the smallest coefficient of friction between the tyre and the road ? [ANS: 18°35′ ; 0·336]

(2) A cyclist moving at 8 mph wants to turn about. What is the radius of the smallest circle he can take if the coefficient of friction between the road and the tyres is 0·4 ? [ANS: 10.8 ft]

(iii) In the case of a railway train, the rail on the side of the circle away from the centre is raised. The forces are as for a motor car on an inclined road bed. When the track is tilted to an angle $\tan \theta = v^2/gr$, the stress between flange and rail vanishes. At other speeds either flange must press against the corresponding rail.

(iv) When a car suddenly rounds a curve a passenger seated in the car *feels* a force pushing him radially outwards. This subjective feeling of a force directed outwards is the centrifugal force in its original sense. It exists relative to one who shares the motion

of the car. *To a stationary observer outside the car* the passenger tends to move tangentially to the track due to inertia, while the car moves in a circle. This reduces the distance between the passenger and the outer side of his seat, which soon reaches him, and presses him inwards, thus supplying the necessary centripetal force to make him move in the required circle. Some prefer to call the radial thrust exerted by the passenger on the body of the car as centrifugal force.

(*v*) In a grinding machine specks appear to fly off tangentially to the wheel. They do so due to inertia, as the force holding them to the wheel is not strong enough to supply the necessary centripetal force. The flight of particles of mud and water from the wheels of a running car on a wet road is due to the same cause.

(*vi*) A flywheel must not be driven too fast as it may burst. This will happen to the earth if it rotated about 17 times faster around its axis.

(*vii*) Planets move in approximately circular paths around the sun, and so do the satellites around their planets. In these cases the centripetal force is supplied by the gravitational attraction of the sun for the planet or of the planet for its satellite, as the case may be.

Exercises

1. Explain the terms *angular displacement*, *angular velocity*, *angular acceleration*, *periodic time* and *frequency* in angular motion.

For motion in a circle, how are the first three related respectively to the linear displacement, linear velocity and linear acceleration ?

2. What is meant by *angular momentum* of a particle about a point ? Find how it is related to the *moment of the force* acting. What is the importance of of the moment of a force ?

3. What do you understand by the term *torque* ? What is the *moment of a force* about a point ? What is the importance of a torque in angular motion ?

4. Explain the terms *couple*, *moment of a couple*, *arm of a couple*. Give examples of couples. How can a couple destroy the action of another ? What is meant by the *axis* of a couple ?

5. What kind of force is necessary to make a particle move in a circle with a constant angular velocity ? What is the name of this force ? Find its magnitude and direction.

6. Distinguish between *centripetal* and *centrifugal* force. Why do we call the former a true force, and the latter a 'pseudo' or 'fictitious' force

Distinguish between a centrifugal force and a centrifugal reaction, giving examples of each.

7. Prove that a constant force directed towards the centre of a circle is required to make a particle move with a constant speed in the circle.

Calculate the magnitudes of the centripetal force and the centripetal acceleration.

How will the particle move when the centripetal force ceases to act ?

8. (a) A stone is tied to a string 1 m long and made to move in a harizontal circle with a speed of 3 m/s. What is the accelaration of the stone ? If the string were inclined to the vertical at an angle of 30° and the speed of the stone were the same as before, what will be the aceeleration ? If the stone weighed 250 g, what are the values of the centripetal force in the two cases ? [Ans : 900 cm/s^2 ; 1800 cm/s^2, $2{\cdot}25 \times 10^5$ dyn ; $4{\cdot}5 \times 10^5$ dyn.]

(b) A string snaps under a tension of 10^7 dyn. A stone weighing 500 g is tied to it and made to revolve in a horizontal circle of radius 50 cm. At what frequency of rotation will the string snap ? [Ans : $10/\pi$ per second].

9. Explain how a cyclist gets the necessary centripetal force when taking a bend on a level road.

If the speed of the cycle is 4 m/s and its coefficient of friction with the road is 0·3, what will be the shortest diameter of the circle the cyclist can take ? ($g = 980$ cm/s^2). What will be his angle of tilt ? [Ans : 10·9 m ; 16°42′].

10. (a) Why is a road made sloping at a sharp bend ?

(b) In the case of a curved railway track, the line away from the centre of the curve is slightly raised. What advantage does it give ?

11. A motor car moving with a speed of 20 m/s takes a bend of 30 m radius. If the driver weighs 72 kg, what is the centrifugal force acting on him ? How many times is this force greater than his weight ? [Ans : $9\ 6 \times 10^7$ dynes ; about 1·4 times.]

12. What should be the angular speed of diurnal rotation of the earth if a person on the equator is to have no weight ? How long will a day be then in hours ? (Earth's radius=6400 km.) [Hint : The pull of the earth on the person should be fully spent in supplying the necessary centripetal force. The man moves in a circle equal to the radius of the earth. Therefore, acceleration due to gravity will be equal to the centripetal acceleration, that is, $g = v^2/r = w^2 r$. $w = 1{\cdot}24 \times 10^{-3}$ rad/s ; Duration of day about 1·4 hours.]

CHAPTER

3 STATICS

3-1. Introduction. Statics deals with particles and bodies which are at rest in equilibrium. By the term *system of particles* we mean a collection of particles which we treat as being isolated from the rest of the universe. A *rigid body* is a collection of an infinite number of particles ; it has the special property that the distance between any two of its particles does not change when forces are applied to it. All bodies are treated as rigid unless we say otherwise.

A particle (or body) is said to be *at rest* when it has *no velocity* (relative to its surroundings). It is said to be *in equilibrium* when it has *no acceleration.*

Motion may be one of *translation* (that is, linear motion), or of *rotation.* A body at rest in equilibrium must have neither translational nor rotational motion. An unbalanced force produces translational motion. An unbalanced torque produces rotational motion.

For simplicity, we shall, at this stage, consider all acting forces and couples to be confined to a single plane.

3-2. Conditions of equilibrium. A body which has no acceleration is said to be *in equilibrium.* It may be at rest or in uniform motion in a straight line. The concept of equilibrium is very important in physics.

In order that a body, subjected to the action of more than one force, may remain in equilibrium *two conditions* must be satisfied. These are the following :—

Condition 1. The vector sum of all the forces acting on the body must be zero. (This prevents linear acceleration).

Condition 2. The torques acting upon the body shall be balanced, the clockwise torques being equal to the anticlockwise torques. (This prevents rotation.) In other words, we may take any point on the body and calculate the moments of all the forces about the point. The algebraic sum of the moments must vanish.

In applying the first condition to the solution of problems, it is often convenient to resolve the forces into two components in mutually perpendicular directions, say the horizontal and the vertical (See Sec. 1-8). Representing such components by the symbols F_x and F_y, we shall have, for equilibrium,

$$\Sigma F_x=0 \text{ and } \Sigma F_y=0. \qquad (3\text{-}2.1)$$

The above conditions lead to the following results :—

(*i*) When a body is in equilibrium under **two forces**, they must be equal, opposite and collinear (that is, act along the same line).

(*ii*) For a body to be in equilibrium under **three nonparallel forces**, *the resultant of any two of the forces must be equal and opposite to the third.* Since the resultant of two forces passes through the point of intersection of the lines of action of its components and lies in the same plane with them, all the three forces must pass through the same point and lie in the same plane, *i.e.*, they must be *concurrent* and *coplanar*.

Triangle of forces rule. It follows from (*ii*) above that *three non-parallel forces in equilibrium can be represented in magnitude and direction by three sides of a triangle taken in order.* This is known as the *triangle of forces rule.*

Example. A body weighing 12 kg is suspended vertically by a string. Find the tension in the string when the body is pulled to one side by a horizontal force of 5 kg. (See Fig. 3.1.)

Solution : The body is in equilibrium under the action of three forces, namely, (*i*) its weight of 12 kg acting vertically downward, (*ii*) the horizontal force of 5 kg, and (*iii*) the tension T in the string. Since the body is in equilibrium, the resultant of any two forces acting on it must be equal and opposite to the third force. Hence T is equal and opposite to the resultant of the perpendicular forces (*i*) and (*ii*). Its magnitude is $\sqrt{12^2+5^2}=13$ kg (kg, wt).

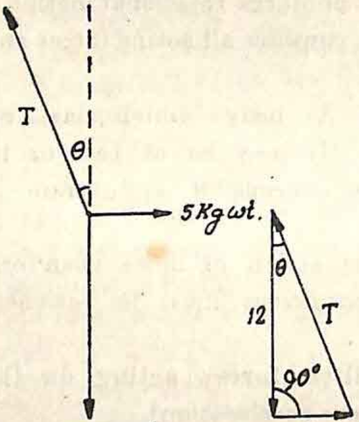

Fig. 3.1

Lami's theorem. It is a property of triangles that the ratio of any two sides is equal to the ratio of the sines of their opposite angles. Let F_1, F_2, F_3, be three forces in equilibrium forming a triangle, and let θ_1, θ_2, θ_3 be respectively the angles opposite F_1, F_2 and F_3.

Then $\dfrac{F_1}{F_2}=\dfrac{\sin\theta_1}{\sin\theta_2}$ and $\dfrac{F_2}{F_3}=\dfrac{\sin\theta_2}{\sin\theta_3}$. It follows,

therefore, that $$\frac{F_1}{\sin\theta_1}=\frac{F_2}{\sin\theta_2}=\frac{F_3}{\sin\theta_3} \qquad (3\text{-}2.2)$$

This is the mathematical form of Lami's theorem, which states that *when a body is held in equilibrium by three forces, each force is proportional to the sine of the angle between the other two.*

To summarise : For a body to be in equilibrium under *three non-parallel forces*,

(*a*) the forces must lie in a plane, *i.e.*, be coplanar ;

(*b*) the forces must meet at a point, *i.e.*, be concurrent ;

(*c*) the force polygon must be a closed triangle, *i.e.*, the forces must satisfy Lami's theorem.

3-2.1. Principle of moments. Condition 2 of equilibrium stated above is also known as the *principle of moments*. This may be stated as follows :

When a body is in equilibrium, the sum of the anticlockwise moments about any point is equal to the sum of the clockwise moments.

It therefore follows that **when a number of parallel forces are in equilibrium,**

(i) the sum of the forces in one direction is equal to the sum of the forces in the opposite direction ;

(ii) the sum of the anticlockwise moments about any point is equal to the sum of the clockwise moments about the point.

This result is applied below for the calculation of the resultant of two parallel forces.

3-2.2. A special application of the conditions of equilibrium : Resultant of parallel forces. When the forces to be added are parallel, the parallelogram law of addition cannot be directly applied. We may however use the general conditions of equilibrium to get the resultant. A force equal and opposite to the resultant of two forces is called the **equilibrant**. The given forces together with the equilibrant form a system in equilibrium.

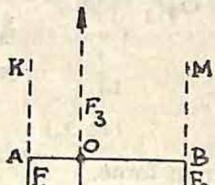

Fig 3.2

A. Two like parallel forces. In Fig. 3.2 let the forces F_1 and F_2 act along the parallel lines *KL* and *MN* in the same sense. Let the line *AB*, perpendicular both to *KL* and *MN*, intersect them at *A* and *B* respectively. Suppose these two forces along with a third force F_3 acting at *O* keep a body in equilibrium. Then F_3 is the equilibrant of F_1 and F_2, and is equal and opposite to their resultant *R*.

Since the forces F_1, F_2 and F_3 are in equilibrium, the sum of their components along AB must be equal to zero. F_1 and F_2 have no component in this direction. Therefore F_3 also will have no component along AB and will be perpendicular to AB. Further, since the algebraic sum of the components perpendicular to AB is zero, we shall have

$$F_1+F_2=F_3 \qquad (3\text{-}3.1)$$

Applying the second condition of equilibrium and taking moments about O, we have

$$F_1 \times OA - F_2 \times OB + F_3 \times 0 = 0 \text{ or } F_1 \times OA = F_2 \times OB$$

or

$$OA/OB = F_2/F_1 \qquad (3\text{-}3.2)$$

Thus *the point O divides the line AB internally in the inverse ratio of the forces.*

The resultant R of the forces F_1 and F_2 is equal and opposite to F_3. Hence its magnitude is $R=F_1+F_2$. It acts through O parallel to the forces, where O is a point dividing the distance between F_1 and F_2 internally into two parts inversely proportional to the forces.

B. Two unlike parallel forces. When the parallel forces F_1 and F_2 act in opposite senses (Fig. 3.3) an application of the above method shows that the resultant R

(*a*) has a magnitude equal to the difference of the forces,

(*b*) acts in the same sense as that of the larger force and

(*c*) the line of action of the resultant divides the distance between F_1 and F_2 *externally* in the inverse ratio of the respective forces, R lying nearer to the larger force.

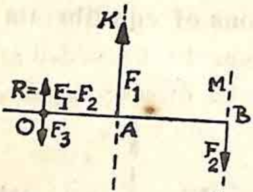

Fig. 3.3

C. Any number of parallel forces. The same principle may be applied to find the resultant of any number of parallel forces. The system of forces along with its equilibrant keeps a body in equilibrium. Therefore, the given forces along with their equilibrant satisfy the conditions $\Sigma F=0$ (vector sum of forces equals zero) and

$\Sigma T=0$ (anticlockwise torques equal clockwise torques).

From these the magnitude, direction and line of action of the equilibrant may be found out. The resultant is equal and opposite to the equilibrant.

Examples. (1) Two weights 10 g and 20 g hang vertically from the ends of a rigid horizontal bar of length 12 cm and negligible weight. Find where and how the rod must be supported so as to be in equilibrium.

Solution: The student is advised to draw his own diagram. Let P be the force producing equilibrium. Let it act at a distance x from the 10 g weight.

Since $\Sigma F = 0$ we must have $10+20+P=0$ or $P=-30$ g.

The negative sign means that P acts in a sense opposite to that of the weights.

Taking moments about the end of the bar where the 10 g weight hangs, we have, since $\Sigma T=0$,

$$10\times 0-30x+20\times 12=0 \quad \text{or} \quad x=8.$$

$\therefore$ P acts at a distance $x=8$ cm from the 10 g end of the bar.

(2) Two persons carry a load of 100 kg suspended from a rigid bar of length 5 m and of negligible weight, at a distance of 3 m from one end. Calculate the thrust on each.

Solution: Let F_1 be the thrust at the end of the bar more remote from the load and F_2 that at the nearer end. Then the resultant of F_1 and F_2 is the 100 kg force. The equilibrant of F_1 and F_2 is equal and opposite to the load.

$\therefore$ From $\Sigma F=0$, we have $F_1+F_2-100=0$ or $F_1+F_2=100$.

Further, taking moments about the point of application of the load,

$$F_1\times 3-F_2\times 2+100\times 0=0 \quad \text{or} \quad 3F_1=2F_2$$

Solving for F_1 and F_2 we have $F_1=40$ kg and $F_2=60$ kg.

3·3. Centre of gravity. Any body whatsoever may be considered to be made up of a large number of particles, each of finite mass. Each particle is attracted towards the centre of the earth by the force of gravity which is proportional to the mass of the particle. Owing to the large radius of the earth the forces of gravity on the particles may be taken to be parallel. The weight of a body thus consists of a system of parallel forces acting upon the individual particles which make up the body. The resultant of this system of parallel forces always passes through a point, fixed relative to the body, whatever the position of the body. This point is called its *centre of gravity* (abbreviated *c.g.*) of the body. The weight of a body, although actually a system of parallel forces acting upon all parts of the body, can be correctly represented by a single force acting downward at the centre of gravity.

Definition. The point, fixed with respect to a given body, through which the resultant force of gravity on it acts, no matter how the body is oriented, is called the **centre of gravity** of the body.

It follows from the definition of the *c.g.* that *when a body is supported at its centre of gravity, it will have no tendency to turn* *under the action of gravity alone.* The position of the centre of gravity is independent of the position of the body.

Centre of gravity of bodies of simple shapes can often be located by inspection. For a thin uniform rod the *c.g.* is at the centre of the rod. The turning moment about the centre due to the weight of any particle is balanced by that of a particle similarly placed on the other side of the centre. Hence the total turning moment of the rod about its centre is zero. The centre of the rod is therefore its centre of gravity.

The centre of gravity of some uniform, homogeneous bodies having simple geometrical shapes are given below :

Body	*Position of centre of gravity*
Circular lamina	Centre of circle.
Annular disc	Centre of the annulus.
Triangular lamina	Point of intersection of the medians.
Rectangular lamina	Point of intersection of the diagonals.
Sphere	Geometrical centre.
Spherical shell	Geometrical centre.
Cylinder	Midpoint of the axis.
Cone	On the axis at a distance $= \frac{1}{4} \times$ height above the base.

3·3.1. Stable, unstable and neutral equilibrium.

The behaviour of a body when slightly displaced from its position of rest, depends upon the position of its *c.g.*

A body is said to be in **stable equilibrium** if, when slightly displaced, it tends to fall back to its original position. *Equilibrium*

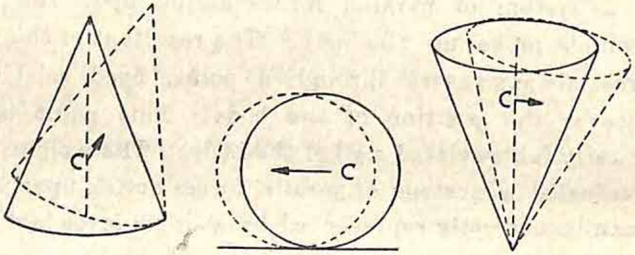

Fig. 3.4

is stable when the displacement raises the c.g. A cone standing on its base (Fig. 3.4), a cube resting on a face, a weight suspended by a string, a ball lying in a spherical cup (Fig. 3.5), a chair or table standing on its legs etc. are examples of stable equilibrium.

A body is said to be in **unstable equilibrium** if, when slightly displaced, it tends towards further displacement. *Equilibrium is unstable when the displacement lowers the c.g.* A chair balanced on two legs, a cube balanced on an edge, a cone balanced on its vertex (Fig. 3.4), an egg balanced with its long axis vertical, a ball on the top of a sphere (Fig. 3.5) etc., are examples of unstable equilibrium.

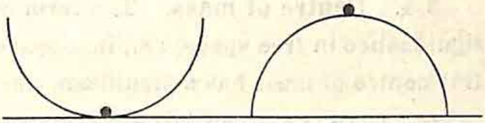

Fig. 3.5

A body is said to be in **neutral equilibrium** if, when slightly displaced, it remains in its new position. *Equilibrium is neutral when the displacement neither raises nor lowers the c.g.* A sphere resting on a horizontal surface (Fig. 3.4), a pencil, a cylinder or a cone resting on its side etc., are examples.

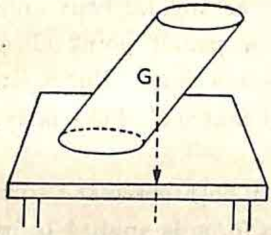

Fig. 3.6

3-3.2. Toppling of a body. A body will topple unless the vertical line through its *c.g.* passes through the base on which it is supported. The cylinder in Fig. 3.6 cannot be kept standing on an end. Try piling up bricks one above the other, but slightly displaced to one side. Soon the pile will topple. This occurs when the vertical line through the *c.g.* of the pile moves outside the base offered by the lowest brick. Place a cylinder with its axis vertical on a board, and gradually tilt the board. The cylinder will topple as soon as the vertical line through its *c g.* moves out of the base (Fig. 3.7).

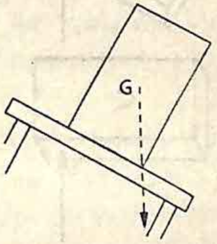

Fig. 3.7

The stability of a body depends on how far it may be tilted without toppling. High stability is obtained by keeping the *c.g.* low and making the base large. This is an important point to remember in the design of structures.

Extra passengers should not be allowed on the upper deck of double-decker buses. If they are allowed, the *c.g.* will rise, and the risk of the bus toppling over while taking a bend will increase. Racing cars are built low for the same reason.

3-4. Centre of mass. The term centre of gravity will have no significance in free space, *i.e.*, in a space where no force of gravity acts. But centre of mass has a significance under all circumstances.

If a body is thrown spinning into the air it will rotate smoothly about its centre of gravity. Out in free space it will spin about the same point. *The two points coincide when g is the same all over the body.* Physically, we may say that the **centre of mass** of a rigid body is the point in it which is such that if the line of application of a force passes through this point, it will produce only translational motion of the body, but cause no rotation.* The existence of such a point may be easily tested. Place a smooth rectangular body on a smooth horizontal table and apply a push with a pencil point (Fig. 3.8). In general it will move and rotate at the same time. But when the line of action passes through the geometrical centre C of the body, it will move without rotation.

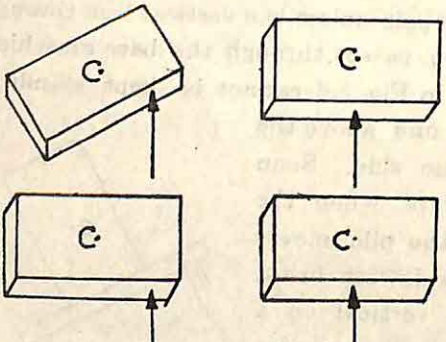

Fig. 3.8

It is easy to understand how a rigid body may both move and turn when a force is applied to it. Fig 3.9 shows a body to which a force F is applied. M is its centre of mass. Consider two equal and opposite forces P_1 and P_2, each equal to F, to be applied at M. These two forces do not affect the motion of the body in any way; the three forces are equal to the single force F. F and P_2 form a couple which causes the body to rotate about an axis through M, P_1 acting at the centre of mass causes translation. If m is the mass of the body, its linear acceleration in the direction of F is $P_1/m = F/m$.

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*This statement may serve as the definition of centre of mass for a beginner. A rigorous definition is given on different lines as follows : The centre of mass of a body (or system of particles) is that point the vector sum of the mass moments with respect to which vanishes. (If r_i is the vector distance of the i th particle of mass m_i from the centre of mass, then $\Sigma m_i r_i = 0$ the summation extending over all the particles. m_i r_i is the vector mass moment of the particle.)

In all problems of translational motion of a rigid body, the entire mass of the body may be taken as concentrated at its centre of mass.

When a diver jumps from a spring board his centre of mass describes a parabolic path regardless of how he may twist and turn. In astronomy it is often necessary to locate the centre of mass of two bodies considered as a single unit or system. The centre of mass lies along the line joining the two and divides the line in the inverse ratio of the masses. Binary stars rotate about their centre of mass, while this point hurtles through space. So also in the case of the solar system.

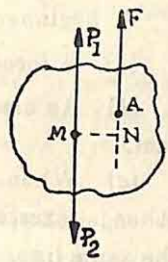

Fig 3.9

Difference between Centre of Gravity and Centre of Mass.

C. G.	C. M.
1. It is the point through which the resultant force of gravity passes, whatever the position of the body.	1. It is such a point in the body that if a force acts through this point, it causes only translation but no rotation of the body.
2. If g does not vary over the body, the two points are the same.	
3. In the absence of any force of gravity, *c.g.* has no meaning.	3. Has its own meaning under all circumstances.

3·4 1. Procedure in problem work involving forces. In solving problems involving the action of more than one force on a body or a system of bodies, much difficulty may be avoided by following the procedure suggested below :

(*i*) Select one of the bodies for consideration, usually the one whose rest or motion is to be discussed. Consider this body to be isolated from the rest of the system by an imaginary closed surface. Mark all forces which act on this body from *outside the surface.*

(*ii*) Construct a 'force diagram' in which the selected body is represented by a point, and represent by suitable vectors all of the forces acting *on* this body from outside the imaginary surface. Be careful not to omit any force, but *never include any force which this* *body exerts on other bodies*. If any force is unknown, represent it by a vector, but mark it as unknown.

(*iii*) From the force diagram find the resultant of the forces. If it is zero, the body is at rest. If, otherwise, this is the unbalanced force which gives the body an acceleration in its own line of action.

A beginner may avoid much confusion if he remembers that

(*a*) A force like a push or a pull is exerted by contact.

(*b*) An electric, magnetic or gravitational force acts without contact.

(*c*) When two bodies are in contact, the reaction of one on the other is exerted at the point of contact. Action and reaction act in the same line.

(*d*) In a flexible cord stretched between two points the tension is the same throughout. The tension acts on the body at the end of the cord, and exerts a pull on the body along the cord.

Exercises

1. Under what condition will a number of forces acting on a body fail to produce an acceleration in it?

Under what condition will a number of forces acting on a body fail to produce a rotation of the body?

What is the difference between a body being *at rest* and being *in equilibrium*?

State the conditions of equilibrium of a body.

2. Find the relation between three coplanar forces which, acting on a particle, keep it in equilibrium.

3. Distinguish between *centre of gravity* and *centre of mass* of a body. What characteristic properties do these two points have? When are these two points the same? When is it not proper to speak of a centre of gravity?

4. Do being 'at rest' and being 'in equilibrium' mean the same thing? A parachute is falling with a constant velocity. Is it in equilibrium?

5. Find the conditions of equilibrium of three coplanar parallel forces acting on a rigid body. When will three concurrent forces (that is, passing through the same point) produce equilibrium?

6. Forces of 30, 20, 60 and 60 gram-weights act on a particle in directions south, east, north and west respectively. Find the magnitude and direction of their resultant. What is meant by the 'equilibrant' of these forces ? [ANS : 50 g-wt inclined at an angle θ with the north toward the west, with tan $\theta = 4/3$.]

7. A body weighing 500 g is suspended from the middle of a straight horizontal string 200 cm long. As a result the centre of the string is depressed by 25 cm. What is the tension in the string with the load on ? [ANS : About 2060 g-wt].

8. A 3 metre long plank is supported horizontally at the ends by two strings. A man weighing 72 kg sits on the plank at a distance of 1 metre from one end. What are the tensions in the strings ? Ignore the weight of the plank. [ANS: 48 kg-wt in the nearer string, and 24 kg-wt in the other.]

9. A lever 2 m long and weighing 4 kg has its fulcrum 50 cm from one end. If a weight of 16 kg is suspended from the end nearer the fulcrum, what force at the other end will be required to keep the lever in equilibrium ? [*Hint* : The clockwise torques of the forces about the fulcrum will be equal to the anticlockwise torques about the same point. The weight of the rod acts at its midpoint.] [ANS : A force of 4 kg-wt.]

10. A man buys 3 kg of rice. To check that the weight is correct he collects a metre scale and a spring balance reading up to 1 kg. How can he check the weight of the rice with this equipment ? [*Hint* : Use the scale as a lever with fulcrum at 25 cm. Suspend the rice from the nearer end and apply a force of 1 kg with the spring balance at the other end.]

CHAPTER

4

WORK, POWER AND ENERGY

4-1. Work. In our everyday language the term 'work' means some kind of labour, physical or mental, producing a result. In the language of mechanics, or science in general, the term is used in a special sense. *Work is said to be done when the point of application of a force moves in the direction of the force.*

Work, so defined, involves two quantities, *viz.*, (i) a force and (ii) a distance in the direction of the force. When a man pushes a cart or lifts a load from the floor, he not only exerts a force, but exerts it through a distance in the direction of the force. Under these conditions the man is said to do work. When, however, he supports a bucketful of water in his hand without moving it, he exerts a force but does not exert it through any distance in the direction of the force. Whatever muscular fatigue he may feel in the act of holding the bucket, he does *no work* in the technical sense of the term.

The amount of work (W) done is measured by the product of the force (F) and the distance (s) through which the point of application of the force moves in the direction of the force. In symbols,

$$W = Fs \tag{4-1.1}$$

or **Work = Force × Distance.**

When the directions of the force and displacement are not the same (Fig 4.1) the component of the force in the direction (x) of displacement is $F \cos \theta$, where θ is the angle between the two. Since $F \cos \theta$ is the *effective part* of the force in the direction of displacement, the work done is given by

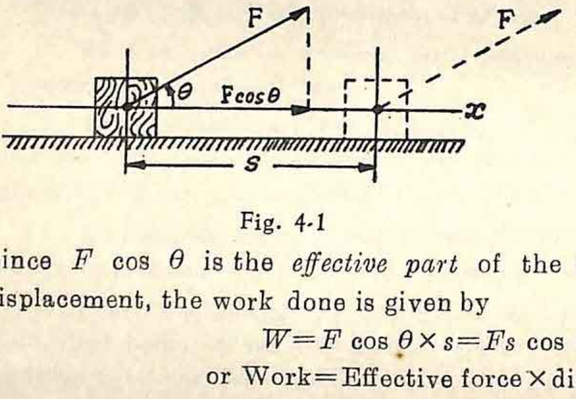

Fig. 4-1

$$W = F \cos \theta \times s = Fs \cos \theta \tag{4-1.2}$$

or Work = Effective force × distance

Alternatively, we may resolve s into components $s \cos \theta$ and $s \sin \theta$ in the direction of F and perpendicular thereto. The displacement of F in its own direction is $s \cos \theta$; hence the work done by it is $Fs \cos \theta$. For the displacement $s \sin \theta$ no work is done by F.

You get an example of application of Eq. 4-1.2 when a body moves down an inclined plane under gravity. The force of gravity and the motion it produces are not in the same direction.

Work done 'by', 'on' or 'against'. When a body A exerts a force F on another body B making it move through a distance in the direction of F, we say

(i) A (the agent which applies the force) does work *on* B.

(ii) Work is done *by* A or the force F *on* B.

(iii) When an agent moves a body against and opposing force, work is said to be done *against* the opposing force. Thus when a man lifts a load from the floor, he does work *on* the load *against* gravity. When a body falls under gravity, work is done *by* gravity.

Whenever work is done, it is done *against a resistance.* In lifting a weight, work is done against gravity. In pulling a load over a rough surface, work is done against friction. When an unbalanced force causes a body to move with acceleration it does work against the force of inertia.

4-1.1. Work done in rotation.

Let a force F (Fig. 4.2) turn a body about an axis while acting at a distance r from the axis. We may imagine a rope to be wound round the body, and the force applied at one end of the rope. If the body is turned through θ radians by the action of the force, the point of application of the force moves through a distance $s = r\theta$, the length of the rope unwound. Hence the work done is

$$W = Fs = Fr\theta \qquad (4\text{-}1.3)$$

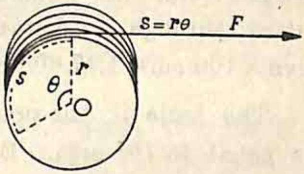

Fig. 4 2

But Fr is the moment of the force about the axis of rotation, which is also called its *torque.* θ is the angular displacement (in radians). Denoting torque by the symbol T, we may write

$$\text{Work} = \text{torque} \times \text{angular displacement} = T\theta \qquad (4\text{-}1.4)$$

4-2. Units of work.

The unit of work involves the units of force and distance. Force may be expressed in absolute or gravitational units (see Sec. 1-10.4). Besides, there are the metric and the British systems. All these lead to a multitude of units of work. The more important of these units are listed below.

Nature of unit	*Unit of force*	*Unit of distance*	*Corresponding unit of work*
MKS absolute (SI)	newton	metre	1 newton × 1 metre = 1 newton-metre or 1 **joule** (symbol **J**)
,, gravitational	kg-wt or kg-f	metre	1 kg-wt × 1 metre = 1 kg-m
CGS absolute	dyne	cm	1 dyne × 1 cm = **1 erg**
,, gravitational	gm-wt or gf	cm	1 gm-wt × 1 cm = **1 gram-centimetre** (g-cm)
FPS absolute	poundal	ft	1 poundal × 1 ft = **1 ft-poundal**
,, gravitational	lb-wt or lbf	ft	1 lb-wt × 1 ft = **1 foot-pound** (ft. lb)

A definition in words may be easily provided for each unit of work. Illustrating with the erg, we may say that the *erg* is the work done when the point of application of a force of one *dyne* moves through a distance of one *centimetre* in the direction of the force. For other units appropriate words may be substituted in the above statement.

The **joule**: The erg is a very small unit. The Indian Paisa weighs 1·5 g. Its weight is, therefore, 1·5 × 980 = 1470 dyn. When lifted through a distance of one metre the work done = 1470 dyn × 100 cm = 1,47,000 erg.

The joule is the practical unit of work in the cgs system, and is equal to 10^7 ergs. But in the mks system or in SI units it is the absolute unit of work.

$$\textbf{1 joule} = \mathbf{10^7}\ \textbf{ergs}.$$

It is not difficult to understand the meanings of such terms as 'a kilogram-metre' or 'a ton-foot'. The former is the work done when a weight of one kilogram is lifted through one metre against gravity. When a mass m is moved vertically through a distance h, the force of gravity on it is mg and the displacement in the direction of the force, h. Hence the work $W = mgh$. This is done by gravity when the body falls, but against gravity when the body is lifted. mgh will be in absolute units, and mh in gravitational units.

4·2 1. Conversion of units. We have seen in Sec. 1·10.4 that the newton may be replaced by its equivalent kg m/s^2, the dyne by g cm/s^2 and the poundal by lb ft/s^2. In converting a unit of work from one system (of units) to another, we shall find this very useful. A few examples are given below.

$$1 \text{ joule} = 1 \text{ newton} \times 1 \text{ metre} = 1 \text{ kg m/s}^2 \times 1\text{m} = 1 \text{ kg m}^2/\text{s}^2$$
$$= 1000 \text{ g} \times (100 \text{ cm})^2/\text{s}^2 = 10^7 \text{ g cm}^2/\text{s}^2 = 10^7 \text{ erg}.$$

The equivalent of the joule is kg m^2/s^2 ; and of the erg, the equivalent is g cm^2/s^2. The foot-poundal$=$1 poundal $\times$ 1 ft $=$ 1 lb. ft/s^2 $\times$ 1 ft $=$ 1 lb ft^2/s^2. The absolute units of work in any system of units is thus formed by the quantity (unit mass)$\times$(unit length)$^2 \div$ (unit time)^2.

We may now try some conversions, remembering that if the quantities are not in absolute units, they must first be so converted.

To convert the ft. lb into joules.

$$1 \text{ ft. lb} = 1 \text{ lb-wt} \times 1 \text{ ft} = 32{\cdot}2 \text{ poundals} \times 1 \text{ ft}$$
$$= 32{\cdot}2 \text{ lb ft}^2/\text{s}^2 = x \text{ kg m}^2/\text{s}^2 \text{ (say)}.$$

$$\text{Then } x = 32{\cdot}2 \frac{\text{lb}}{\text{kg}} \times \left(\frac{\text{ft}}{\text{m}}\right)^2 = 32{\cdot}2 \times 0{\cdot}4536 \times (0{\cdot}3048)^2 = 1{\cdot}356 \text{ J}.$$

[N.B. This also shows how the unit symbols are treated as algebraic quantities. (Sec. I-6)]

Examples. (1) A man weighing 150 lb ascends a flight of 36 steps each 8 inches high. What is the work he does against gravity ?

Solution : Work done$=150$ lb-wt $\times \frac{36 \times 8}{12}$ ft $= 3600$ ft. lb.

(2) A force of 1 megadyne ($= 10^6$ dynes) acts on a body weighing 1 kg. Find in joules the work done in 2 seconds.

Solution : To calculate the work done we require the distance over which the force acts.

The acceleration of the mass$=\frac{\text{force}}{\text{mass}} = \frac{10^6 \text{ dyn}}{1000 \text{ g}} = 10^3$ cm/s^2

∴ The distance traversed in 2 s $= \frac{1}{2} ft^2 = \frac{1}{2} \times 10^3 \times 4$ cm.

∴ The work done$=$force $\times$ distance$=10^6$ dyn $\times 2 \times 10^3$ cm.

$= 2 \times 10^9$ erg$=200$ J.

4·3. Power. When work is done, its amount is not the only item of importance. The time in which the work is done is also of great importance. Suppose a 100 gallon tank is to be filled by drawing water from a well, 50 ft deep. 100 gallons of water weigh 1000 lb. Hence the work to be done in drawing it from the well will be $1000 \times 50 = 50{,}000$ ft. lb. If the work is done by manual labour a stronger man will be able to do it more quickly than a weaker one. The former would then have done more *work per unit time* than the latter. If a motor-driven pump were employed, a more powerful motor would complete the job in a shorter time than a less powerful motor. The former does more work per unit time than the latter.

The time rate of doing work is called power, *i.e.*, *power is the work done per unit time.*

$$\textbf{Power} = \frac{\textbf{Work}}{\textbf{Time}} = \frac{\textbf{Force} \times \textbf{Distance}}{\textbf{Time}} = \textbf{Force} \times \textbf{Velocity}.$$

When a person does work his sense of fatigue is determined more by the rate at which he does the work than by the total work done. Suppose a boy weighing 120 lb ascends a flight of steps which carries him 15 ft above the ground. The work he does against gravity is $15 \times 120 = 1800$ ft. lb. When he completes the ascent in 20 seconds he feels little fatigue. But if he tries to do it in 4 seconds, he will feel much more fatigued. In the latter case his rate of doing work *i.e.*, power, is 5 times greater than that in the former case.

Two railway trains of the same weight move at 30 mph and 50 mph respectively. If the friction of the rails and the resistance due to air are the same for both, the latter does more work than the former, because it moves a greater distance in the same time. So its rate of doing work, or power, is greater than that of the former.

The power delivered by a machine may be constant, or it may fluctuate. In the latter case we shall be concerned with the *average power*.

Units of power—Watt ; Horse-power. Any of the units of work combined with a suitable unit of time will provide a unit of power. Two practical units of power are, however, of great importance. They are (i) the watt and (ii) the horse-power.

When one joule of work is done per second, the power is one **watt** (symbol W). A power of thousand watts is a **kilowatt** (kW). In other words, when 1000 joules of work are done per second, the power is one kilowatt. Electric machines are rated in watts and kilowatts.

In mks units the watt is the absolute unit of power : in cgs units it is the practical unit. The watt is the SI unit of power. 1W=1J/s.

The practical unit of power used in engineering practice in the fps system is the horse-power (hp). When work is done at the rate of 550 foot-pounds per second (or 33,000 ft. lb per minute), the power is called one **horse-power :**

To express the horse-power in watts.

1 horse-power=550 foot-pounds/second

$$=550\times 1{\cdot}356 \text{ joules/second}=746 \text{ J/s (Sec. 4-2.1)}$$
$$=746 \text{ W (watts)}.$$

A hp is thus approximately $\frac{3}{4}$ kW.

Some units of power and their conversion factors are summarised in the table below :

$$1 \text{ watt}=1\frac{\text{joule}}{\text{second}}=10^7\frac{\text{ergs}}{\text{second}}$$

$$1 \text{ hp}=550\frac{\text{ft. lb}}{\text{second}}=33000\frac{\text{ft. lb}}{\text{min}}=746 \text{ watts}=0{\cdot}746 \text{ kW}$$

$$1 \text{ kW}=1000 \text{ watts}=1{\cdot}34 \text{ hp.}$$

$$1 \text{ ft. lb per second}=1{\cdot}356 \text{ watts.}$$

The kilowatt and the horse-power have given rise to two more *units of work*—the **kilowatt-hour** and the **horse-power-hour.** The kilowatt-hour is the work done when power is used at the rate of one kilowatt over a period of one hour. This is the unit on which the cost of electricity is based. It is also known as the B. O. T. (Board of Trade) unit.

Similarly, the horse-power-hour is the work done when power is used at the rate of one horse-power over a period of one hour.

Examples. (1) A 1000 gallon tank at a height of 60 ft above the water level kas to be filled with water. If a motor pump rated at $\frac{1}{2}$ hp is used to do the work, find the time required to fill the tank. (Given, 1 gallon of water weighs 10 lb).

Solution : Weight of 1000 gallons of water = 10000 lb.

$\therefore$ the total work to be done = 10000×60 ft. lb.

The motor has a power = $\frac{1}{2}$ hp, *i.e*,, it can do 550/2 ft. lb of work in 1 sec.

$$\therefore \text{ The time required to fill the tank}=\frac{10000\times 60 \text{ ft. lb}}{\frac{550}{2}\ \frac{\text{ft. lb}}{\text{s}}}$$

$$=\frac{10000\times 60\times 2}{550}\text{ s}=36 \text{ min } 22 \text{ sec (approximately).}$$

(2) A tractor can exert a horizontal force of 1000 kg and travel at 5m/s. Find it a power in kilowatts.

Solution : The work done by the tractor in 1 s = 1000 kg-wt × 5 m.

$= 1000 \times 9{\cdot}8$ newton × 1 metre.

∴ The work done in 1 s = 9800 newton-metres = 9800 joules

∴ its power = 9800 J/s = 9800 W = 9·8 kW.

4-4. Energy. When a body can do work, we say it possesses energy. *Energy of a body is defined as its capacity for doing work.* It is measured by the amount of work the body can do. Energy and work are essentially the same kind of quantity, and are measured in the same units.

A man or a horse can do work ; so he possesses energy. Steam can push the piston within the cylinder of a steam engine ; so it possesses energy. A moving body possesses energy since it can make other bodies move when it collides with them. An elevated body possesses energy since, while falling, it can pull other bodies up. If such a body pulled a mass of 1 kg through a height of 1 m before reaching the ground, we may say its energy was 1 kilogram-metre or 9·8 joules, or that the body lost 9·8J of energy in pulling the weight up.

Different forms of energy. There are many forms of energy. An electrically charged body attracts other bodies, and can make them move. It possesses what we call electrical energy. Magnets possess magnetic energy. Heat of steam makes locomotives move. Heat is a form of energy. Radio waves, light and X-rays are also forms of energy. An important form of energy is chemical energy, as is possessed by coal. When coal burns it combines with oxygen releasing chemical energy in the form of heat.

In recent times man has harnessed a vast source of energy. It is derived from the conversion of matter into energy under the action of nuclear forces in the atom, and is called *atomic energy*. The atom bomb, the hydrogen bomb and the atomic reactor are devices for the release of atomic energy.

In mechanics, we are however interested in what we call *mechanical energy*. (The definition of energy we gave at the beginning of this section, strictly relates to mechanical energy. In the wider sense, energy is that which can bring about any change in matter.) A body may possess mechanical energy due to either one or both of two causes, viz., (i) by virtue of its *motion*, and (ii) by virtue of its *position* or *configuration*. The former is called *kinetic energy* and the latter, *potential energy*.

4-4.1. Kinetic energy of a body is defined as the energy it possesses by virtue of its motion. It is measured by the amount of work a moving body can do against an opposing force before it comes to rest. A moving hammer possesses kinetic energy, which enables it to do work in driving a nail into a wall against the resistance.

To obtain an expression for the kinetic energy of a body of mass m moving with velocity v, let us impress on the body a force F which opposes the motion and finally brings the body to rest. The acceleration (or rather the deceleration) produced by the force is $-f=-F/m$. If the body moves through a distance s before it comes to rest, then from the relation that

$$(\text{final velocity})^2 - (\text{initial velocity})^2 = 2 \times \text{acceleration} \times \text{distance (Eq. 1-3.3)}$$

we have $0^2 - v^2 = -2fs$ or $v^2 = 2(F/m)s$ or $Fs = \frac{1}{2}mv^2$.

Now Fs is the work done by the moving body against the opposing force before coming to rest. Hence this is also the magnitude of its kinetic energy. Since $Fs = \frac{1}{2}mv^2$, the latter expression gives the kinetic energy of the moving body in terms of quantities which are known. Note that it does not depend on the opposing force F.

$$\textbf{Kinetic energy} = \tfrac{1}{2}\ \textbf{mass} \times (\textbf{velocity})^2$$

$$\mathbf{K} = \tfrac{1}{2}\ \mathbf{mv^2} \tag{4-4.1}$$

It is easy to see that if a force F acting through a distance s changed the velocity of a body from u to v, then

$$Fs = \tfrac{1}{2}m(v^2 - u^2)$$

or *work done by the force = change in kinetic energy*.

Examples. (1) A body of mass 10 kg moves with a velocity of 10 metres per second. Calculate its kinetic energy in ergs and in joules.

Solution : 10 kg = 10,000 g ; 10 m/s = 1000 cm/s.

$$\therefore \text{ Kinetic energy} = \tfrac{1}{2} \times 10^4 \text{ g } (10^3)^2 \frac{\text{cm}^2}{\text{s}^2} = \tfrac{1}{2} \times 10^{10} \frac{\text{g. cm}}{\text{s}^2} \times \text{cm.}$$

$$= \tfrac{1}{2} \times 10^{10} \text{ ergs} = \tfrac{1}{2} \times \frac{10^{10}}{10^7} = \tfrac{1}{2} \times 10^3 \text{ joules} = 500 \text{ joules.}$$

To get the work in absolute units of the cgs system the mass must be in grams and the velocity in cm/s. The values in the problem are in mks units. So kinetic energy $= \frac{1}{2} \times 10$ kg $\times (10 \text{ m/s})^2 = 500$ kg m²/s² $= 500$ J (see Sec. 4-2.1)

(2) A bullet weighing 4 oz is fired with a speed of 1200 ft/s from a gun weighing 20 lb. Calculate the kinetic energies of the shot and the gun.

Solution : K. E. of the bullet $= \frac{1}{2} \times \frac{4}{16}$ lb $\times (1200 \text{ ft/s})^2$

$= \frac{1}{8} \times 1200 \times 1200$ lb. $\text{ft}^2/\text{s}^2 = 18 \times 10^4$ ft. poundals.

To calculate the K. E. of the gun we must know its velocity. This can be found from the fact that the momentum of the shot and the gun are equal (vide principle of conservation of momentum ; Sec. 1-11).

Now, momentum of the shot $= \frac{4}{16}$ lb $\times$ 1200 ft/s $=$ 300 lb ft/s.

$\therefore$ the velocity of the gun $= \dfrac{\text{momentum}}{\text{mass}} = \dfrac{300 \text{ lb. ft/s}}{20 \text{ lb}} = 15$ ft/s

$\therefore$ K. E. of the gun $= \frac{1}{2} \times 20$ lb. $\times (15 \text{ ft/s})^2$

$= 10 \times 225$ lb $\text{ft}^2/\text{s}^2 = 2250$ ft. poundals.

[Note that the kinetic energies are not equal, but are in the inverse ratio of the masses.]

4-4.2. Potential energy is the energy which a body possesses by virtue of its position relative to the surroundings, its condition, or configuration (*i.e.*, the relative position of its parts). It is measured by the work the body can do in passing from the given position, condition or configuration, to some standard position condition or configuration. Potential energy is always measured by the difference from the standard position. The P.E. in the standard position is always taken as zero. We can never find the absolute value of the potential energy. The choice of the standard position is *arbitrary*.

Consider a body of mass m raised to a height h above the ground. In falling it can pull another body up. The work it can do is given by the product of its weight and height above the ground (generally the standard position). This is the measure of its gravitational potential energy.

Gravitational potential energy $= mgh$. (4-4.2)

Compressed air can drive machinery. It possesses energy by virtue of its condition of being compressed. The standard condition is generally that under normal atmospheric pressure. The work that the compressed air can do in expanding to a pressure of one atmosphere is a measure of its potential energy.

The main spring of a clock, when wound, drives the hands of the clock. It acquires potential energy in winding which changes the relative position of the different portions of the spring. As it is unwound it loses energy.

4·4.3. The sum of the kinetic and potential energies of a freely falling body remains constant throughout its motion. Let m be the mass of a body falling freely from rest from a height h above the ground. Let v be its velocity after it has travelled a distance x from its initial position. From the relation

(final velocity)^2 $-$ (initial velocity)^2 $= 2 \times$ acceleration $\times$ distance

we have

$$v^2 = 2gx$$

$\therefore$ The kinetic energy of the body at this height is

$$\tfrac{1}{2}mv^2 = mgx.$$

Its potential energy at this height, *i.e.*, $h-x$ above the ground, is $mg\,(h-x)$.

$\therefore$ The total energy = kinetic energy + potential energy

$$= mgx + mg\,(h-x) = mgh.$$

Since x may have any value from 0 to h, we find that the total energy of the falling body remains constant and is *equal to its potential energy at the topmost position.*

It is also clear from the analysis that *the gain in kinetic energy is equal to the loss in potential energy.*

If v_0 is the velocity of the body when it reaches the ground, then $v_0^2 = 2gh$. Hence the kinetic energy when the body just touches the ground $= \frac{1}{2}mv_0^2 = mgh =$ its potential energy at the top.

4·4.5. Relation between the work done and the potential energy of a system. Let us consider a body of mass m at rest on the surface of the earth. The potential energy of the system consisting of the body and the earth in this condition is *arbitrarily* taken as zero. When an agent raises the body to a height h, the work done *by* him *against* gravity (*i.e.*, against the force operating between the bodies comprising the system) is mgh. The potential energy of the system is also mgh now.

If, on the other hand, the body fell from the height h to the surface of the earth, the work done *by* gravity, *i.e.*, force operating between the bodies comprising the system, is mgh, and this is also its loss of potential energy.

We therefore find that *when work is done by the forces operating between bodies comprising a system, the potential energy of the system diminishes exactly by the amount of work done*, while

the potential energy of a system increases when an external agent *does work on it* (*i.e., against the forces operating within the system*), *the increase of energy being equal to the work done.* The result applies to gravitational, electric and magnetic cases, forces of the nature of friction being neglected.

4-5. Conservation of energy. In all examples of transformation of energy the thing to be noted is that *no body or system of bodies can acquire energy except at the expense of energy possessed by other bodies.*

Energy is never made from anything that is not energy, nor turned into anything that is not energy. Energy can only change form, or pass from one body into another. Whenever one system loses energy, another system gains it. Wherever measurement is possible it is found that the loss of energy of one system is exactly equal to the gain in the energy of the other. Hence we are led to believe that energy is an entity of nature which is conserved. The law of conservation of energy may be stated as follows :

Energy can neither be created nor destroyed. Hence *the total amount of energy in the universe remains constant.*

A system of bodies which can exchange energy only among themselves, no energy leaving the system nor any energy entering it from outside, is called an *isolated system.* The principle of conservation of energy may then be stated in the form that **the total energy of an isolated system remains constant.**

The law was first formulated by the German physicist Robert Mayer in 1842, and was firmly established by Helmholtz. It forms the foundation on which the whole structure of physical science has been built.

Exercises

1. Define *work.* Distinguish between work done *by* a force and *against* a force, giving two examples of each.

In a tug-of-wear team A is defeated by team B. Which team did work against the other ? Explain your answer,

2. A man is rowing a boat upstream, but is unable to avdance relative to the bank (that is, a man seated on the bank sees that the boat does not move). Explain whether the man who is in the boat is doing work or not.

3. What is meant by *energy ?* What is *mechanical energy ?*

Prove that kinetic energy of a body $= \frac{1}{2} mv^2$.

Calculate the increase in kinetic energy of a body which moves a distance s in the direction of an applied force F.

What is *potential energy* ? What do you understand when it is said that the gravitational potential energy is mgh ?

4. Prove that a body moving freely under gravity has a constant mechanical energy (that is, the sum of its kinetic and potential energies is a constant). Consider the case of both rise and fall.

[*Hint :* During rise, the total energy E at a height $x = mgx + \frac{1}{2} mv^2$ where v is the velocity at x. If u=initial velocity, then $u^2 - v^2 = 2g\,x$ or $\frac{1}{2} mu^2 - \frac{1}{2} mv^2 = mgx$ $\frac{1}{2} mu^2$ is the initial total energy. This equals $\frac{1}{2} mv^2 + mgx$. x is any point on the path of ascent]

5. Explain what is meant by 'Conservation of energy'. What is an *isolated system* ?

6. Explain the equation $W = Fs \cos\theta$ retating to work done by a force. What will be the interpretation of the equation if θ is greater than 90° ?

7. (a) Define the terms *joule*, *erg* and *watt*. How are they related ? What is a *kilowatt-hour* ?

(b) What is the relation between application of a force and work done by it ? Does an applied force always do work ?

(c) When a gun fires a bullet, who does work—the powder or the bullet ? What is the kind of transformation of energy that occurs in this case ?

(d) A motor car is moving with a constant velocity on a level road. In such a case there will be no unbalanced force on the car. Is work being done anywhere ? Explain your answer.

8. A train weighing 10^5 kg starts from rest and acquires a speed of 12 m/s in one minute. How much work has been done on it ? If the engine can pull the train with a constant speed of the above value, what is the power of the engine in kilowatts ? [Ans : $\frac{1}{2} mv^2 = \frac{1}{2} \times 10^5$ kg $\times (12 \text{m/s})^2$ joules : 240 kW.].

9. A cloud 5 km above the ground condenses into rain which collects 1 cm deep over an area of 10^4m^2. What is the loss of potential energy of the cloud ? [Ans : $9{\cdot}8 \times 10^8$ joules].

10. A boy weighing 60 kg ascends 32 steps, each 25 cm high, in 10 seconds. What power in kilowatts did he expend ? [Ans : 0·59].

11. A 5 kW motor lifts water to a height of 10 m above the water level. If the efficiency of the pump in 80% how much water will be lifted per minute ? (Efficiency=output/input, that is work done by the pump ÷ work done on the pump. 5 kW is the power the pump draws). [Ans : 2570 litres].

12. Distinguish between momentum and kinetic energy. Forces of 1 kg each act on masses 10 kg and 40 kg. Find the ratio of the times required to give them (a) the same momentum, (b) the same kinetic energy. [Ans : (a) Equal times, (b) 1 : 4.].

13. When a pile driver drops a ball weighing 250 kg from a height of 5 m on a pile, the pile sinks 2·5 cm into the ground. What is the average resisting force of the ground ? [*Hint* : $mgh = Fs$. Ans : 5×10^4 kg-wt.]

VIBRATIONS AND WAVES

CHAPTER 1

VIBRATIONS

1-1. Vibrations. A particle or body is said to vibrate if it describes a to-and-fro motion along a path. If it reaches the same point of its path at equal intervals of time, its motion is called **periodic motion.** When the motion is in a straight line, as in the case of the needle of a sewing machine or the piston of an engine, it is *rectilinear* vibration. Periodic motion may as well be along a circle or an ellipse, such as the motion of the earth round the sun or of an electron round the nucleus in an atom.

A rectilinear vibration is also called an *oscillation.* There is no hard and fast difference in their meanings. Generally, when the motion is fast we call it a vibration. A relatively slow vibration is often called an oscillation.

Certain terms are very useful in describing a periodic vibration. These are

(i) **Periodic time** (T). It is the time required by the vibrating particle (or body) to execute one complete vibration, say, in going from one end of its path to the other end and back to the first end.

(ii) **Frequency** (n). The number of vibrations executed in one second is called the frequency. From the definitions, it follows that

$$nT=1 \text{ or } n=1/T \qquad (1\text{-}1.1)$$

(iii) **Amplitude.** The maximum distance through which the particle moves from its normal position of rest is called the amplitude of the vibration.

(iv) **Particle displacement.** The distance, at any moment, of the vibrating particle from its normal position of rest is the *particle displacement.* We shall illustrate the meanings of these terms by reference to the motion of a pendulum (Fig. 1.1). The pendulum is suspended from O. A is the normal position of rest of the bob. It oscillates between the extremes B and C. $AB=AC$ is the amplitude of the motion. The *angular*

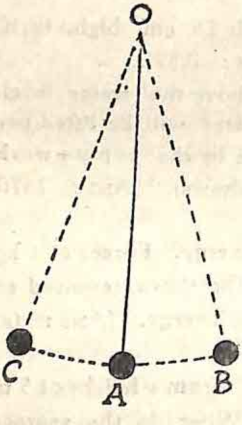

Fig. 1.1

amplitude is $\angle AOB = \angle AOC$. The periodic time (or *period*) is the time required by the bob in going from B to C and coming back to B (or from A to B, B to C and back from C to A). The reciprocal of the period (T) is the frequency.

1·2. Simple harmonic motion. The relation between the particle displacement and the time required for it may be quite complex even in a periodic motion. It depends on the forces acting. The motion in which the above relation is simplest is called simple harmonic motion (or S.H.M. in brief). Besides, any periodic motion, however complex, may be expressed as the sum (or superposition) of the right number of S.H.M.s with appropriate amplitudes and frequencies. (This result is known as Fourier's theorem; but the theorem cannot be intelligently discussed at this stage of our study.)

Definition of S.H.M. It can be defined in *two* ways :

(i) The projection of a uniform circular motion on any diameter is called simple harmonic motion.

(ii) If a particle is acted on by a force proportional to the particle displacement and is always directed towards the normal position of rest of the particle, the motion that occurs is called simple harmonic motion. (A force of the above kind is called a *linear*, *restoring* force. By the term 'linear' we mean proportional to particle displacement. 'Restoring' means trying to bring the particle back to its normal position).

Though the two definitions appear very different, both represent the same motion, and we can reach one from the other. The first definition gives the characteristics of the motion very easily, but does not say anything about the nature of the force that causes the motion. We may call it the *kinematic* or *geometrical* definition. The second definition tells us about the nature of the force causing the motion. We may call it the *dynamical* or *physical* definition. To the beginner, the first definition is more helpful. But to the student of physics who has advanced a little in his subject, the second definition is more helpful.

Projection. Perhaps from geometry you have already learned the meaning of 'projection' of a line segment on any straight line. You drop two perpendiculars from the two ends of the given line segment on the given straight line. The distance between the feet of these two perpendiculars is the projection of the line segment on the given line. If l is the length of the line segment and θ is the angle it makes with the given straight line, the projection of l is $l \cos \theta$. Remember this relation.

1-2.1. Projection of a uniform circular motion on a diameter. Refer to Fig. 1.2. Suppose a particle is moving round the circle CADB with constant angular velocity ω in a clockwise direction. We want the projection of this motion on the diameter AB. Let P be the position of the particle at any moment on the circle. Drop PX perpendicular on AB. As P goes round and round in the circle, the point X executes a to-and-fro motion between A and B. The motion of X is the projection of the uniform circular motion on diameter AB.

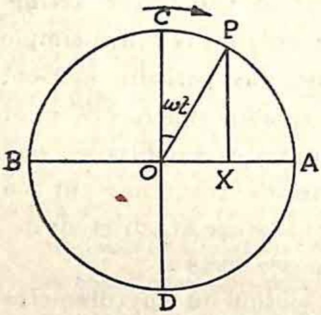

Fig. 1.2

1-2.2. Equation for particle displacement in S.H.M. Let us consider the motion from the moment when the rotating particle crossed the point C (Fig. 1.2). CD is the diameter perpendicular to AB. When the particle was at C, X was at O, the centre of the circle. Let us measure displacement from this point. (The second definition will tell us that O is the normal position of rest of the particle in S.H.M.) If after a time t from the moment considered the rotating particle reaches P (Fig. 1.2), X moves from O to X. So the particle displacement x in time t is $x=OX$. If $OP=a$ is the radius of the circle, then

$$x=OP \sin COP=a \sin \omega t \qquad (1\text{-}2.1)$$

If at the initial moment the rotating particle was at A, and we measured particle displacement from O, Fig. 1.3 will show that

$$x=OX=OP \cos AOP=a \cos \omega t \quad (1\text{-}2.2)$$

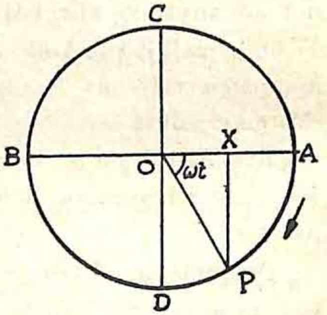

Fig. 1.3

Eq. 1-2.1 or 1-2.2 represents the same S.H.M. The difference in form lies in the choice of the initial moment. If the motion is considered from the moment the particle crossed the midpoint of its path, Eq. 1-2.1 will apply. If we considered the motion from the moment the particle was at the end of its swing, Eq. 1-2.2 will apply. So we may use Eq. 1-2.1 or 1-2.2 according to our convenience.

We might have considered the motion from any other moment, say, when the particle was at E (Fig. 1.4). If $\angle EOC=\epsilon$ (Greek letter, pronounced 'epsilon'), the figure would show that

$$x=OX=a\sin(\omega t-\epsilon) \qquad (1\text{-}2.3)$$

If we took the position of E relative to the line OA then $\angle EOA=\epsilon$ and

$$x=OX=OP\cos(EOA-EOP)$$
$$=a\cos(\omega t-\epsilon) \qquad (1\text{-}2.4)$$

Eq.s 1-2.1 to 1-2.4 represent the same S.H.M. The difference in form is due to the choice of the initial moment. ϵ may be positive or negative. We shall ordinarily use Eq. 1-2.1 to represent an S.H.M., unless special consideration makes use of one of the other forms more convenient.

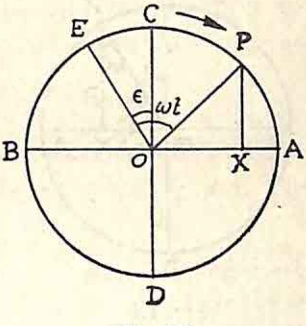

Fig. 1.4

a is called the *amplitude* of the S.H.M. ω is called its *angular frequency*. The periodic time T is the time in which the rotating particle goes once round the circle. This gives us

$$T=2\pi/\omega \qquad (1\text{-}2.5)$$

The frequency n of the S.H.M. is $n=1/T=\omega/2\pi$ (1-2.6)

1-2.3. Particle velocity in S.H.M. Suppose the constant speed with which the circle is being described is $v_o=a\omega$. The velocity v_o of the particle at P is perpendicular to OP, and is represented by a line segment, say, PR. The projection of PR is the velocity v of the particle in S.H.M. when it is at X. From Fig. 1.5

$$v=SR=PR\cos\omega t$$
$$=v_0\cos\omega t=a\omega\cos\omega t \qquad (1\text{-}2.7)$$
$$=a\omega\sqrt{1-x^2/a^2}=\omega\sqrt{a^2-x^2} \qquad (1\text{-}2{,}8)$$

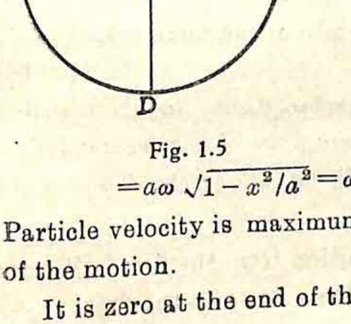

Fig. 1.5

Particle velocity is maximum when $x=0$, that is, at the midpoint of the motion.

It is zero at the end of the swing $(x=a)$.

1-2.4. Acceleration of particle in S.H.M. We know (Sec. 2-6.1; MECHANICS) that the acceleration of a particle moving in a circle of radius a with constant angular velocity ω is $a\omega^2$, and that the acceleration is directed towards the centre of the circle (Fig. 1.6).

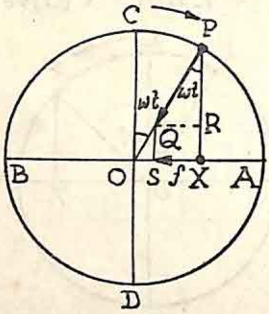

Fig. 1.6

The projection of this acceleration on the diameter AB is the acceleration of the particle moving simple harmonically along AB. If in Fig. 1.6, the line segment PQ represent the acceleration at P, its projection XS on AB is the acceleration of the particle in S.H.M. when it is at X. From Fig. 1.6, $XS=RQ=PQ \sin \omega t$ $=a\omega^2 \sin \omega t$. But XS is directed towards O and is opposite to the particle displacement. If we take particle displacement x as positive, we should give particle acceleration f a negative sign to take care of the fact that their directions are opposite. Therefore, the acceleration f at X should be given by

$$f=-a\omega^2 \sin \omega t=-\omega^2 x \tag{1-2.9}$$

Acceleration in S.H.M. is directed opposite to the displacement. This means that as soon as the particle is displaced from its position of rest, an acceleration (and therefore a force) acts on the particle which tends to bring back the particle to its normal position of rest. The acceleration is greatest when x is largest, that is, when $x=a$. It is zero when $x=0$, that is, when the particle is in the mid-point of its motion. Comparing with the case of v, we find

when $x=0$, $v=a\omega$ (maximum), $f=0$ (minimum)
and when $x=a$, $v=0$ (minimum), $f=-a\omega^2$ (maximum).

1-2.5. Force on particle in S.H.M. If m is the mass of the moving particle and $f=-\omega^2 x$ is its acceleration, the force acting is

$$F=mf=-m\omega^2 x \tag{1-2.10}$$

Clearly, the force is proportional to the displacement and is directed opposite to it. This is the *linear, restoring force* of our second definition. If there is no force, the particle will be at $x=0$. This point is its normal position of rest.

We have come to the second definition from the first. We are not interested at this stage in going from the second to the first.

1·3. Characteristics of S.H.M. The analyses in Sec.s 1-2.1 to 1-2.5 tell us that S.H.M. has the following characteristics :

(i) The motion is periodic and rectilinear.

(ii) The force acting, and hence the acceleration, is proportional to the particle displacement measured from the mid-point of the motion. Besides, force and acceleration are opposite in direction to the displacement. They try to bring the particle back to the midpoint of the motion.

(iii) The particle velocity is maximum when particle displacement or acceleration is zero. This occurs at the midpoint. At the end of the swing, velocity is zero, but force and acceleration are maximum.

Examples. (i) The motion of the bob of a long pendulum oscillating with a *small* amplitude is practically simple harmonic. Its path may be taken as practically straight. The motion is repeated at equal intervals of time. At the end of a swing the bob is momentarily at rest ($v=0$). It starts moving towards the position of rest with gradually increasing velocity, crosses the rest-position with maximum velocity, then gradually slows down and ultimately comes to rest at the other end of the swing. This is half the motion. The other half is exactly similar.

(ii) If a mass m is attached to a long vertical spring (Fig. 1.7), pulled downward a little and released, it executes an up and down simple harmonic motion. In the figure a is the amplitude of the S.H.M.

Problems. The velocity of a particle in S.H.M. when 8 cm from the centre is 72 cm/s. At 18 cm from the centre it is 24 cm/s What are the maximum displacement and acceleration ? (*Hint* : Use Eq. 1-2.8. Substitute values of v and x and solve for a and ω.) [Ans. : 18·9 cm ; 335 cm/s².]

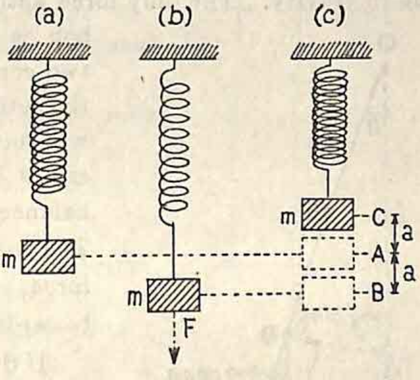

Fig. 1.7

1·4. Periodic time in S.H.M. Eq. 1-2.5 gave us that the periodic time $T=2\pi/\omega$. Eq. 1-2.9 shows that the *magnitude* of the acceleration is $f=\omega^2 x$ whence $\omega^2=f/x$. Thus $\omega=\sqrt{f/x}$=square root of the acceleration at unit displacement. Therefore

$$T=2\pi/\omega=2\pi/\sqrt{f/x}=2\pi/\sqrt{\text{acclrn. at unit displacement.}} \qquad (1\text{·}4.1)$$

(See also Eq. 1-7.4)

Problem. In the problem of Sec. 1·3, find the period of the S. H. M. (*Hint* : Find acceleration at unit distance. It is 335 cm s^{-2}/18·9 cm. 2π divided by the square root of this quantity is the period.)

1·4.1. Examples of S.H.M. In many physical problems we come across linear restoring forces. It will be convenient to remember that the motion in all such cases will be simple harmonic with periodic times given by Eq. 1-4.1. Two well known examples are discussed below.

(i) The simple pendulum. *A heavy particle suspended by a weightless, inextensible and perfectly flexible string constitutes a* **simple pendulum**. You will realize that we cannot have any real string of the above description, nor any real particle (because a particle has position only and no extension). Still, the idea of a simple pendulum is very helpful in treating oscillations mathematically. The simple pendulum, also called the *ideal pendulum*, is an imaginary thing like a particle or perfectly smooth plane in mechanics. It is realized in practice by suspending a spherical metal ball by a thin and light but strong string.

Consider *a* simple pendulum (Fig. 1.8) of length l displaced through an angle θ. Let m be the mass of the bob and g, the acceleration due to gravity. The only force acting vertically downwards on the bob is its weight mg. Resolve it into two components, one along the string and the other perpendicular thereto. They will have magnitudes mg cos θ and mg sin θ respectively. mg cos θ will be balanced by the tension T in the string. The component mg sin θ is the unbalanced force which causes the bob to move towards its position of rest A.

If θ is *very small* we may, without appreciable error, write θ for sin θ. The distance of the bob from A will then be $l\theta = x$ (say). The very small arc in which the bob moves may, in such a case, be taken to be a straight line. The force acting on the bob then comes to be equal to $mg\theta = mgx/l$.

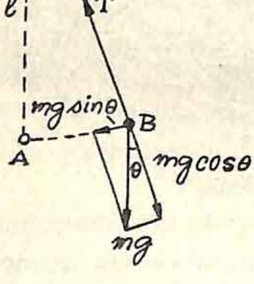

Fig. 1.8

Clearly, it is a restoring force proportional to the displacement of the bob from its position of rest. The force at unit distance from A is mg/l. The acceleration there is g/l. Hence from Eq. 1-4.1, we get the periodic time

$$T=2\pi/\sqrt{g/l}=2\pi\sqrt{l/g}. \quad (1\text{-}4.2)$$

Laws of simple pendulum. Certain statements are generally known as the laws of simple pendulum. They are all included in Eq. 1-4.2. The statements are as follows :

(*i*) *Law of isochronism.* At a given place, the oscillations of a simple pendulum of given length are executed in equal times provided the amplitude is small.

(*ii*) *Law of length.* The periodic time of a simple pendulum of small amplitude varies as the square root of its length at a given place. ($T \propto \sqrt{l}$ at a given place)

(*iii*) *Law of gravity.* For a given pendulum, of small amplitude, the periodic time varies inversely as the square root of acceleration due to gravity at the place. ($T \propto 1/\sqrt{g}$)

(*iv*) *Law of mass.* The periodic time of a simple pendulum does not depend on the mass or material of the bob.

(ii) Motion of a vertical loaded spring. Consider a spring of length L and negligible weight hanging vertically from a fixed support and having a load of mass m at its lower end. The spring extends a little, and the increased tension in it supports the load.

Suppose the load is pulled downwards through a small additional vertical distance x from this position (Fig. 1.9). The spring will exert a restoring pull on the load which, for small extensions, is proportional to the extension. The additional upward pull on the load will therefore be sx, where s is the pull per unit extension of the spring. If the mass M stretched the spring initially by a length a, then $Mg=sa$. s is the force per unit displacement. The acceleration at unit distance is $s/M=g/a$. Hence from Eq. 1-4.1, the period

$$T=2\pi/\sqrt{g/a}=2\pi\sqrt{a/g} \quad (1\text{-}4.3)$$

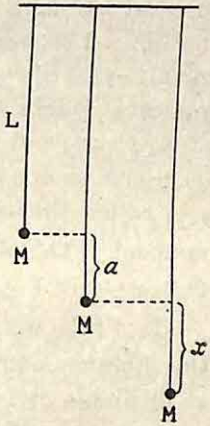

Fig. 1.9

The mass M executes an S.H.M. about its normal position of rest with this periodic time. Note that this period is the same as that of a simple pendulum of length a.

Examples. (1) A body in S.H.M. has an amplitude of 5 cm and a mass of 10 g. The restoring force acting on it at the end of its swing is 1000 dynes. Calculate the periodic time.

Solution : The restoring force per unit displacement $= 1000/5 = 200$ dynes. Hence the acceleration at unit distance $= 200/10 = 20$ cm/s^2.

$\therefore$ Time period $= 2\pi/\sqrt{20} = 1{\cdot}4$ s.

(2) A body of mass 100 g causes a vertical spiral spring to stretch by 0.5 cm. If it is set into vertical oscillations of small amplitude, what is the periodic time of the oscillation ?

Solution : Assuming that the extension of a spring is proportiona to the force, the force required to produce an extension of 1 cm is 100 gm-wt/0·5 $= 200$ gm-wt $= 200\,g$ dynes, where $g =$ acceleration due to gravity. Hence the acceleration of the body at unit distance $=$ force/mass $= 200g/100 = 2g$.

$\therefore$ Time period $= 2\pi/\sqrt{2g}$

1-5. Phase in S.H.M. The concept of *phase* is very important in connexion with propagation and superposition of vibrations. Try to understand its meaning clearly.

In S.H.M. we find that the state of motion of the particle, that is, its displacement, velocity and acceleration are changing continuously. They come back to the same set of values after a definite interval of time. So, there is a continuous cycle of changes in the motion. The word **phase** means the state of motion of the particle in this cycle of changes. (Compare the 'phases' of the moon.) Any quantity which can give the displacement and velocity of the particle at any moment, may be taken as the measure of phase. The angle $(\omega t \pm \epsilon)$ is such a quantity. It is called the **phase angle**. The angle ϵ is called the *initial phase* or *epoch* ; it gives the phase at the initial moment. The phase angle is measured with reference to either radius OA or OC of Fig 1.2.

Two S.H.M.s of the same frequency may be in different phases at the same moment. The difference between their phase angles is called their **phase difference**.

Note the following relations of S.H.M. :

$$\left.\begin{aligned}
&\text{(i) Particle displacement } x = a \sin \omega t\\
&\text{(ii) Particle velocity } \quad v = a\omega \cos \omega t = a\omega \sin(\omega t + 90^\circ)\\
&\text{(iii) Particle acceleration } f = -\omega^2 x = -a\omega^2 \sin \omega t = a\omega^2\\
&\qquad\qquad\qquad\qquad\qquad\qquad \sin(\omega t + 180^\circ)
\end{aligned}\right\} \quad (1\text{-}5.1)$$

Comparing the phase angles of the three expressions, we can say

(a) Particle velocity *leads* particle displacement in S.H.M. by 90°. (This means that when one is maximum, the other is zero.)

(b) Particle acceleration and particle displacement differ in phase by 180°, or that they are in *opposite phases.* This means that they are oppositely directed, and reach the maximum or zero value at the same instant.

1·6. Time-displacement graph in S.H.M. The time-displacement graph (x vs. t) in S.H.M. can be easily obtained geometrically from the uniform circular motion of which the S.H.M. is the projection. Refer to Fig. 1.10. The circular motion is along $DBEC$.

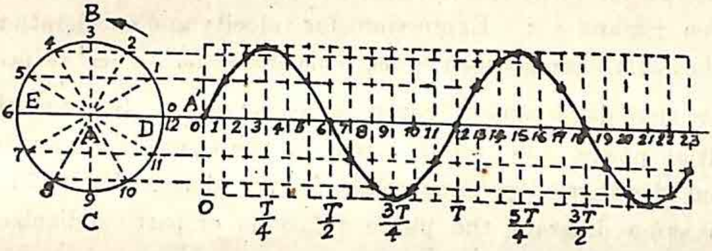

Fig. 1.10

The projection is taken on the vertical diameter BC. At the initial moment the particle is at D where ED is perpendicular to BC. Divide the circumference into a convenient number (here 12) equal parts. Mark the points of division successively as 0, 1, 2, 3 etc. starting with D as zero. (The mark 12 falls on D or O.) Extend the line ED to the right and mark along it a number of equidistant points, taking the first of these points (A') as zero. Number the points successively. (There are altogether 24 such points in the figure.)

Suppose at $t=0$ the moving particle was at D (*i.e.* zero) and its motion was anticlockwise. Its angular velocity is $\omega=2\pi/T=2\pi n$. The foot of the perpendicular dropped on BC from the moving point describes the S.H.M. along BC.

From the marked points on the circumference drop perpendiculars on BC. As the moving particle on the circle crosses a particular mark, the foot of the perpendicular drawn from this point on BC gives the position of the point in S.H.M. Its displacement at the moment is the distance of the foot of the perpendicular from the centre A of the circle. At each marked point on ED draw a perpendicular line. Intersect these lines by lines drawn parallel to ED from the marked points on the circle. Mark those points of intersection which correspond to the same number on ED produced as on the circumference (such as 0. 0 ; 1, 1 ; 2, 2 ; ...... ; 5, 5 ; ...... ; 7, 7 ; etc.). Draw a smooth, continuous line through the points so marked. This is the time-displacement graph of the given S.H.M. The ED-produced axis is the time axis, each interval along which is $T/12$. The ordinates are the corresponding displacements. The amplitude is the maximum of the curve. If a is the radius of the circle, the displacement is confined between the limits $+a$ and $-a$. This is also evident from the relation $x=a \sin \omega t$. *x alternates* between $+a$ and $-a$. Expressions for velocity and acceleration show that they *alternate* between $+a\omega$, $-a\omega$ and $+a\omega^2$, $-a\omega^2$ respectively.

The time-displacement curve is a *sine curve*. If the motion has an initial phase, the initial point on the curve is not on the $x=0$ axis, but elsewhere depending on the initial phase. Fig. 1.11 shows in the same diagram the phase relations of particle displacement velocity and acceleration (Refer to Eq.s 1-5.1). For convenience, the amplitudes have been made equal. It is clear that the values recur as the phase angle increases by an integral multiple of 2π, for $\sin \omega t = \sin(\omega t + 2m\pi)$ where m is an integer.

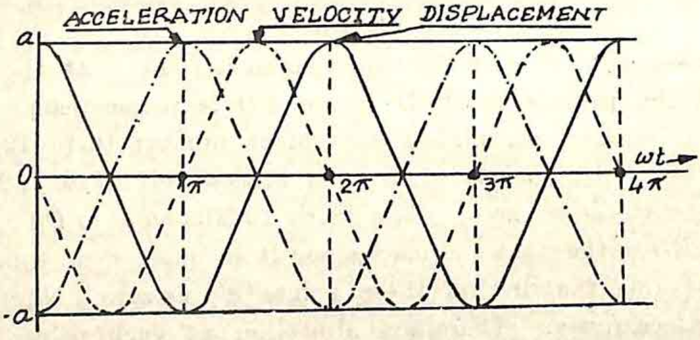

Fig. 1.11

1·7. Energy in S.H M. Let us take $x=a \sin \omega t$. Then $v=a\omega \cos \omega t$. If m is the mass of the moving particle in S.H.M. its kinetic energy is

$$K=\tfrac{1}{2}mv^2=\tfrac{1}{2}ma^2\omega^2\cos^2\omega t.$$
$$=\tfrac{1}{2}m\omega^2(a^2-x^2) \qquad (1\text{-}7.1)$$

K is maximum when $x=0$, that is, when the particle crosses its normal position of rest. At the end of a swing $x=\pm a$ and $K=0$.

The particle also has a potential energy V. Since the force on the particle is proportional to displacement, we can write the force as $F=-sx$ where s is the factor of proportionality. It is sometimes called the *spring factor*, and is the *restoring force per unit displacement.*

If the particle moves a very short distance x' when its displacement is x, the work done against the restoring force (that is, the work done on the particle) is sxx'. The force increases uniformly from 0 to sx as the displacement increases from 0 to x. We may therefore assume that the displacement x has occurred under an average force $\frac{1}{2}sx$. Hence the work done on the particle for displacement x is $\frac{1}{2}sx.x=\frac{1}{2}sx^2$. This is its potential energy V at x. Eq. 1-2.10 tells us that $s=m\omega^2$.

$$\therefore \quad V=\tfrac{1}{2}sx^2=\tfrac{1}{2}m\omega^2x^2=\tfrac{1}{2}ma^2\omega^2 \sin^2 \omega t \qquad (1\text{-}7.2)$$

$$\text{The total energy}=K+V=\tfrac{1}{2}ma^2\omega^2(\cos^2\omega t+\sin^2\omega t)$$
$$=\tfrac{1}{2}ma^2\omega^2. \qquad (1\text{-}7.3)$$

The energy is obviously constant, since m, a and ω are constants in a given case. Eq. 1-7.3 also shows when kinetic energy is maximum, potential energy is minimum. The potential energy is zero when $x=0$, and maximum at the end of a swing ($x=a$). S.H.M. is an example of the conservation of mechanical energy.

Note that since s is the restoring force for unit displacement, s/m is the acceleration at unit distance. By Eq. 1-4.1 we shall then have

$$T=2\pi/\omega=2\pi/\sqrt{s/m}=2\pi\sqrt{m/s} \qquad (1\text{-}7.4)$$

1-8 Superposition of two simple harmonic motions in the same direction : Let a particle have two S.H.M.'s at the same time. Its displacement at any instant will be the *vector* sum of the displacements due to each S.H.M. at the moment considered. If both S.H.M.'s are in the same direction vector addition becomes simple *algebraic* addition. We shall consider two such cases. The superposed S.H.M.'s will be in the *same direction* and have the *same frequency*. In one case (Fig. 1.12) they will be in the *same phase*. In the other case (Fig. 1.13) they will be in *opposite phases*. In both figures, the continuous curves represent the S.H.M.'s to be added. Mathematically, they are represented by $x_1 = a_1 \sin \omega t$ and $x_2 = a_2 \sin \omega t$.

In Fig. 1.12, the two motions are *in phase*. Hence they always produce displacements in the same direction. Their resultant is, therefore, the algebraic sum of the two. This means that the resultant displacement at any time t is $x = x_1 + x_2 = a_1 \sin \omega t + a_2 \sin \omega t = (a_1 + a_2) \sin \omega t$. It represents an S.H.M. of the same frequency (ω) as the other two, and has an amplitude $(a_1 + a_2)$ equal to the sum of the amplitudes of each. In the figure, it is given by the broken curve.

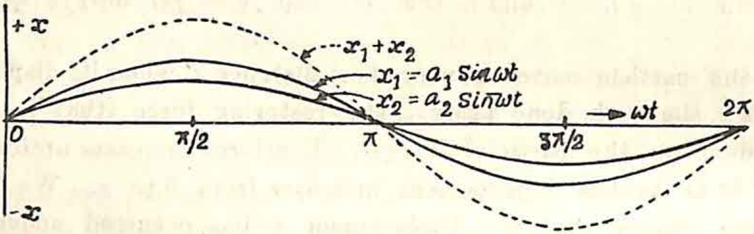

Fig. 1.12

In Fig. 1.13, the same S.H.M.'s are added but *in opposite phases*. If the phases are opposite, the phase angles will differ by 180°.

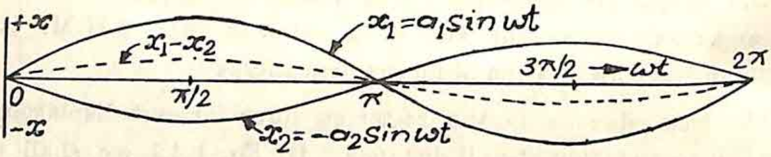

Fig. 1.13

Suppose $x_1 = a_1 \sin \omega t$ has the larger amplitude. The other motion will be $x_2 = a_2 \sin (\omega t \pm 180°) = -a_2 \sin \omega t$. This means that the resultant motion will be given by $x = x_1 - x_2 = a_1 \sin \omega t - a_2 \sin \omega t = (a_1 - a_2) \sin \omega t$. This is an S.H.M. in phase with the S.H.M. of larger amplitude but an amplitude equal to the difference of the two amplitudes. If $a_1 = a_2$, the superposition of the two motions (two S.H.M.'s of the same amplitude and frequency but in opposite phases) will *cancel* each other. No motion will be produced by their combined effect.

In both figures, the broken curves represent the resultant motion.

The importance of these cases will be understood later.

1-9. Free vibration. Any elastic body may be made to vibrate under suitable conditions. The vibration may be *transverse*, *longitudinal* or *torsional*. For example, the motion of a pendulum bob or of a metal strip clamped at one end is transverse ; that of a stretched spring is longitudinal, while the oscillatory motion of a twisted wire carrying a load at the free end is torsional. The vibrations of a tuning fork is transverse. All of these vibrations will be simple harmonic if the restoring force or couple is proportional to the displacement. It is generally so when the displacements are small. After vibrations have been excited in an elastic body and the exciting force removed, the body continues to vibrate with a frequency characteristic of its own. Such vibration is called **free vibration.** The periodic time of vibration is called the *free* or *natural period*. It depends upon the mass and the elastic properties of the vibrating body. Its reciprocal is the frequency, and is called the *natural frequency*.

In all practical cases, it is found that when left to itself the vibration of a body gradually diminishes in amplitude and finally dies away. This is so because the motion is resisted by various frictional effects, internal as well as external. When a tuning fork is vibrating its different layers are sliding over one another. This brings into play a resisting force due to viscosity or internal friction. This internal friction as well as resistance to motion offered by air, gradually diminishes or *damps* the amplitude and finally brings the body to rest. A vibrating pendulum is similarly damped because of resisting forces between its suspension and support, and between the air and the bob. Vibrations thus opposed from inside and outside gradually diminish in amplitude. They are called *resisted* or *damped vibrations*.

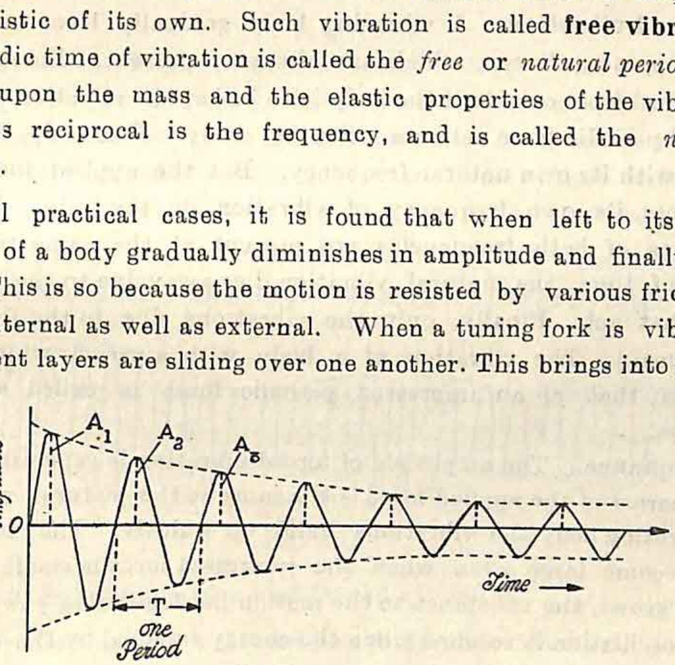

Fig. 1.14

Fig. 1.14 represents a case of *damped simple harmonic motion*. At each vibration some energy is lost in overcoming the resisting forces. If the resisting forces are large the rate of loss of energy is also large. In such a case the body comes quickly to rest. With small damping, the vibrations continue for a long time, the amplitude diminishing slowly.

1-10. Forced vibration and resonance. The study of forced vibration and resonance is of special interest in wave motion. Sound waves and radio waves are detected by the forced vibration or resonance they produce in the receiver. Resonance is a special case of forced vibration.

Forced vibration. A vibrating body gradually loses amplitude due to opposing forces, which are always present. Energy must be supplied from outside if the body is to be kept in vibration. Let an external periodic force act on a vibrating body. The body tends to vibrate with its own natural frequency. But the applied force tries to impress its own frequency of vibration on the body. Initially, vibrations of both frequencies are present at the same time. In course of time, the natural vibration dies away due to the resisting forces that act. Finally, only the vibrations due to the impressed force remain. The vibration of a body with a period which is the same as that of an impressed periodic force is called a *forced vibration.*

Resonance. The amplitude of forced vibration is generally small. If the period of the applied force is the same as the natural period of the vibrating body the vibrations build up quickly. The amplitude may become large even when the impressed force is small. As the motion grows, the resistance to the motion increases also. A state of steady oscillation is reached when the energy supplied by the external source is fully used up in overcoming the resistance to the motion. The particular case of forced vibration where the applied force has the same period as the natural period of *unresisted* vibration of the body is known as **resonance.** A resonant vibration is also called *sympathetic vibration.*

It can be shown mathematically that if resonance occurs for a body whose vibration is undamped—an ideal case never realised in practice—the amplitude would be infinitely large. In an actual case, damping forces limit the amplitude.

There are, in fact, two cases of resonance to distinguish, namely, (*i*) *amplitude resonance* and (*ii*) *velocity resonance*. In the former, the amplitude of the forced vibration acquires a maximum value. In the latter, the velocity is a maximum. This corresponds to maximum energy transfer between the forcing system and the forced system. When damping is negligible both resonances occur at practically the same frequency, which is the natural frequency of the system. When there is appreciable damping, the two resonances occur at slightly different frequencies. Velocity resonance occurs at the frequency equal to the undamped frequency of the body in forced vibration ; this is the more important case.

Characteristics. Forced vibration shows the following characteristics :

(1) Initially, vibrations with the natural frequency of the vibrating body and the frequency of the impressed force are both present. If the frequencies are close enough they may form 'beats', that is a rise and fall in amplitude (Fig. 1.15). In course of time the natural vibration dies out and the body vibrates with the forcing frequency.

(2) The amplitude of forced vibration is generally small except in the case of resonance. This amplitude may then be quite large.

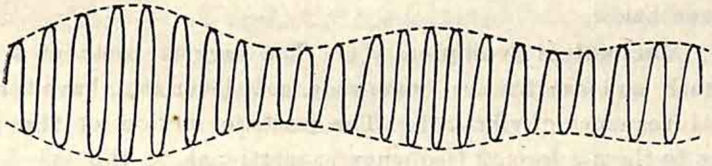

Fig. 1.15

Difference between free and forced vibration.

(*i*) Free vibration is executed by a body under the action of its own elastic forces without being subject to any external force. But forced vibration is executed by a body under the action of an externally applied periodic force.

(*ii*) Amplitude of free vibration may have any value, large or small, depending on the initial supply of energy. If damping is present the amplitude diminishes exponentially. Amplitude of forced vibration is generally small except when resonance occurs. Under resonance conditions the amplitude of vibration is large. Near resonance the amplitude increases rapidly as the frequency of the applied force approaches the frequency of the unresisted free vibration.

(*iii*) Frequency of free vibration of a body depends on its mass and elasticity. Frequency of forced vibration is equal to that of the applied force.

(*iv*) Free vibration finally ceases due to action of resisting forces. But forced vibration continues so long as the applied force acts.

Fig. 1.16 shows an arrangement to demonstrate in a simple way the difference between forced vibration and resonance. A,B,C,D are four pendulums of different lengths suspended from the same horizontal stick.. A′ is another pendulum having the same length as A. A and A′ therefore have the same natural frequency ; but the frequencies of the others are different. When A is set into oscillation A′ gradually picks up an increasing amplitude, while B,C,D vibrate with very small amplitudes. A′ resonates with A, but the others are thrown into forced vibration. The vibration of A is transmitted to the others through the stick.

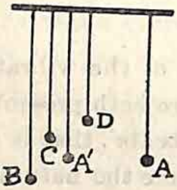

Fig. 1.16

1-11. Some examples of forced vibration and resonance.

Forced and resonant vibrations are fairly common. A few examples are given below.

A. Mechanical examples. (1) The various parts of a motor car, such as loose fittings, brake rods, gear lever etc., have their own natural frequency of vibration. The periodic motion of the pistons applies to them a forcing frequency proportional to the speed of the car. As the speed alters, the frequency of the pistons may match the natural frequency of some part so that it is thrown into resonant vibration and rattles vigorously.

(2) A child on a swing applies periodic impulses to the motion, keeping time with the swing. The swings gradually increase in amplitude. It is a case of resonance.

(3) If one leans on one side of a heavy boat and then on its other side, a considerable roll can be built up if the motions of the body have the same period as the swing of the boat.

(4) Resonant vibration is of great practical importance to structural and mechanical engineers. If quite a small periodic force operates on some structure or machine having the same natural period, vibrations of large magnitude develop. Vibrations produce stresses and, for large magnitudes, the resultant stresses may exceed

the elastic limit and damage the structure. Hence in designing a structure it is necessary to examine what external periodic forces may act on the structure. The structure is then built up so as to have a different natural frequency.

(5) Soldiers break steps while marching over a suspension bridge. Bridges have been known to vibrate dangerously and even to break when resonance occurs between the period of steps of the soldiers' march and the natural period of vibration of the bridge.

Jointless rails are used to prevent railway bridges being affected by the periodic impact of wheels crossing the joints.

B. Acoustical examples. (1) A vibrating fork held in the hand produces a feeble sound. When its stem is pressed on a table-top the sound is greatly increased. The fork's vibration is transmitted to the table-top. The top is thrown into forced vibration with a frequency equal to that of the fork. The large vibrating surface of the table sets a large mass of air into vibration. This increases the volume of the sound.

In this as well as in all other cases of forced vibration the kinetic energy of the sounding body is transferred to the forced vibrator. Because of the large surface of the table the *rate of loss of energy* to the surrounding air is much greater than when the fork vibrated alone. That is why the vibration of a fork dies away much more quickly when the fork is pressed on a table-top.

(2) Take a tall empty glass jar. Hold a vibrating tuning fork just above the upper end of the jar and and slowly pour water into it. When the water reaches a particular level, a loud sound may be heard. Once the level is crossed the loud sound ceases. The loud sound is due to resonance between the fork and the air column in the jar above the particular level of water.

(3) Stretch two identical wires side by side under the same tension, between two bridges on a wooden board. Place a third wire under a different tension beside them. The first two have the same natural frequency while that of the third is different. If one of the identical wires be plucked its vibration will be transmitted to the other two wires. In the identical wire resonant vibration will occur and grow to a large amplitude. A paper rider placed on it may be thrown off. But this will not occur on the third wire where the vibration is forced and consequently the amplitude is small.

(4) In stringed instruments like the sitar, esraj, guitar, violin, etc., strings are stretched on a thin wooden board. The vibration of a string produces forced vibration of the board and thence of air. This makes the emitted sound strong.

Many of these instruments have several strings tuned to different frequencies. When a note is sounded on the principal wire resonant vibrations are excited in the strings which have the same note as the sounded note. This increases both the intensity and the pleasantness of the note played. In 'percussion' instruments like drums, the intensity of sound is increased by the forced vibration of air inside them.

(5) It has at times been noticed that when some particular note is played on an organ or a piano, some object inside the room, such as a vase, may produce a sound. This happens when the frequency of the note agrees with some natural frequency of the object. (An object may have several natural frequencies corresponding to the various modes in which it can vibrate.)

Loudspeakers of poor quality sometimes give a magnified response to certain parts of the musical scale because of such resonance. The result is boomy reproduction.

C. Electromagnetic examples. The principle of resonance is utilised in tuning a radio receiver. A circuit of low resistance has a natural frequency of oscillation depending on its inductance and capacitance ($n=1/2\pi\sqrt{LC}$). To receive the radio waves from a given station the frequency of the oscillatory circuit of the receiver is adjusted to the value of the incoming waves. The latter then sets up resonant electrical oscillations in the receiver. These are amplified by valves and operate the loudspeaker. Reception of radio signals is thus brought about by resonance. This is why one particular station (*i.e.* frequency) can be "tuned in" at a time.

The phenomena of forced and resonant vibration of electrons under the impact of electromagnetic waves have been utilised to explain scattering, dispersion and absorption of light.

Exercises

1. Explain the terms *periodic motion, period, frequency and amplitude* of vibration. Give two examples of periodic motion.

2. Define simple harmonic motion in two ways. What are the characteristics of S H.M ?

3. From the definition of simple harmonic motion, derive expressions for (*i*) the displacement of the particle, (*ii*) its velocity, and (*iii*) its acceleration.

4. Take an example of S.H.M., and explain the terms *periodic time, frequency, angular frequency* and *amplitude* of the motion.

Find the relations between periodic time, frequency and angular frequency.

5. What is an *auxiliary circle* in S.H.M. ? How is S.H M. related to it ?

6. What do you understand by *phase* in S.H.M. ? What are *phase angle* and *phase difference* ? Illustrate them with the help of the auxiliary circle.

7. How does energy in S.H.M. depend on the mass of the particle and the amplitude and frequency of the motion ? At which places in an S.H.M. are the kinetic energy and the potential energy of the particle a maximum ? Where are they zero ?

What is the displacement of a particle in S.H.M. when its kinetic and potential energies are equal ? [Ans. : amplitude$\div\sqrt{2}$].

8. Distinguish between free and forced vibrations, giving examples. Explain the nature of damped oscillation with an example.

9. Distinguish between forced vibration and resonance, giving examples.

10. A mass m is suspended vertically by a spiral spring. When the mass is pulled down a distance x a restoring force kx acts on it (k is a constant). What will be the nature of the motion of the mass when it is pulled down a little and released ? Find the frequency. [*Hint* : see Sec. 1-4.1 (ii)].

11. Show that when a particle in S.H.M. is at a distance of $\sqrt{3}/2$ times its amplitude, its velocity is half the maximum velocity.

12. What is a simple pendulum ? How would you realize it in practice ? Calculate its periodic time.

State the laws of a simple pendulum.

13. Explain the terms phase and phase difference in S.H.M. Two particles have S.H.M. of the same frequency. If one of them is at the midpoint of its motion and the other at the end of the swing, show that their phase difference is $\pi/2$ or $3\pi/2$.

14. How can you represent an S.H.M. graphically ?

CHAPTER
4

PROPAGATION OF VIBRATIONS : WAVE MOTION

2-1. Introduction. In this chapter we shall consider the 'propagation' (that is, transmission) of vibrations in an 'elastic medium'. This is wave motion.

In chapter 1, we have considered the simplest kind of vibration, which is simple harmonic motion. To keep matters simple, we shall consider transmission of simple harmonic vibrations only through an elastic medium. Now, what is an 'elastic' medium ? It is a material medium in which elastic forces of some kind or other act between neighbouring particles. In propagation of vibrations through an elastic medium, *two factors* are necessary. One is *inertia*. It is necessary for vibration, for the particle must not stop at its normal position of rest. There is no force on it in that position. It must be carried through this position by 'inertia'. Ths other is *elasticity*. There must be elastic forces acting between neighbouring particles. This will enable a vibrating particle to cause neighbouring particles also to vibrate in the same way. It is through such elastic forces that vibrations are transmitted from particle to particle through the medium. So *a material medium that will propagate elastic vibrations must have inertia and elasticity distributed throughout it.*

A medium of which the properties are the same in all directions, is called an *isotropic* medium. Air, water, glass are isotropic. So are many other homogeneous materials. In an isotropic medium, vibrations are transmitted with the same velocity in all directions. Again, for simplicity, we shall confine ourselves to the transmission of simple harmonic vibrations in *one direction* only. The resulting motion in space and time is known as *plane, progressive, harmonic wave motion.* We shall come back to it later. But in the mean time let us add a few words in general about wave motion.

2-2. Wave motion. Wave motion in matter is the kind of motion that takes place in time and space when vibrations are being propagated through an extended material medium. The commonest kind of wave motion that we see is when we throw a stone on the still surface of water in a pond. A portion of the water where the stone has been thrown rises and falls a few times, that is, it vibrates. This rise-and-fall spreads out on the surface of water in all directions in the form of concentric circles. If you have a piece of floating cork on the water surface, you will find that it simply rises and falls with the moving water surface, but does not go on advancing with the waves. This is a characteristic of wave motion. *In wave motion, no portion of the medium advances with the waves ; it executes a vibratory motion about its normal position of rest.*

You will naturally ask, 'What is it then that advances in wave motion' ? The answer to this question is that *it is the state of vibration, that is, the phase of motion that advances.* In the given case, the waves advance along radial lines from the centre of disturbance. Any such radial line is a direction of wave propagation in this case. The particles of water along any such line are thrown into vibration one after another as the wave advances. A particle more remote from the centre of disturbance acquires at some later moment the same state of motion (same phase) as a particle nearer to the centre. All particles lying on a circle concentric with the centre of disturbance are in the same state of motion at a given instant. They all acquire their maximum elevation over the still water surface, or maximum depression below it, at the same time. They are all in the same phase at any given moment. But this phase (that is, the state of vibration) changes with time.

In the above case, the rise-and-fall (or vibration of a particle) occurs only a few times and then stops. Besides, the amplitude of vibration dimininishes with increasing distance from the centre. Such factors complicate any elementary discussion of wave motion. We shall, therefore, *idealize* our waves for simplicity of discussion. This is like introducing idealizing conditions in other fields of physics. Think, for example, of the point particle, the perfectly smooth surface, the weightless and perfectly flexible string of a simple pendulum, etc. They don't really exist ; but they are of great help in treating problems.

2-2.1. The idealized wave : Plane, progressive, harmonic wave. In our idealized wave, the vibrations of all particles of the medium will be simple harmonic. That is why we call it a *harmonic* wave. We call it *plane* because all particles on a plane perpendicular to the direction of propagation are taken to be in the same phase at any given instant. It should be clear that the waves propagate (move forward) in *one direction only*. (They do not spread out in all directions in a plane as in the case of water waves just described.) The waves are called *progressive* because they do not come across any boundary on their way which will modify the waves.

The fact that you cannot have a real wave satisfying these conditions does not matter. You cannot realize a simple pendulum, you cannot realize a point particle or a ray of light or a frictionless surface. Nevertheless, these idealized concepts help you in getting useful results. The same applies to our idealized wave.

In Sec. 2-5 we shall take up the properties of these idealized waves. Before that I should tell you what are meant by 'transverse' and 'longitudinal' waves.

2-2 2. Transverse waves. Waves in which the direction of vibration of the particles of the medium is perpendicular to the direction of propagation, are known as *transverse waves*. You may think that the water waves mentioned earlier are transverse. But strictly it is not so. The vibration of particles has components in directions perpendicular and parallel to the direction of propagation. (The particles move in circles or ellipses.)

Visible examples of transverse waves are few in number. If the free end of a long suspended heavy cord is jerked perpendicular to its length, transverse vibrations move towards the upper end in the form of a wave (Fig. 2.1). A stretched string plucked or struck laterally executes transverse vibrations. Light, heat and radio waves are transverse in nature, but we cannot see their vibrations.

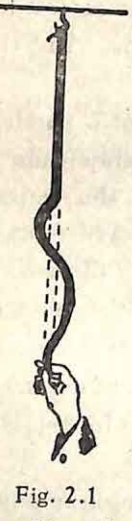

Fig. 2.1

When a wave advances along a string (Fig. 2.1) it will be seen that in one half of a wave the particles are displaced to one side and in the other half, to the other side. A complete wave is made up of two such halves in which the displacements are in opposite directions. In a transverse wave travelling horizontally, the part of the wave above the undisturbed part is called a **crest** and the part below is called a **trough**. *A crest* *and a trough together make up one wave.* The words are often used to represent two halves of any transverse progressive wave. Particles on a crest and a trough, which are at equal distance from the equilibrium position but move in opposite directions, are in opposite phases. A succession of waves of finite number is called a **wave train.**

2-2.3 Longitudinal waves. Waves in which the direction of particle vibration is along the direction of propagation of the waves, are called *longitudinal waves*. To visualize such a wave, take a long, vertical, closely wound spring (Fig. 2.2) and load it with a weight at the lower end. Pull it slightly downward and then release it. The turns of the spring will vibrate up and down along the length of the spring. If you mark a point on the spring with a short piece of white thread, you will find the mark moving up and down. You may treat the spring as the elastic medium, the marked point as a particle and the motion of the spring as wave motion. The layers of the spring may be considered as layers of the medium, all particles of a layer being in the same phase of motion.

Since the particles in this kind of waves vibrate in the direction of wave propagation, their displacement does not produce crests and troughs. In that part of the wave in which the particles *move in the direction of wave propagation*, they come nearer together than their normal separation in the medium and form a **compression.** A layer of medium in this portion is in a state of compression. (Longitudinal waves are also called *compressional waves.*) If the medium is a fluid (that is, a liquid or a gas), *pressure* in the compressed layer will be higher than the normal value (that is, when there are no waves). If the medium is a solid, compression will cause the *stress* (PROPERTIES OF MATTER ; Sec. 2.1) in the layer to increase. In that part of the wave where the particles *move opposite to the direction of wave propagation*, they are further apart than their normal separation and form a

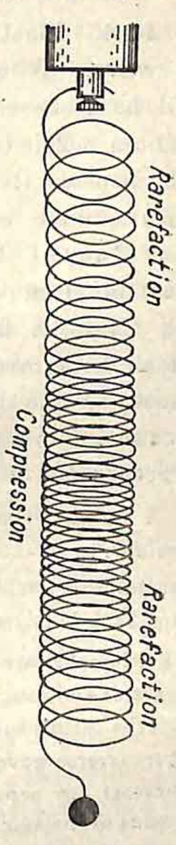

Fig. 2.2

rarefaction. The medium in this portion is rarefied. The 'pressure' or the 'stress' in a rarefied layer will be lower than normal. A complete longitudinal wave consists of one compressed and one rarefied region. *The pressure or stress therefore alternates about the normal value in a longitudinal wave.* Sound waves are longitudinal waves.

Don't think that all waves are either transverse or longitudinal. In many kinds of waves, particle displacements may have components parallel and perpendicular to the direction of wave propagation. But it need not bother us. We are not concerned with such waves now.

2-2.4. Elastic properties of a medium determine the nature of wave. Whether a wave generated by the vibration of a particle will be transverse or longitudinal depends on the nature of the medium. This is so because the direction in which a vibrating particle will displace its neighbour depends on the acting elastic forces. Since a fluid cannot sustain a shearing stress the displacement of any of its layers cannot drag a parallel layer in its own direction of motion. Hence *a transverse wave cannot be produced in a fluid.* A fluid, however, resists a volume change. Hence a sudden compression or rarefaction applied in a given direction to a gaseous or a liquid layer is transmitted along the line of the applied force to adjoining parallel layers. *A longitudinal wave, therefore, can be propagated in a gas or a liquid.*

A solid resists deformation of both size and shape. So, if a particle in a solid is disturbed, it displaces others lying in directions parallel and perpendicular to its own direction of motion. Hence *in a solid both longitudinal and transverse waves are possible.*

Earthquakes are caused by large vibrations in the interior of the earth. Both transverse and longitudinal waves are produced during an earthquake. As they reach the earth's surface with different velocities (longitudinal waves at 7·2 km/s and transverse waves at 4 km/s) their responses on the seismograph (earthquake indicator) are separate. From this time interval, it is possible to estimate the distance of the epicentre of the earthquake.

When the *displacement of the disturbed particle is small*, the restoring force is proportional to displacement and acts towards the equilibrium position of the particle. Under such conditions the motion is simple harmonic. Such waves are called *simple harmonic waves.* In our discussions we shall assume this condition to hold.

2·2 5. Waves transmit energy. From where do the particles that vibrate in wave motion get their energy ? Waves spread out through a medium only when a portion of the medium is displaced. The agent that causes this displacement is the supplier of energy. The energy that the agent transfers to the medium spreads out in the medium in the form of waves. *Wave motion is the most important form of energy transmission, particularly over long distances.* (You may have heard of 'beam transmission' in radio.)

Summary of Sec. 2-2. *In material wave motion, the particles on a line of propagation vibrate about their normal position of rest. A point on a line of propagation more remote from the source of disturbance acquires at some later instant the same state of motion (phase) as a point nearer to the source. Wave motion means propagation of the phase of vibration. What advances in wave motion is the phase of motion, and not any portion of the medium.*

Though waves may be of various kinds, we shall consider only longitudinal and transverse waves. Longitudinal waves can pass through all kinds of media, solid, liquid or gas. Transverse waves are possible in solids only. *In longitudinal waves, layers of the medium are alternately compressed and rarefied. Compression occurs in the layer in which particle velocities are in the direction of wave propagation. When particle velocity is opposite to the direction of wave propagation, rarefaction occurs. In a longitudinal wave, pressure (in fluids) and stress (in solids) alternate about their normal values at any point. Sound waves are longitudinal. Waves carry energy received from the source.*

2-3. Propagation of vibrations. Let us consider the particles of a medium lying on a straight line (Fig. 2.3). Let one of them be displaced perpendicular to the line and made to execute a simple harmonic motion along its line of displacement. Its neighbour will follow suit but will start a little later. Between these two vibrating particles there will be a small phase difference because of this time lag. The vibration is then transmitted from particle to particle with a small phase difference between two successive particles. *The phase difference between any two particles on the line of propagation will be proportional to the distance separating them.*

Individual particles will vibrate with a constant periodic time T. In Fig. 2.3 the particles are shown after intervals of time $T/12$. Each particle starts to

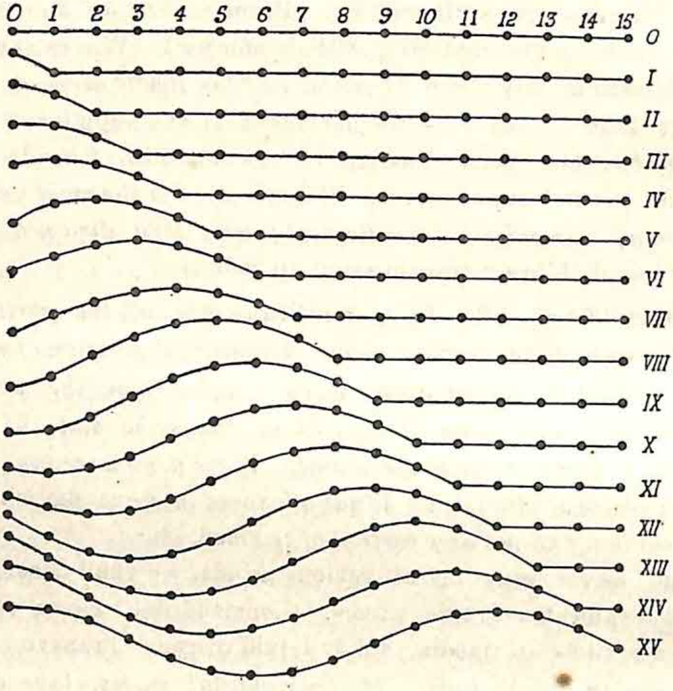
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Fig. 2.3

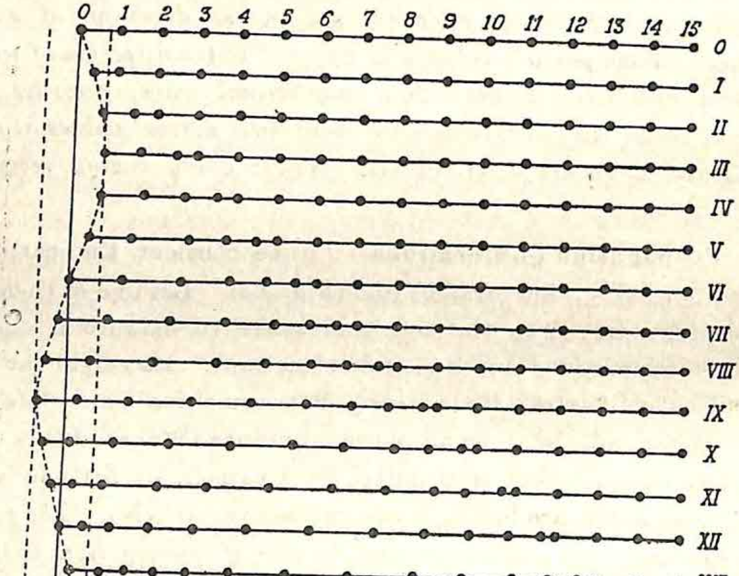
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Fig. 2.4

move $T/12$ after the particle preceding it. The figure indicates that the vibrations gradually pass on from particles on the left to those on the right. The direction along which the vibrations are transferred is the direction of wave propagation.

When the 'zero' particle has completed one vibration (*i.e.* when it is passing through its initial position with the initial velocity in the initial direction) the 12th particle just starts moving. These two particles therefore vibrate in phase. But since the former has completed one vibration when the latter begins to move, the phase difference between them will be 2π radians. The phase difference of any particle relative to the zero (zeroth) particle will increase with its distance from O, as has already been noted, till it grows to 2π radians at particle 12.

The above analysis indicates that the difference in phase angles of the vibrating particles will increase linearly with distance. For some particular distance it will be 2π radians. Such particles are in the same phase of vibration ; they have the same displacements and are moving in the same direction. *The minimum distance, measured along the line of propagation of a wave, between two particles in the same phase of vibration is called the* **wavelength** (λ) *of the wave.* Particles on the line of propagation separated by a distance that is a multiple of λ will differ in phase by the same multiple of 2π radians.

The displacements of particles in a longitudinal wave can be investigated exactly as above and are shown in Fig. 2.4. The particles marked 0, 1, 2, etc. are on the line of propagation. The figures marked I, II, III, etc. indicate the positions of particles at intervals of $T/12$ where T, as before, is the time period of vibration of a particle. A particle executes an S.H.M. about its mean position of rest in the direction of wave propagation. AB and AC represent the amplitude of this vibration. Any two neighbouring particles have a small relative phase difference.

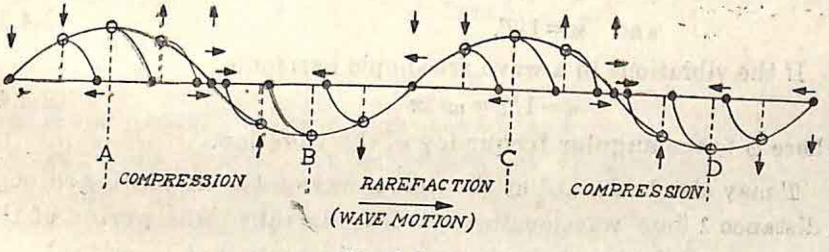

Fig. 2.5

Fig. 2.5 represents the *wave form* (or *wave profile*) of a longitudinal wave. The horizontal axis of the figure represents the equilibrium position of the particles. The vertical axis represents the displacement of particles along the line of propagation. It resembles the wave form of a transverse wave exactly.

2-4. Some important definitions in connexion with waves. In speaking of waves we have to use some special terms. You must know them well and be clear about their meaning. These are given below.

(i) Wave velocity or phase velocity (c). It is the velocity with which the phase of vibration advances along a line of wave propagation. We shall denote it by the letter c.

(ii) Wave length (λ). It is the shortest distance between two particles on a line of wave propagation which are in the same phase of vibration. Particles on a line of propagation which are separated by distances equal to integral multiples of λ are all in the same phase.

(iii) Frequency of a wave (n). Frequency of a wave is the number n of wavelengths contained in a distance equal to the wave velocity c. Hence the relation between c and λ is

$$c = n\lambda. \tag{2-4.1}$$

We can look at it from another angle. As a particle in a wave starts vibrating, it at the same time starts disturbing the particle ahead of it on the line of propagation. By the time the first particle has executed one complete vibration, the disturbance has advanced a certain distance. This distance is the wave length. If T is the periodic time of vibration of the particle (and they are all alike in a medium), we must have $\lambda = cT$. The reciprocal of T is the frequency of the particle. So $c = \lambda/T$. But $c = n\lambda$ also. Therefore $n = 1/T =$ frequency of vibration of the particle as well. Thus

$$c = n\lambda = \lambda/T, \tag{2-4.2}$$

$$\text{and} \quad n = 1/T. \tag{2-4.3}$$

If the vibrations in a wave are simple harmonic,

$$n = 1/T = \omega/2\pi \tag{2-4.4}$$

where ω is the **angular frequency** of the vibration.

T may also be defined as the time a wave takes in moving through a distance λ (one wavelength). It may be called the **period of the wave** as well (besides being the period of vibration of a particle).

(iv) Amplitude of a wave. The maximum displacement that a vibrating particle undergoes in a wave is called the *displacement* *amplitude* of the wave. Similarly, we shall have 'velocity amplitude', 'pressure amplitude', etc.

(v) **Wave front and ray.** When a wave is passing through a medium it is always possible to find at any instant a surface through any point (of the medium) on which (surface) the particles are in the same phase of vibration. The continuous locus of points in the same phase of vibration in a medium through which a wave is travelling is called a **wave front.** A wave front does not remain stationary but advances through the medium with a definite velocity called the **wave velocity** or **phase velocity.**

A normal to a wave-front is called a **ray.** The energy that a wave carries, moves along the rays. In a homogeneous medium, a disturbance produced at a point inside it, spreads out in all directions with equal velocity. Particles equidistant from the source will vibrate in the same phase. Hence the wave front will be spherical. Such waves are called **spherical waves.** A wave in which the wave fronts are plane surfaces is called a **plane wave.** A finite portion of a wave front coming from a source at a very great distance is practically a plane surface and may be treated as a plane wave front.

(vi) **Wave form or wave profile.** In Fig. 2.3, if 0 executes an S.H.M. at any instant the space-displacement curve of the particles lying between 0 and 12 will be a complete sine curve (Fig. 2.6). The space-displacement curve (for a complete wave at any moment) is known as the **wave form** or **wave profile.** It pictorially represents the displacements at any instant of all the particles lying within a distance of one wavelength along the line of propagation. (Compare curve XII Fig. 2.3)

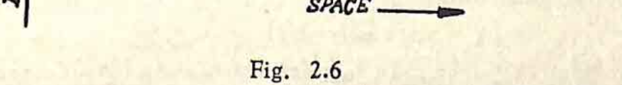

Fig. 2.6

In a simple harmonic wave, if we plot the displacement curve of any single particle for a complete period T, then also a sine curve results (Fig. 2.7). This means that the time-displacement curve of a single particle of the medium traversed by simple harmonic waves is exactly similar to the wave form (i.e., the space-displacement curve)

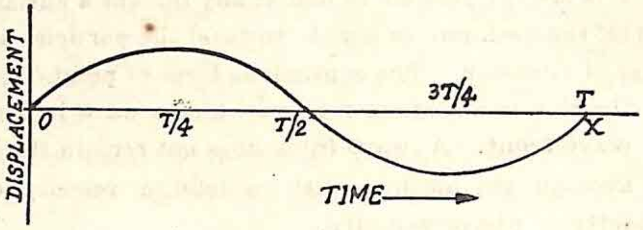

Fig. 2.7

at any instant. In other words, the cinematographic picture of displacements (*i.e.*, a record of successive positions) of a single particle of the medium in time T agrees with an instantaneous photograph of displacements of all the particles within a distance λ.

2-5. Equation of plane, progressive, harmonic waves. In Sec. 2-2.1, we have introduced the *idealized* wave, which we called a *plane, progressive, harmonic wave.* Let us derive a mathematical expression for particle displacement in such a wave. For simplicity, we often call it a plane, harmonic wave.

Take the positive x-axis as the direction of propagation, and any convenient point on it as the origin ($x=0$). The equation of particle displacement at the origin will be $y=a \sin \omega t$. (The waves may be transverse or longitudinal.) A particle at a distance x from the origin will acquire after an interval t' the phase that the particle at the origin has at time t. Hence the displacement equation for the particle at x will be $y_{x,t}=a \sin \omega\,(t-t')$, If c is the wave velocity $x=ct'$, and we shall have

$$\begin{aligned} y_{x,t} &= a \sin \omega(t-x/c) \\ &= a \sin (\omega/c)(ct-x) = a \sin (2\pi/\lambda)(ct-x) \qquad (2\text{-}5.1) \end{aligned}$$

Introducing the various quantities n, T, λ of Sec. 2-4 into this equation and writing $k=2\pi/\lambda$ (k is called the *wavelength constant*), we can throw Eq. 2-5.1 into various equivalent forms. Dropping the subscripts x, t from y for convenience (since the sense is clear,) we have

$$\begin{aligned} y &= a \sin 2\pi n(t-x/c) = a \sin 2\pi(t/T - x/cT) \\ &= a \sin 2\pi(t/T-x/\lambda) = a \sin (\omega t-kx) \qquad (2\text{-}5.2) \end{aligned}$$

The last is the most compact form of writing the equation. We shall use it more often. a is the amplitude of the wave. Remember $k=2\pi/\lambda$.

Properties of the wave : (1) *The waves are plane*, because the phase angle depends on x alone. For a given x, all points on the y-z plane perpendicular to the x-axis are in the same phase. Hence the wave front is plane.

(2) *The waves are progressive.* If in Eq. 2-5.1 we increase t by unity and x by c, we get the same value for y. This means that y at $x+c$ and $t+1$ is the same as y at x and t. In other words, the phase of the vibration has moved a distance c in one second. Using symbols, we find

$$\begin{aligned} y_{x+c,\,t+1} &= a \sin \omega\{(t+1)-(x+c)/c\} \\ &= a \sin \omega(t-x/c) = y_{x,t}. \end{aligned}$$

The phase advances along the direction of the positive x-axis with velocity c. The equation

$$y = a \sin \omega(t+x/c) = a \sin (2\pi/\lambda)(ct+x) \tag{2-5.3}$$

when treated as above, will show that it represents a wave moving in the direction of the *negative* x-axis with velocity c.

(Note that the waves represented by Eq. 2-5.1 or 2-5.3 have no ending. They do not indicate any stoppage of motion.)

(3) That *the vibrations in the wave are simple harmonic* is our basic assumption.

(4) *The waves have a two-fold periodicity, one in time and one in space.* The *time periodicity is* T, for as t increases by T the vibrations repeat themselves. They are also repeated at intervals of space equal to λ, the wavelength. *λ is the space period.*

Examples. (1) A wave has an amplitude of 0·1 mm, velocity of 350 m/s and frequency 500/s. What are the wavelength and period ? Write down its displacement y at the point x.

[Ans. : Wavelength $= \dfrac{\text{velocity}}{\text{frequency}} = \dfrac{350 \text{ m/s}}{500/\text{s}} = 70$ cm.

Period $= 1/\text{frequency} = 1/500\text{s}^{-1} = 0{\cdot}002$ s.

Equation : $y = 0{\cdot}01 \sin 2\pi \times 500(t - x/35000)$ cm.

Here y and x are in cm and t in seconds. If y and x are in metres

$y = 1.10^{-4} \sin 1000\,\pi\,(t - x/350)$ m.]

(2) The displacement equation in a wave is $y=1\cdot10^{-5}\sin 2\pi\left(\frac{t}{0\cdot01}-\frac{x}{200}\right)$ metre. If t is in seconds and x in centimetres, find (a) the amplitude, (b) period, (c) frequency, (d) wavelength and (e) wave velocity.

[ANS. : (a) $1\cdot10^{-5}$m ; (b) 0·01 s ; (c) 100/s ; (d) 200 cm ; (e) 200 m/s]

(3) If the wavelength is 1 metre, what is the phase difference in the vibration of two particles which are 10 cm apart on the line of wave propagaion ? [ANS. : 36°. Note that a distance λ corresponds to a phase difference of 2π].

2-5.1. Periodic waves. Vibration of particles in a wave may be periodic, but not simple harmonic (Fig. 2.8c). Such a wave is a *periodic wave*, but not a harmonic one. We have said that a periodic vibration may be expressed as the sum of a suitable number of S.H.M.s of appropriate amplitudes. Similarly, any periodic wave may be expressed as the sum of a suitable number of harmonic waves of appropriate amplitudes. Curve 2.8(c) is the combination of pure simple harmonic waves 2.8(a) and 2.8(b).

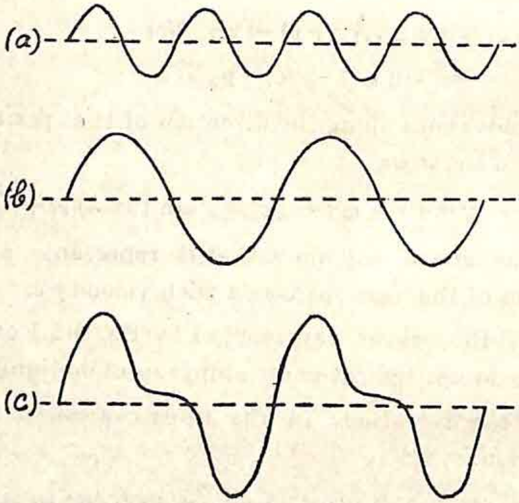

Fig. 2.8

2-5.2. Characteristics of progressive wave motion. We may now summarize the characteristics of progressive wave motion as follows :

(i) Progressive waves are produced by continuous vibration of a a portion of the medium. They advance through the medium with a velocity determined by the elasticity and density of the medium.

(*ii*) Each particle of the medium executes the same vibration about its equilibrium position with identical frequency and amplitude. The vibrations may be transverse, longitudinal (or torsional) relative to the line of propagation of the waves.

(*iii*) The state of vibration, *i.e.*, the phase, of one particle is transmitted to the next along the line of propagation. The phase difference between two particles along this line is proportional to their separation.

(*iv*) *Progressive wave motion is thus a disturbance recurring both in space and time.* The space-displacement curve at any instant (of the particles on a line of wave propagation) agrees with the time-displacement curve of any single particle. Both are sinusoidal in a simple harmonic wave.

The wavelength λ gives the periodicity in space and $T=1/n$ gives the periodicity in time. In time T the wave moves a distance λ, giving a velocity of wave propagation $c=\lambda/T$.

(*v*) Progressive wave motion carries energy from one point to another along the wave-normals, *i.e.*, the rays, without any bodily transfer of the medium.

(*vi*) As a compressional (*i.e.*, longitudinal) wave proceeds, every point of the medium suffers the same changes in pressure and density (in fluids) or stress and density (in solids).

There are various kinds of wave apparatus to demonstrate the above characteristics.

2-6. Doppler effect. It is an effect associated with waves when the source or the receiver is moving. When any one or both are in motion, the apparent frequency as found is different from that emitted by the source. *The effects upon the apparent frequency of a wave produced* (*i*) *by the motion of the source toward or away from an observer at rest, and* (*ii*) *by the motion of the observer toward or away from a stationary source, are known as* **Doppler effects.**

The whistle of a railway engine or the sound emitted by a moving aeroplane appears to have a higher frequency as it approaches an observer at rest. As the source moves away from the observer, the frequency appears to be reduced. When a moving observer approaches a source of waves, the apparent frequency increases. When the observer moves away from the source, the apparent frequency falls. These are all cases of Doppler effect. The effect is common to all wave motions including sound and light. In order that the effect may be appreciable the velocity of motion of the source or observer must be an appreciable fraction of the wave velocity.

We may do a little bit of calculation. c will stand for wave velocity.

(i) **Source moving : Observer at rest.** Let the source S be moving *towards* the observer with velocity v_s. The n waves emitted by the source S in one second will be contained in a distance $c-v_s$

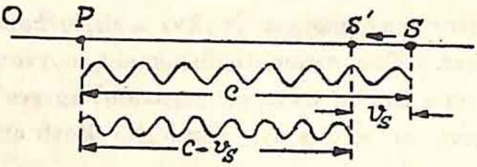

Fig. 2·9

(Fig. 2.9) and will have a wavelength $\lambda'=(c-v_s)/n$. The observer will take them as waves of frequency $n'=c/\lambda'=nc/(c-v_s)$. The change in frequency is $n'-n=nv_s/(c-v_s)$.

If the source is moving *away* from the observer, the sign of v_s will be negative. In that case we shall have $n'=nc/(c+v_s)$ and $n-n'=nv_s/(c+v_s)$.

(ii) **Observer moving ; source at rest.** When the observer moves towards the source with a velocity v_0, he receives in one second all waves contained in a distance $c+v_0$. This number is $n'=(c+v_0)/\lambda=(c+v_0)/(c/n)$ as n waves are contained in a distance c. Hence the apparent frequency is $n'=n(c+v_0)/c$. It is greater than n.

If the observer is moving away from the source with velocity v_0, the sign of v_0 in the above relations will be negative. Thus $n'=n(c-v_0)/c$. It is less than n.

Remember that in all these cases n' is the frequency as it appears to the observer.

2-7. Characteristic properties of waves. Waves have the following characteristic properties :

(a) Through a homogeneous, isotropic medium waves travel with the same velocity in all directions.

(b) When a wave travelling in a medium falls on a surface (*large compared with the wavelength*) separating it from another, part of it turns back into the first medium while the rest passes into the second. The first phenomenon is known as **reflection** and the second as

refraction. They occur at any boundary, the elastic properties or densities on the two sides of which are different. In both cases there is a change in direction of wave propagation. The laws governing reflection and refraction are already familiar to the students of light.

(c) When a wave meets an obstacle of *size comparable with its wavelength* or passes through an opening of similar size, it bends round the corners to some extent and encroaches into the region of geometrical shadow. This phenomenon is known as **diffraction**.

(d) If two *identical wave trains* passing simultaneously through a medium *continue to meet* in opposite phases at some points, the particles at these points will remain permanently at rest. Two waves in this way neutralise the effects of each other at these points. The phenomenon is called **interference**.

(e) If a wave meets a small body in a medium, the body is thrown into forced vibration by the wave. The body (of *size small compared with the wavelength*) absorbs energy from the wave and spreads out this energy in the form of spherical waves through the medium. The phenomenon is known as **scattering**. It weakens the waves.

(f) Transverse waves have a special property known as **polarization** which longitudinal waves do not possess. The transverse vibrations may be confined to a plane containing the direction of propagation. With appropriate devices, these vibrations may be prevented from advancing. But this cannot be done to longitudinal waves. Light waves show polarization ; sound waves do not. Polarization distinguishes between transverse and longitudinal waves.

2·8. Comparison of transverse and longitudinal waves.

Transverse wave	*Longitudinal wave*
1. The vibration of particles is at right angles to the direction of wave propagation.	1 Vibration of particles is in the direction of wave propagation.
2. Can be produced only in solids but not in gases or liquids.	2. Can be produced in solids, liquids and gases.
3. A crest and a trough make up a complete wave.	3. A complete longitudinal wave consists of a compressed and a rarefied region.
4. Exhibits polarization.	4. Does not show polarization.

Exercises

1. Take any material wave as an example and illustrate with its help what happens in wave motion as also the nature of wave motion. What advances in a wave?

2. What is a *longitudinal wave*? What is meant by *compression* and *rarefaction* in a longitudinal wave? In which directions do particles of the medium move in two such portions of a wave? What kind of motion does a particle of the medium execute in a longitudinal wave?

3. While a longitudinal wave can move through all kinds of material media, a transverse wave can move through only a solid. Why is it so? Where does the energy of wave motion come from?

4. Discuss the characteristics of wave motion. (See Sec. 2-2)

5. What is a simple harmonic wave? What do you understand by the *amplitude*, *periodic time*, *frequency*, and *wavelength* of such a wave?

Find the relation between the wave velocity and wavelength. [*Hint :* Use their definition to find the relation].

6. Explain the terms *periodic wave* and *wave form*. What importance would you attach to wave form? What relation does a periodic wave bear to simple harmonic waves?

7. What is meant by a *plane, progressive, harmonic* wave? Explain the italicized words. What is the importance of such a wave?

Establish an equation for particle displacement in such a wave, and explain the meanings of the symbols used.

8. Show how the equations $y=a \sin 2\pi n\,(t-x/v)$ and $y=a \sin 2\pi n\,(t+x/v)$ represent two waves moving in opposite directions along the x-axis. Explain the symbols used in the equations.

9. Take any of the equations in the above question and show that the displacement is repeated at intervals of time $T=1/n$ and distances v/n.

10. State the common properties of waves and briefly explain their nature.

11. What is a *wave front*? How are rays and wave fronts related?

12. What is Doppler effect? Illustrate it with an example. If the source advances towards an observer with velocity u what will be the apparent frequency of the waves?

13. (*a*) A tuning fork vibrates 200 times per second. If the velocity of the waves generated in air by these vibrations have a wave velocity of 340 m/s, what are the periodic time and wavelength of the waves? (Ans. : 1/200 s ; 1·7 m].

(*b*) If the waves are simple harmonic, what will be the equation of particle displacement in the waves if the amplitude is 10^{-6} cm? [Ans. : $y=10^{-6} \sin 400\,\pi(t-x/34000)$ cm.]

(*c*) If the frequency of the fork vibrating in water is 256 cps (cycles per second) and the velocity of longitudinal waves in water is 1024 m/s, how many vibrations will the fork execute before the wave moves 100 m? [Ans. : 25].

CHAPTER

3

SOUND WAVES, VELOCITY OF SOUND, REFLECTION AND REFRACTION

3-1. Sound waves as elastic waves. (1) It is well known that sound is due to vibration of a body and that it requires a *material medium* for propagation. Sound cannot pass through vacuum.

(2) As sound travels through air, the air does not advance with the sound. Even in a loud sound, there is no air current. This behaviour is like the behaviour of a wave. An advancing wave does not carry the medium with it.

(3) In all media (solid, liquid or gas), sound travels with a definite velocity characteristic of the medium. The velocity is determined by the *elasticity* and *density* of the medium. Waves behave similarly. So sound must be wave-like in nature—waves in a material medium.

(4) Only longitudinal (also called compressional) waves can move through gases and liquids. Transverse waves cannot do so. Hence sound waves must be *longitudinal* (that is, compressional) in character.

(5) All waves have the common properties of reflection, refraction, diffraction and interference. Sound shows all these properties. This supports the wave nature of sound.

(6) Only transverse waves show the phenomenon of polarization. Longitudinal waves do not have this property. Sound does not show polarization. This supports the longitudinal character of sound waves.

These facts convincingly prove that sound waves are elastic waves, longitudinal (compressional) in character.

3-1.1 Definition of sound. Sound has been defined by the American Standards Association as follows :

Sound is an alternation in pressure, stress, particle displacement or particle velocity, which is propagated in an elastic material, or the superposition of such propagated vibration.

The definition is objective ; it does not depend on whether any body hears the sound or not. Sound is audible when the frequency of alternation lies within the approximate limits 20 and 20,000 cps. Dogs, bats, birds can respond to higher frequencies.

When you get the opportunity to go through different aspects of sound, try to notice how the aspects agree with the definition. For practical purposes, the most important aspect is the change of pressure in sound waves. Most methods of detection and measurement in sound depend on the changing sound pressure. (See the Chapter on sound recording and reproduction.)

3·2. **Audible and inaudible sound.** Whether compressional waves will produce the sensation of sound in the human ear depends upon the frequency of the waves. Ordinarily, the human ear responds to compressional frequencies in the approximate range 20 to 20,000 cycles per second (cps). The range, however, varies from person to person. In the same person, the range is reduced with age.

Compressional waves of frequency lower than 15 cps are inaudible to most people. Still they are sound waves and are called *infrasonic waves.* Compressional waves of frequency higher than 20,000 cps are called *ultrasonic waves.* They are also inaudible to the human ear. Study of ultrasonic waves now forms a very important branch of physics. It has found useful applications in industry and other fields.

3·3. **Sources of sound.** Any solid, liquid or gaseous body which can vibrate between 20 and 20,000 times a second, and transfer the vibrations to the medium in which it is situated producing compressional waves in the medium, is a source of sound. If the frequency is beyond this range, the sound is inaudible.

Clearly, a classification of sources of sound does not carry any sense, unless you say on what basis you want to make the classification. When we hear a sound we can try to identify the source of sound. Strike a solid ; you will hear a sound. The solid is the source. When rain-drops fall on the water in a pond, the sound that you hear is due to the vibrations of portions of water. These vibrating portions are the sources. In musical instruments we can indentify the sources often easily. In stringed instruments, such as the sitar, esraj, piano, guitar, etc., the vibrating string is the source. In percussion instruments, such as drums etc., the vibrating membrane is the source. In flutes or clarionets, the vibrating air column inside is the source.

Our *voice box*, a small organ inside the throat, is the most important source of sound to us. It has two thin membranes (vocal cords) nearly closing the gap in the box. Air from the lungs is forced through this gap, causing the membranes to vibrate. These vibrations give rise to the sounds we emit.

For experiments on sound we often require a source which gives rise to simple harmonic vibrations in air. There are complicated electrical devices for the purpose. But a very simple one which you will use in the laboratory is the *tuning fork*. It is described below.

3-3.1. The tuning—fork a special source of sound. In considering the action of waves it is very helpful to have a source which produces waves of a single frequency. In the case of sound waves, the tuning fork is such a source.

A tuning fork consists of a rectangular bar bent into the form of a U with a stem attached at the bend (Fig. 3.1). If any of the arms of the fork (also called *prongs*) is struck, both arms begin to vibrate. *The vibrations (unless very strong) are simple harmonic.* The nature of the vibrations has been shown in the figure. Both prongs simultaneously move inward (as at 1,1) or simultaneously outward (as at 2,2). Two points (*N*, *N*) near the bend do not vibrate. They are called *nodes*. The bend between the nodes rises and falls as the fork vibrates. So the stem executes an up-and-down motion.

A tuning fork has the special characteristic that it *gives out a sound of only one frequency*. A sound having a single frequency is called a '*pure tone*'. Ordinarily any sound has several frequencies in it at the same time. Because of its ability to produce a sound of a single frequency, the tuning fork is an essential equipment in a sound laboratory.

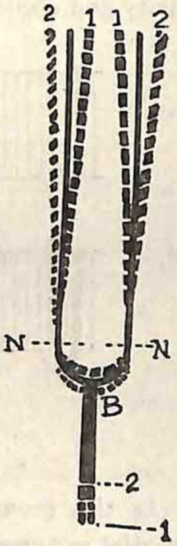

Fig. 3.1

The frequency of a tuning fork can be lowered by loading one of its prongs. Some forks are provided with sliding weights. If the weight is fixed near the free end of the fork, the lowering of frequency is greater than if the weight is fixed nearer the nodes. Instead of sliding weights we can use one or more turns of thin or thick wire wound round a prong. For very small lowering a little wax can be stuck to a prong.

If the mass of a prong is reduced, say, by filing, its frequency increases.

3-4. Mechanism of sound propagation. Let us now study the state of motion of air when sound is propagated through it. Consider a vibrating tuning fork emitting sound (Fig. 3.2). Imagine the undisturbed layer in front of the prong to be divided into thin vertical layers of equal thickness. As the arm moves outward it compresses the layer in front of it. This layer tends to expand and presses the next layer into a smaller volume. This second layer in its turn compresses the third layer and so on. In this way *the compression advances to the right from layer to layer with a definite velocity.* By the time the arm moves from the left end to the right end of its swing, the state of compression advances through a certain distance depending on the period of vibration of the fork and the density and elasticity of the medium.

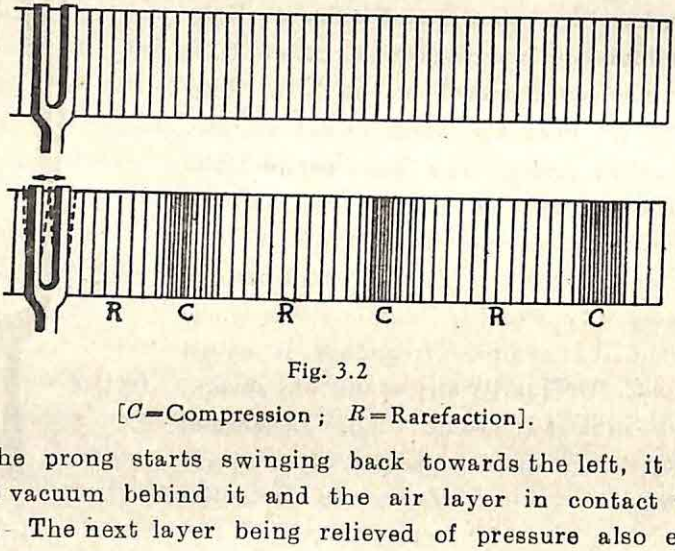

Fig. 3.2
[C=Compression ; R=Rarefaction].

As the prong starts swinging back towards the left, it creates a partial vacuum behind it and the air layer in contact with it expands. The next layer being relieved of pressure also expands. The following layers follow suit one after another. Thus *a state of rarefaction is passed on from layer to layer travelling with the same velocity as that of the compression.* A compression together with the rarefaction following it, forms a complete longitudinal wave of sound. The particles vibrate simple harmonically since the vibrations of the fork are so.

So long as the fork vibrates, alternate compressions and rarefactions occur in the surrounding medium. *The moving pattern of alternate compressions and rarefactions of the medium constitute sound waves.* From a small source in a homogeneous medium the generated sound waves spread spherically as shown in Fig. 3.3.

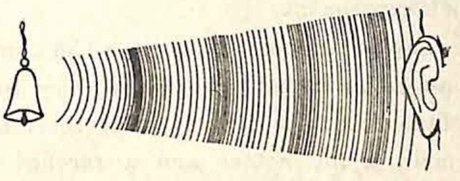

Fig. 3.3

Particle of air. We might make two things clear in this connexion. One is the meaning of the word 'particle' when the medium is air (or a gas). A particle of air does not mean a molecule of air, but a small volume, say 10^{-15}cm^3, of air. At S.T.P. it contains about 10^4 molecules. Air molecules in this volume may be supposed to have the same displacement from the mean position of the volume. In the displaced position, the volume will not contain exactly the molecules it had at the rest position. Many molecules will leave the volume due to their thermal motion, while new ones will enter. Density over the volume is supposed to remain the same. The same consideration applies to liquids. In spite of their discrete structure, *we treat gaseous and liquid media as continuous* so far as wave propagation is concerned.

Particle velocity and compression. The other is the relation between particle velocity and compression. As the right prong of the fork moves from its extreme left to its extreme right position, it gives the air particles a velocity in the direction of wave propagation. The particles disturbed in this half of the motion of the fork constitute a *compression*. Note that during the first half of the left-to-right motion of the right prong, the air particles in contact with it were to the left of their rest position. Hence particle displacements during this quarter of the fork's motion were opposite to the direction of propagation.

3-5. Velocity of sound. The velocity of sound in a medium depends on its elasticity and density. We shall not try to establish the relation between them, but shall be satisfied with a mere statement of the relation.

In solid rods, the sound velocity $c = \sqrt{E/\rho}$, where E is Young's modulus of the material and ρ its density.

In liquids and gases $c = \sqrt{K/\rho}$ (3-5.1)

where K is the bulk modulus and ρ the density.

3-5.1 Newton's equation and Laplace's correction. When volume changes are *isothermal*, it can be shown that the bulk modulus of a perfect gas is equal to its pressure P. Newton calculated the velocity of sound in air on this basis. But his results did not agree with experiment.

Laplace argued that air is a bad conductor of heat and the volume changes in sound take place very quickly. For these reasons the volume changes cannot be isothermal. A compressed layer must remain a bit hotter and a rarefied layer a bit cooler than normal. He considered the volume changes to be *adiabatic*. This means that heat cannot leave a compressed layer nor enter a rarefied layer to bring their temperatures to the same value due to the bad conductivity of air and quickness of changes.

Under adiabatic conditions the bulk modulus of a gas is γP where γ is a constant for a gas. For air $\gamma = 1{\cdot}40$. Laplace found that when we take $c = \sqrt{1{\cdot}40\, P/\rho}$, the value agrees very well with the experimental value. So, in a gas

$$c = \sqrt{\gamma P/\rho} \tag{3-5.2}$$

[γ of a gas is the ratio of its specific at constant pressure (c_p) to that at constant volume (c_v). $\gamma = c_p/c_v$.]

3-6. Velocity of sound in other media. Velocity of sound in solids is in general higher than that in liquids, and that in liquids higher than that in gases. Some values are given in the table below.

Velocity of sound in different media

Gases at S. T. P.		*Solid rods and water*	
	metres/sec.		metres/sec.
Air	331.5	Copper	3790
Hydrogen	1286	Steel	5150
Oxygen	314.8	Glass (crown)	4710-5300
Sulphur dioxide	211	Brass	3130-3450
Carbon dioxide	260.3	Water (25°C)	1497
Ammonia	415	Sea-water (18°C)	1516

You may find the following problem interesting.

Problem. An explosive on a rail is detonated by the passage of a train passing over it. A listener one kilometre away with one ear on the rail hears two reports. Explain the phenomenon and calculate the time interval between the two reports. Given, Young's modulus E for steel $= 2 \times 10^{12}$ dyn/cm^2, ρ of steel $= 7.8$ g/cm^3, ρ of air $= 0{\cdot}0013$ g/cm^3, γ for air $= 1.4$ and atmospheric pressure $= 10^6$ dyn/cm^2.

Solution : With one ear on the rail, the listener receives the sound of the signal through the rail and with the other ear in air he hears the sound coming through air. Velocity in the two media being different two reports are heard.

Velocity of sound through the rail,

$$c'=\sqrt{E/\rho}=\sqrt{2\times 10^{12}/7.8}=5064 \text{ m/s.}$$

$\therefore$ Time required for this sound to travel through 1 km of steel is $t'=10^3/\ 5064=0.2$ s.

Velocity of sound through air,

$$c=\sqrt{\gamma P/\rho}=\sqrt{1.4\times 10^6/0\cdot 0013}=328.4 \text{ m/s.}$$

$\therefore$ Time required for sound to travel through 1 km of air is $t=10^3/328.4$ $=3.0$ s. Hence time-interval$=(t-t')=2.8$ s.

3-7. Reflection.

Like all waves, sound waves are reflected under proper conditions. Reflection of sound plays an important part in a number of phenomena. Echoes, rolling of thunder, reverberations in auditoria etc., speaking tubes, stethoscopes, and wind pipes provide examples of reflection of sound.

For regular reflection, the dimensions of a surface must be large and its irregularities small compared with the wavelength. Since sound waves are long we require large reflectors to produce regular reflection. A big enough brick wall can reflect sound but not light, because its irregularities are small compared with the wavelengths of sound but quite large for those of light.

A surface of limited dimensions does not reflect all wavelengths equally. Shorter waves are more strongly reflected.

The geometrical laws of reflection of sound waves are the same as those of light waves. But this is not ordinarily appreciated because the sound waves are long and it requires large reflectors to reflect sound in the same way as light. Sound is reflected from walls, hills, rows of trees etc. The most familiar example of the reflection of sound is the *echo*.

3-7.1. Echoes.

A sound heard by reflection and clearly distinguished from the original is called an **echo.** For its formation an echo requires a suitable reflector such as an extended wall, a cliff etc., whose irregularities are small compared with the wavelength of the sound. To hear the sound and its echo *separately* the reflector must be at a sufficient distance, which may be calculated as follows :

A sensation of sound persists in the brain for almost 1/10th of a second after it is heard. Hence to recognise a sound and its echo as separate, at least this interval must pass between them. In our country, the velocity of sound under ordinary conditions (a mean temperature of $30^\circ C$ and a fair amount of moisture in the air), is about 350 m/s. In 1/10th of a second sound travels 35 m in the air. Thus to hear distinctly the echo of a sound of *short duration*, the distance between the source and the reflector must be at least 17·5 m. The source and listener are supposed to be close together.

The rumbling and rolling of thunder is due to the echoing of a peal of thunder from a number of reflecting surfaces such as clouds, rocks, mountain sides, forests, surface of separation between air currents, etc. The continuous rumbling is due to the fact that echoes reflected from different sources enter the ear at intervals of less than 1/10th of a second.

In big halls it is often noticed that a continuous *rolling* of sound persists for some time after a loud sound has ceased. This is known as **reverberation** and is due to multiple reflections from the walls. At every reflection there is some loss of energy due to absorption. If absorption at the walls is small, the sound waves last long enough before they become inaudible, and give rise to the reverberation. Open windows are very effective in reducing reverberation, since sound waves incident on them pass into the outer atmosphere and are completely lost to the room. It is to prevent these reverberations and increase the absorption of undesired sounds that large halls and cinema houses are carpeted and part of the walls is thickly cushioned with soft, sound absorbing materials.

Echoes are used by whalers in Arctic waters to locate an iceberg (or land) in a dense fog or at night. For this purpose a whistle is sounded or a short sharp sound made. By timing the interval between the signal and the receipt of the echo an iceberg may be roughly located and boat steered clear of it.

If a sharp sound is made near a flight of steps, the reflections from adjacent steps will reach the listener in a regular succession. This may be fast enough to give rise to the sensation of a musical note.

Example. A man standing between two parallel hills fires a gun and hears two echoes, one $2\frac{1}{2}$ seconds and the other $3\frac{1}{2}$ seconds after the firing. If the velocity of sound is 330 metres/sec, find the distance between the two hills. How long will it take him to hear the third echo?

Solution: The first echo is received after $2\frac{1}{2}$ seconds. Hence the sound has been reflected $1\frac{1}{4}$ seconds after the firing. In that time the sound travels $330 \times 1\frac{1}{4}$

$=412\cdot5$ metres. Hence the distance of one of the hills is $412\cdot5$ metres. Similarly the distance of the other hill is $330\times3\frac{1}{2}\times\frac{1}{2}=577\cdot5$ metres. So the separation of the two hills is $412\cdot5+577\cdot5=990$ metres.

The first and the second echoes will again be re-echoed from the second and the first hills respectively. For this the sound must altogether travel the distance between the hills twice. Hence the third echo will be heard $2\times990\div330=6$ seconds after the firing. Note that this echo is formed by one reflection at each of the two hills.

3-7.2. Echo depth-sounding. Depths in the sea are often measured by timing the reflection of a sound from the sea-bed. There are many compact devices used for the purpose. Though they differ in details they employ the same principle, which is as follows: A sharp sound, now-a-days, generally a strong pulse of ultrasonic waves, is produced near a hydrophone (a device similar to a microphone) submerged in the water. The hydrophone responds twice to the sound, once to the original one and again to the waves reflected from the sea-bed. The time interval between the two events is automatically recorded on a device which also measures short time intervals accurately. If the time between the two responses of the hydrophone is t and the velocity of sound in sea-water is c, then the depth at that place is $d=\frac{1}{2}ct$.

The fraction (I_r/I) of the energy reflected depends upon the angle of incidence. At normal incidence the value is

$$\frac{I_r}{I}=\left(\frac{\rho_2c_2-\rho_1c_1}{\rho_2c_2+\rho_1c_1}\right)^2 \tag{3-7.1}$$

where ρ_1 and ρ_2 are the densities of the media and c_1 and c_2 are the velocities of sound in them. Now, both ρ and c for liquids and solids are much greater than those for air. For example, ρc for air is about 42 cgs units while that for water is about $1\cdot43\times10^5$. For most solids ρc is of the order of 10^6 cgs units. Hence sound waves incident normally on an air-water or air-solid interface will be *almost completely reflected*, from whichever side they be incident. Sound started in a relatively thin layer of water, such as a shallow lake, will therefore remain confined to the water by a process of internal reflections.

3-8. Refraction of sound. When soundwaves fall on a surface separating two media, a part of it enters the second medium and travels with a different speed. Unless the incidence is normal there is also a change in the direction of travel. If i is the angle of incidence and r the angle of refraction (considered as in light) then

$$\frac{\sin i}{\sin r}=\frac{c_1}{c_2}=\mu \tag{3-8.1}$$

where c_1 and c_2 are the speeds of sound in the first and second media respectively. μ is the *refractive index* of the second medium relative to the first. Sound rays bend away from the normal in the medium in which the speed is higher. The medium in which the speed is slower corresponds to the optically denser medium of light. Very little of the sound energy travelling in air enters a solid or liquid medium, because most of the energy is reflected at the boundary due to large difference in the ρc values of the two media (see Eq. 3-7.1). The product ρc is called the *characteristic impedance* of a medium.

Sound, like light, may be refracted by prisms and lenses of suitable materials. The geometrical relations are the same, but owing to the largeness of scale such experiments are inconvenient to perform.

3-8.1. Total internal reflection. In passing from a medium in which the speed of sound is lower the sound rays bend away from the normal. For a particular angle of incidence, called the critical angle (θ), the angle of refraction is 90°. Then from Snell's law we have $\sin\theta=\mu$ where μ is the ratio of the lower speed to the higher. For sound travelling from air to steel,

$$\sin\theta=\mu=\frac{c_1}{c_2}=\frac{332}{5200}\text{ whence }\theta = 4^\circ\text{ (nearly).}$$

Since all 'sound rays' incident on steel from air at a greater angle must suffer 'total internal reflection', the speaking tube can transmit sound with little loss over a long distance (Fig. 3.4).

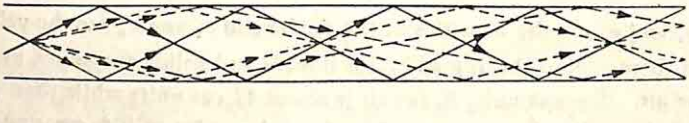

Fig. 3.4

The speeds of sound in air and water are 330 m/s and 1450 m/s nearly. This gives a critical angle of about $13\frac{1}{2}^\circ$. Sound rays in air incident on water at a larger angle will be totally reflected.

3-8.2. Refraction by air of varying temperature. A sound on shore may be heard from a good distance on a boat on an expanse of water at night; but under the same conditions it may not be heard in daytime. This is due to rafraction, arising out of variation of temperature at different altitudes. At night the air just above the water is cooler, but higher up it is warmer. Hence, the sound travels faster in the upper layers than in the lower ones. As a result, the upper part of the wavefront precedes the lower part (Fig. 3.5) and the waves bend downwards. This phenomenon may be compared with a superior mirage (see LIGHT, Sec. 3-8.1, Fig. 3.14). In daytime the entire phenomenon is reversed as the air in contact with the water is warmer. In this case, the sound waves bend upwards and do not reach the boat.

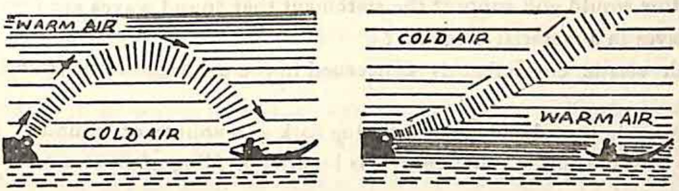

Fig. 3-5

3-8.3. Refraction due to wind. Sound from a source is better heard in the direction in which the wind blows than against it. The whistle of a train or the tolling of a bell from a temple or a church is heard when the wind is from its direction, while it may be quite inaudible at other times. When wind blows, the upper layers of air move more quickly than the lower since the latter are retarded by trees, houses etc. on the ground. The wavefront of the sound moving in the direction of the wind therefore bends downwards. Moving against the direction of the wind they bend upwards (Fig. 3.6).

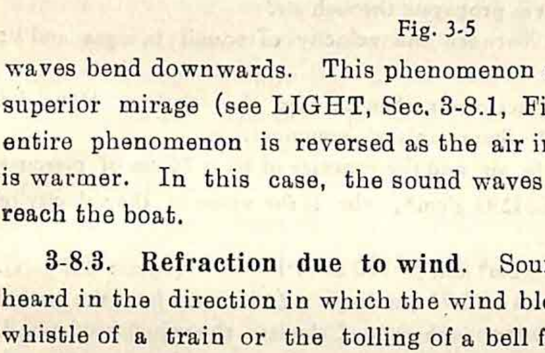
WAVE FRONT
WIND VELOCITY
HIGH
SOURCE
LOW
GOOD AUDIBILITY (LEEWARD SIDE)
POOR AUDIBILITY (WINDWARD SIDE)

Fig. 3.6

Reports of very loud explosions may be heard at distances where the ground wave does not reach. This is due to total reflection from layers of air high up in the atmosphere where the temperature of air begins to increase. In this way the existence of an atmosphere at a height of 25 to 40 miles has been established. The effect is somewhat similar to the formation of superior mirage.

Exercises

1. How would you support the statement that sound waves are *longitudinal, elastic* waves in a material medium ?

Which elastic coefficient is concerned in the propagation of sound through air ?

2. What is the advantage of a tuning fork as a source of sound ? Describe its mode of vibration. What effect has loading or filing of an arm of a fork on its frequency ?

3. Explain how sound waves propagate through air.

4. What is the relation between the velocity of sound in a gas and its pressure and density ?

Newton did not get the correct result when he used the relation. How did Laplace improve the relation ? State Laplace's argument.

5. If $\gamma=(C_p/C_v)=1{\cdot}404$ in air and the pressure of air is 76 cm of mercury and the density of air is $\rho=0{\cdot}001293$ g/cm^3, what is the value of the velocity of sound in air ?

(Density of: mercury$=13{\cdot}6$ g/cm^3 and $g=980$ cm/s^2) [Ans. : 332 m/s].

6. A stone is dropped into a well 78 4m deep. 4·23 seconds later the sound of the stone touching the water is heard. Calculate the velocity of sound. Take $g=980$ cm/s^2. [Ans. : 341 m/s].

7. What geometrical laws of reflection do sound waves obey ? What is the disadvantage of demonstrating these laws ?

State any practical application of reflection of sound.

8. What is an *echo* ? To hear the echo of a sound clearly, the reflector must be at a minimum distance ? Why is it so ? How can you measure the depth of water with the help of echoes ?

9. Distinguish between reflection and reverberation of sound. Give an example of reverberation and state how it is caused.

10. What are the geometrical laws of refraction ? Why do we say that in the case of sound, air is an acoustically denser medium than water ? If the velocity of sound in air is 340 m/s and that in water is 1500 m/s, what will be the critical angle of refraction from air to water ?

11. A sound emitted from the bank of a river at night can be heard farther on the water than a similar sound emitted in the day time. Explain why.

12. When there is a wind, a sound is carried farther down the wind than in the opposite direction. Explain why.

What relation has this phenomenon with the refraction of sound ?

[*Hint* : In refraction, the velocities in the two media are different. But in this case, the medium is the same while the speeds of the medium itself is different at different levels. This makes the apparent velocity of sound different at different levels of the medium.]

CHAPTER

4

SUPERPOSITION OF WAVES

4-1. Principle of superposition. A very simple principle, first enunciated by Huyghens in connection with light waves and called the **principle of superposition**, can be used to explain the effects we find when two waves, such as those of sound, are superposed on each other. It states that *when two or more wave trains are superposed, the resultant displacement is equal to the vector sum of the individual displacements provided they are small.* The principle, though strictly valid for waves of infinitely small amplitudes, is nearly true in all ordinary cases. The principle also holds for light waves.

Many important phenomena in sound are due to superposition of *sonic* waves (that is, waves of audible sound) from two different sources or two wave-trains derived from the same source. In discussing these phenomena on the basis of the principle of superposition, we assume that

(1) Passage of waves through one part of a medium is not affected by the simultaneous passage of other waves through the same part of the medium. For we notice that after the waves have crossed the region of overlapping, they maintain the frequency, amplitude and other characteristics unchanged. (When two persons talk simultaneously, the voice of one is not modified by the voice of the other.)

(2) The resultant displacement is the *vector sum* of those due to individual wave-trains.

The principle applies to all cases regardless of the directions of wave propagation or the frequency of the individual wave-trains. But cases of practical importance are those where the waves have equal or nearly equal frequencies. Two such cases are important to us at present, namely,

(*i*) **Beats**, which occur when two wave-trains of *nearly equal frequencies and amplitudes moving in the same direction* overlap.

(*ii*) **Stationary** or **standing waves**, which occur when *two identical wave trains moving in opposite directions* overlap.

A third effect, called **interference**, has been discussed in connexion with light waves (Sec. 9-6). Sound waves also produce interference (Sec. 4-6).

4·2. Beats. When two notes of nearly equal frequencies and amplitudes are sounded together the *resultant intensity rises and falls at regular intervals.* This phenomenon of rise and fall of sound is known as **beats.** A beat consists of one rise and one fall of the sound intensity.

Take two tuning forks of the same frequency and load a prong of any one of them with a drop of wax, thereby lowering its frequency (Sec. 3·3). Excite both the forks at the same time. Beats will be heard. Beats may also be heard by pressing the two keys to the extreme left of a harmonium while playing it.

4·2.1. Perception of beats. To *hear beats clearly* the following conditions should be satisfied :—

(*i*) The *difference in frequency* of the two notes should be *small.* When this difference exceeds 6 per second the effect on the ear begins to be unpleasant. Beyond 10 per second the beats become too fast to be clearly distinguished.

(*ii*) The *difference in amplitude* of the two notes should be *small.* Loudness of a note depends on the amplitude of vibration. In the case of a large difference in amplitude the beats will not be clearly recognised as the weaker sound will not appreciably affect the louder one.

(*iii*) To produce a clear effect the two sources should be of the *same kind* (*i.e.*, the two notes should have the *same wave-form*).

When there is a large difference in frequency or amplitude, beats are physically present though they may not be clearly distinguished by the ear.

4·3. Beat frequency. The *number of beats per second is equal to the difference of the frequencies of sources.* To prove it in a general way, let the two sources have frequencies n and $n'=n+m$, where m is small compared with m. The corresponding wavelengths are λ and λ', both having the same velocity c. At a given moment, let the displacements be in phase at a particular point a (Fig. 4.1). They will also be in phase at a distance l when the shorter wave has gained one wavelength over the longer. If ν be the number of longer waves contained in the distance l, then we shall have

$$l=\nu\lambda=(\nu+1)\,\lambda' \text{ or } \lambda=\frac{\lambda'}{\lambda-\lambda'}$$

Since both waves are travelling with the same velocity c, c is also the velocity of the resultant maxima (or minima). Therefore the number of maxima (or minima) received by a stationary observer in one second is the number lying within a distance c. Since the distance between two successive maxima (or minima) is l, their number within a distance c is

$$\frac{c}{l}=\frac{c}{\nu\lambda}=\frac{c}{\lambda}\left(\frac{\lambda-\lambda'}{\lambda'}\right)=\frac{c}{\lambda'}-\frac{c}{\lambda}=n'-n=m$$

So the number of beats per second equals the difference in frequencies of the two sounds.

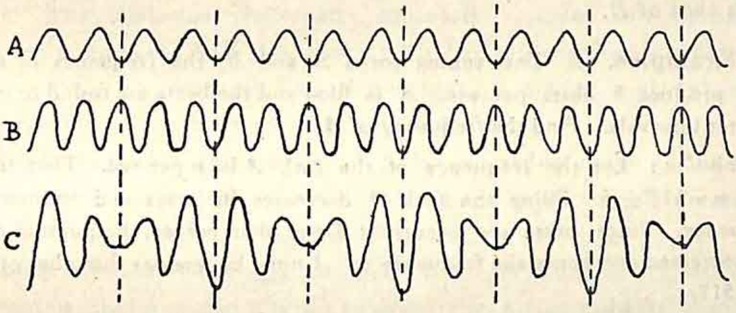

Fig. 4.1

[A and B are sound waves of two notes at a given moment. C is the resultant wave formed by the superposition of A and B. The broken lines in the figure indicate places where the displacements in the two waves are in the same phase or in opposite phases.]

4-4. Uses of beats. Since beats are produced by small differences in frequency of two sources, they provide a sensitive means of detecting whether two notes are in unison or not. This test is used for tuning musical instruments. In almost all acoustic measurements of frequency, beats are used to find exact matching of notes.

(i) Frequency determination. If the frequency of one of the two notes producing beats is known, the frequency of the other can be determined by counting the number of beats. Let two tuning forks have respective frequencies n and n' of which n is known. Provided n and n' are nearly equal easily distinguishable beats will occur. Let their number per second be m. Then $n'=n\pm m$.

To determine whether n' is greater or smaller than n, count the number of beats after putting a little wax on one of the prongs of the unknown fork thereby lowering the frequency of that fork. *If the* *number of beats increase, the required frequency is lower than the known one, i.e.,* $n' = n - m$. If the number of beats decreases the case is the reverse, *i.e.,* $n' = n + m$.

If an increase in mass of a tuning fork A increases the number of beats which it forms with another fork B, then the frequency of A is lower than that of B.

The number of beats will also change if the mass is *decreased* by filing the fork. *A decrease in mass increases the natural frequency of the fork. If decreasing the mass of a fork A decreases the number of beats* it formed with another fork *B*, then *the frequency of A is lower than that of B.*

Examples. (1) Two tuning forks *A* and *B*, the frequency of *B* being 512, produce 5 beats per sec. *A* is filed and the beats are found to occur at shorter intervals. Find the frequency of *A*.

Solution : Let the frequency of the fork *A* be *n* per sec. Then from the data $n = 512 \pm 5$. Filing the fork *A* decreases its mass and so increases its frequency. Since beats are occurring faster than before, the number of beats has increased and hence the frequency of *A* must be greater than that of *B*, *i.e.,* it is 517.

(2) A set of 24 tuning forks is arranged in a series of increasing frequency. If each fork produces 4 beats with the preceding one and the last fork sounds the octave of the first, calculate the frequencies of the first and the last forks.

Solution : Let the frequency of the first fork be *n*. Then that of the last fork will be $2n$. Now, the first fork produces 4 beats with the second, 8 with the third, 12 with the fourth and hence 23×4 or 92 beats with the last one. So between the first and the last the difference in frequency is 92. Since the frequency of the last is also *double* that of the first, the frequency of the first is 92 and that of the last is 184.

4-5. Stationary or Standing waves.

Stationary or standing waves are formed when two wave trains of the same kind, having the same frequency and velocity, and equal (or nearly equal) amplitudes are superposed on each other while moving in *opposite* directions. They can often be seen when the medium is bounded on all sides. If a regular wave train is generated in such a medium the wave train is reflected at the boundary and is superposed on the outgoing wave train. If the reflection at the boundary occurs at normal incidence, and the amplitude is not appreciably reduced, a clear, stationary wave pattern can be seen. We shall discuss this kind of wave pattern.

4-5.1. Characteristics of stationary waves. (Look at Fg. 4.2 as you go through the following. Check the statements with the figure.)

(i) The amplitude of vibration changes from place to place in medium and has a maximum value at certain points. The points of maximum amplitude are called **displacement antinodes** (or often simply, *antiondes*).

(ii) At certain points the medium undergoes no displacement at all. Such points of no displacement are called **displacement nodes** (or simply, *nodes*).

(iii) The distance between successive nodes or successive antinodes is equal to half the wavelength λ of the superposed waves.

(iv) Along the common line of propagation, nodes and antinodes alternate with each other at distances of $\lambda/4$.

(v) All particles lying between adjacent nodes move in the same direction at the same time. They reach maximum or zero displacement all at the same time. They have therefore the same phase. The amplitude increases from zero at a node to a maximum at the antinode. The portion of the medium lying between two nodes is called a *loop*.

(vi) Particles in neighbouring loops vibrate in opposite phases.

(vii) A particle of the medium executes a complete vibration in the same time in which a component wave advances by one wavelength. The frequency of a particle is therefore the frequency of a component wave.

The reason why the disturbance in the medium is called a stationary wave is that the form of the disturbance, *i.e.*, the wave form, remains confined in space and does not advance through the medium as it does in a progressive wave. *The pattern looks as if a wave-train has been suddenly arrested in the course of its motion.*

4-5.2. Demonstration of stationary waves. Stationary waves can be demonstrated as follows :

(i) **Water waves.** Take some liquid in a conical bowl and give it a sharp tap. The water surface will show a circular wave pattern, which will remain steady if the diameter of the liquid surface is appropriate. This may be adjusted by pouring in more liquid. The pattern looks just as if a wave-train has been suddenly arrested in the course of its motion.

(ii) **Melde's experiment.** Stationary waves formed by the transverse vibration of a string under tension can be very elegantly shown by the following experiment due to Melde (Fig. 4.2). An electrically maintained tunning fork of relatively low frequency (say 64 Hz) is clamped to a table as shown in Fig. 4.2. One end of a string is attached to one of the prongs while the other end is attached to a pan after passing over a smooth pulley. The fork is so placed that it vibrates *perpendicular to the length of the string*, which is the so-called *transverse* arrangement. The pan is then loaded and the fork excited. By adjusting the load on the pan or the length of the string the whole string may be made to vibrate transversely in one segment. If the load is reduced to one-fourth the string will vibrate in two segments ; if the load is reduced to 1/9th the string will vibrate in three segments.

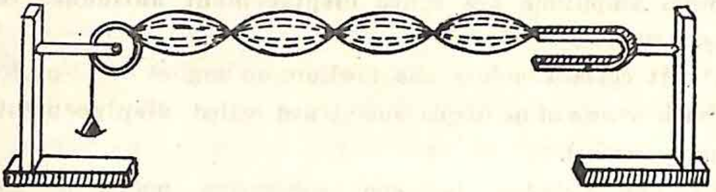

Fig. 4.2

The vibrating fork sends transverse waves along the string. These are reflected at the other end and are superposed on the incident waves. The pattern becomes steady when the length of the string is an integral multiple of half the wavelength of the waves produced.

If an electrically maintained fork is not available, an electric bell may be used in its place. The string is tied to the hammer of the bell.

4·5.3. Formation of stationary waves. We have stated earlier that stationary waves are formed by the superposition of two identical wave trains moving in opposite directions in the same region of a medium. So the individual displacements produced at a given point in the medium at a given instant are given by

$$y_1 = a \sin (2\pi/\lambda)(ct-x), \text{ and } y_2 = a \sin (2\pi/\lambda)(ct+x)$$

(Eqs. 2·5.1 and 2·5.3)

The line of propagation is along the x-axis. The two equations represent the waves travelling along the +ve and −ve directions of x

respectively. In the equations, a is the amplitude, λ the wavelength, c the velocity and x the distance of the point from an arbitrary origin, which generally is a point on the line of propagation where the waves meet in phase. The resultant displacement of the particle due to superposition of the two waves is, therefore,

$$y=y_1+y_2=a \sin (2\pi/\lambda)(ct-x)+a \sin (2\pi/\lambda)(ct+x)$$
$$=2a \sin (2\pi/\lambda)\, ct.\ \cos (2\pi/\lambda)\, x^*.$$

Putting $A=2a \cos (2\pi/\lambda)\, x,$ (4-5.1)

we have $y=A \sin (2\pi/\lambda)\, ct$ (4-5.2)

This represents a simple harmonic motion of the same frequency as the waves but of an amplitude $A=2a \cos 2\pi x/\lambda$. The amplitude depends only on the position x of the particle. It is not a progressive wave as the final expression for the phase angle does not contain any term of the nature $(ct-x)$.

The amplitude $A=2a \cos 2\pi x/\lambda$ changes as x changes. When $\cos 2\pi x/\lambda=0$, then $A=0$. For this we must have

$$\frac{2\pi x}{\lambda}=(2n+1)\frac{\pi}{2} \text{ where } n=0, 1, 2, 3, \text{ etc.}$$

$$\text{or } x=(2n+1)\frac{\lambda}{4}. \qquad (4\text{-}5.3)$$

$\therefore$ When $n=0$, $x_1=\lambda/4$

$n=1$, $x_2=3\lambda/4$

$n=2$, $x_3=5\lambda/4$, etc.

Such points, where the amplitude of displacement is zero, are called **displacement nodes** (points marked N in Fig. 6.3). Obviously, the distance between two successive nodes is equal to $\lambda/2$.

When $\cos 2\pi x/\lambda=\pm 1$, the amplitude is a maximum and has the value $2a$. For this we must have

$$\frac{2\pi}{\lambda}x=n\pi \text{ where } n=1, 2, 3 \text{ etc.} \qquad (4\text{-}5.4)$$

$$\text{or} \quad x=\tfrac{1}{2}n\lambda.$$

Putting values for n we have $x_1=\lambda/2$, $x_2=2\lambda/2$, $x_3=3\lambda/2$ etc. These points of maximum amplitude are called **displacement antinodes**

---

* From the trigonometrical relation $\sin (A+B)+\sin (A-B)=2 \sin A \cos B$.

(marked *A* in Fig. 4.3). As in the case of nodes, the distance between two successive antinodes is also $\lambda/2$. The distance between a node and the next antinode is $\lambda/4$. The amplitude of vibration thus varies from 0 to $2a$ over a distance $\lambda/4$.

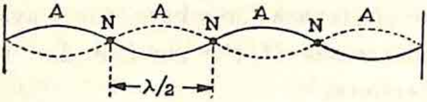

Fig. 4.3

4·5·4. Comparison of progressive and stationary waves.

Progressive wave	Stationary wave
(1) Progressive waves are due to the continuous periodic vibration of a portion of the medium. So long as the waves do not meet any boundary, they are progressive.	(1) Stationary waves are due to the superposition of two identical progressive wave-trains moving along a line in opposite directions through a medium.
(2) The phase of one particle is transmitted to the next along the line of propagation. In this way the wave-form progresses forward. The velocity of wave-motion is determined by the elastic properties of the medium.	(2) The wave-form does not advance through the medium. It is confined to the portion of the medium where the oppositely moving progressive waves overlap.
(3) Each particle of the medium executes the same vibration about its equilibrium position with the same frequency.	(3) The amplitude of the particles is not the same everywhere. At the nodes the amplitude is zero, at the antinodes it is maximum, They are all executed with the same frequency.
(4) Along the line of propagation there is difference of phase between the vibrations of particles. This phase difference is proportional to the distance between the particles.	(4) All the particles between two nodes vibrate in the same phase. Particles on two sides of a node vibrate in opposite phases.
(5) With the progression of the wave every point of the medium undergoes the same change of pressure or density.	(5) Change of pressure or density is not the same at every point of the medium. It is a maximum at the (pressure) nodes and a minimum at the (pressure) antinodes. [Pressure nodes correspond to displacement antinodes and pressure antinodes to displacement nodes.]

4-6. Interference. If two waves of the same frequency and amplitude are superposed, the resultant displacement will be double that due to a single source if the vibrations are in phase. If they are in opposite phases the resultant displacement is zero. Since intensity varies as the square of the amplitude, the effect produced in the first case will be four times that due to either of the waves, but zero in the second case. Thus the combined action of two sound waves may produce a greatly increased effect at places and silence at others. This phenomenon is known as *interference of sound.* There is no annihilation of energy in this case, only a redistribution. Energy removed from places of silence is concentrated at regions of increased sound.

Conditions of interference. To obtain *continuous silence* at a point in a medium under the action of two sound waves the following conditions must be fulfilled :

(1) The waves must have the same frequency and amplitude.

(2) The angle between the displacements due to the two waves at the given point must be small.

(3) The waves must *continue to arrive* at the given point in opposite phases, *i.e.*, the sources must be *coherent.*

If these conditions are fulfilled at a point there will be no displacement and hence no response in a detector placed there. The first two conditions can be satisfied by sound waves from two separate sources. To fulfil the third condition is more difficult. Ordinarily, no constant phase relation exists between two waves arriving at a point from two independent sources. Such sources are said to be *non-coherent.* They will be called *coherent* when there is a constant phase relation between them. The sources may vibrate in the same phase or have a constant phase difference. Waves from coherent sources continue to reach a place with a definite phase difference. So to fulfil the last of the above conditions sound waves from the same source may be made to arrive at the point by *different routes*, thereby acquiring the required phase difference.

Interference due to reflection at oblique incidence. A number of cases of interference of sound occurs when direct waves and waves reflected at oblique incidence are superposed. For example, it has been noticed that as a boat approaches a high cliff on which a fog-siren is sounding it passes through alternate regions of maximum and minimum sounds. [Fog-sirens warn ships off the cliffs obscured in a fog.] Let F be the siren placed on a cliff and B be the boat approaching it (Fig. 4.4). The boatman receives the sound via two paths Fa and FPa. Suppose the difference between these two paths is equal to an odd multiple of $\lambda/2$. Then there will be silence at a. Again, if at b the path difference $(FQb-Fb)$ is an odd multiple of $\lambda/2$,

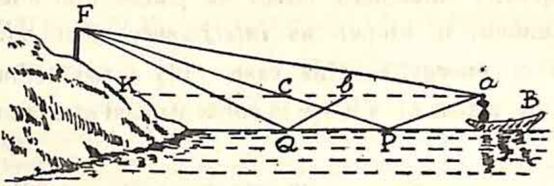

Fig. 4.4

there will be silence at b; similarly at c. Midway between a and b, and also between b and c the path-difference will be an even multiple of $\lambda/2$. There will be reinforcement of sound at such places.

Exercises

1. What is meant by the *principle of superposition of waves*? Describe two phenomena which are due to superposition of two sound waves.

2. What are beats? What conditions must be fulfilled so that beats may be heard clearly?

How do beats enable one to determine the frequency of a fork?

3. Show that the number of beats per second is equal to the difference of frequencies of the superposed sound waves.

4. What are *stationary waves*? Describe their characteristics.

5. Compare *stationary* and *progressive* waves.

6. Calculate mathematically the positions of nodes and antinodes in a stationary wave.

7. Under what circumstances are stationary waves formed? Describe an experiment to demonstrate stationary waves.

8. Two forks produce 4 beats per second. The frequency of one is 312/s. What may be the frequency of the other? How will you determine it?

If a little wax is fixed to an arm of the tuning fork of unknown frequency, it produces 2 beats per second with the other fork. What is the unknown frequency? Give the reason for your answer. [ANS.: 316/s]

9. Two forks produce 6 beats per second. If a prong of one of the forks is slightly loaded the number of beats reduces to 4 per second. Further loading increases the number of beats to 6 per second. Explain how this may happen.

10. (a) Two forks produce 4 beats per second. The frequency of one is 512/s. If one of its prongs is filed the number of beats diminishes. What is the frequency of the other? [Ans : 516/s].

(b) Sound waves of frequency 200/s form stationary waves in air. If the velocity of sound in air is 340 m/s what are the distances between (i) two successive nodes, (ii) two successive antinodes and (iii) a node and the next antinode? [Ans.: (i) & (ii) 85 cm; (iii) 42·5 cm]

11 What is *interference* of waves? Give an example of the interference of sound waves.

CHAPTER

5

TRANSVERSE VIBRATION OF STRINGS

5-1. Transverse displacement waves on a string : A string under tension resists any tendency to displace any point on it from the position of rest. This resistance supplies the restoring force when the displacement occurs. If a point on a long thin stretched string is displaced at right angles to its length and then released, the point begins to vibrate, and starts a *twin displacement wave* on two sides of it which travel along the string (Fig. 5.1). The velocity of wave motion depends on the tension T of the string and its mass m per unit length (also called *linear density*). Mathematically it may be shown that the velocity $c = \sqrt{T/m}$. We need not go into the proof.

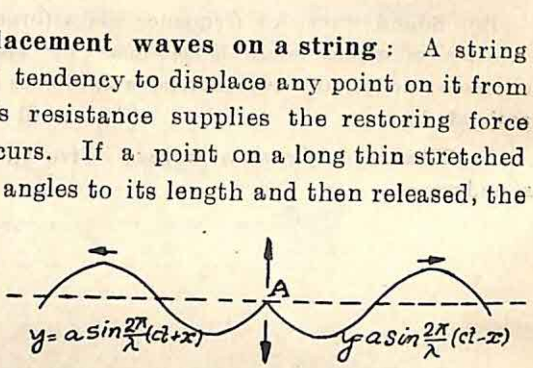
A
$y = a \sin \frac{2\pi}{\lambda}(ct+x)$
$y = a \sin \frac{2\pi}{\lambda}(ct-x)$

Fig. 5.1

If the string is fixed rigidly at its two ends, the waves will be repeatedly reflected from the ends and be superposed on each other. This gives rise to stationary waves on the string. Since the ends are rigidly fixed, **only those stationary waves can persist which have nodes at the ends.** No other stationary wave can persist. If the length of the string is l, persisting stationary waves will satisfy the condition $l = p \cdot \frac{1}{2}\lambda$ where p is an integer and λ the wavelength of a stationary wave. (Remember that the distance between two successive nodes is $\frac{1}{2}\lambda$.) Depending on the value of p, the string can vibrate in 1, 2 or more segments (Fig.s 5.2, 4.2 and 4.3). In the figures, N is a node and A an antinode.

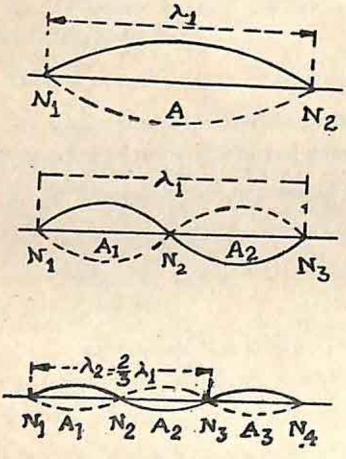

Fig. 5.2

5-2. Fundamental frequency of a vibrating string. The simplest possible mode of vibration of the string is one in which there are only two nodes in the whole string. The antinode is then in the centre of the string. In this mode the entire string moves to and fro in a single segment (Fig. 5.2, Top). The frequency with which this vibration is executed is called the **fundamental frequency**. Since the distance between two adjacent nodes is half the wavelength, the length l of the string is related to the wavelength λ_1 of the fundamental frequency by the relation $l=\frac{1}{2}\lambda_1$ or $\lambda_1=2l$.

If the fundamental frequency is n_1, then we shall have $n_1\lambda_1=c=\sqrt{T/m}$. Since $\lambda_1=2l$, we get

$$n_1=\frac{1}{2l}\sqrt{\frac{T}{m}} \qquad (5\text{-}2.1)$$

5-2.1. Other modes of vibration. If the string vibrates in p loops or segments, each loop will have a length $l/p=\frac{1}{2}\lambda_p$ where λ_p is the corresponding wavelength. Thus $\lambda_p=2l/p$. If the corresponding frequency is n_p, then $n_p\lambda_p=c=\sqrt{T/m}$,

$$\text{or}\quad n_p=\frac{1}{\lambda_p}\sqrt{\frac{T}{m}}=\frac{p}{2l}\sqrt{\frac{T}{m}} \qquad (5\text{-}2.2)$$

Eq. 5-2.2 gives us all the characteristics of transverse vibration of stretched strings (See Sec. 5-3). In this equation p is any integer, 1, 2, 3, etc. When $p=1$, the note emitted by the string is called its **fundamental** or **first harmonic**. It will appear from the equation that the possible notes from the string have frequencies which are integral multiples of the fundamental note. For each value of p, the string vibrates in a characteristic way, that is, with p loops. Each such way of vibration is called a *mode* of vibration.

Theoretically, a string has infinite modes of vibration. These are known as the **natural modes** of vibration of the string.

To make the string vibrate only in a particular mode is extremely difficult. This can be done by resonance with a vibration of that particular frequency. When a string actually vibrates, several modes of vibration are simultaneously present. Their number and nature are determined by the way the string was excited. The emitted note is complex and has several frequencies at the same time. Besides the fundamental, there will be overtones. Since the overtones in this case have frequencies which are integral multiples of the fundamental, they are *harmonics*. The harmonic having $p=2$ is called the *second harmonic*. $p=3$ is the *third harmonic*, and so on.

5-2.2. Quality. The particular harmonics and their relative strengths determine the **quality** (Chap. 7) of the note from the string. If a vibrating string is touched at a point distant d from one end where $d=l/p$, p being an integer, the vibration of that point stops. So it becomes a node, and only those frequencies can be present which have a node at d. These are the harmonics p, $2p$, $3p$, etc. If touched in the middle ($d=l/2$), only $p=2, 4, 6$, etc. harmonics, *i.e*, the even harmonics only, can be present. All odd harmonics will be suppressed. If $d=l/3$, harmonics with $p=3$, 6, 9, etc. can be present. The higher harmonics are generally weaker. In stringed musical instruments the quality of the emitted note can be changed in this way, that is, by touching the vibrating main string at different points.

5-3. Laws* of transverse vibration of strings. These were discovered by the French mathematician Mersenne in 1636. Eq. 5-2.1 was established theoretically by Taylor in 1715. Mersenne's laws are stated as follows : (If you write down Eq. 5-2.1, you can state the laws immediately.)

(*i*) The frequency of vibration varies inversely as the vibrating length when the tension and mass per unit length remain constant. This is known as the **law of length**. In symbols,

$$n \propto 1/l \text{ when } T \text{ and } m \text{ are constant.}$$

(*ii*) The frequency of vibration varies directly as the square root of tension when the length and mass per unit length of the string remain constant. This is the **law of tension**. In symbols,

$$n \propto \sqrt{T} \text{ when } l \text{ and } m \text{ are constant.}$$

(*iii*) The frequency of vibration varies inversely as the square root of mass per unit length provided the length and the tension of the string remain constant. This is the **law of mass**. In symbols,

$$n \propto \sqrt{1/m} \text{ when } T \text{ and } l \text{ are constant.}$$

If the string is of circular cross section we have linear density

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* Results stated as 'laws of transverse vibration of strings', 'laws of simple pendulum', 'laws of falling bodies', etc. were experimentally established in the early stages of the development of physics. Mathematical deductions came at a later stage. The so-called laws automatically follow from the respective mathematical relations.

m=area of cross-section × density $=\frac{1}{4}\pi d^2\rho$ where d is the wire diameter and ρ the density of the material.

The expression for frequency (Eq. 5-2.1) then becomes

$$n=\frac{1}{2l}\sqrt{\frac{T}{\frac{1}{4}\pi d^2\times\rho}}=\frac{1}{l.d}\sqrt{\frac{T}{\rho\pi}} \qquad (5\text{-}3.1)$$

Hence in the case of a *uniform wire of circular section*, we may put the law of mass in the following forms :

(*iii a*) **Law of diameter**. Length, tension and material remaining the same, frequency varies inversely as the diameter.

(*iii b*) **Law of density**. Length, tension and diameter remaining the same, frequency varies inversely as the square root of density.

5·4. Sonometer. A useful instrument based on the vibration of strings is the sonometer. Most of the laboratory experiments on strings are performed with the sonometer or monochord.

A sonometer (Fig. 5.3) consists of a hollow wooden box with a thin horizontal wire, attached to a peg, stretched on the top of it. The wire passes over a grooved pulley and carries a hanger on which weights can be placed so as to apply a known tension to the wire. Wooden bridges, B_1, B_2 and B_3, are placed beneath the wire for adjusting the vibrating length of the wire. To measure these lengths a horizontal scale is sometimes fixed on the box below the wire. Very often an auxiliary wire is provided alongside the main wire, stretched from another peg and passing over a second pulley. It also carries a hanger for weights. When there is only one string the apparatus is called a *monochord*.

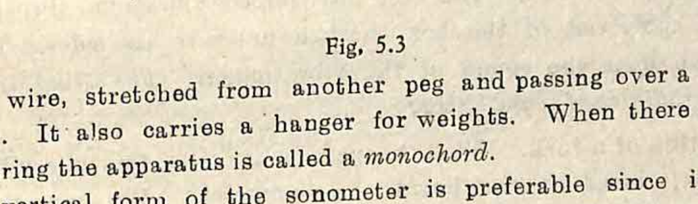


Fig. 5.3

A vertical form of the sonometer is preferable since in the horizontal form there is friction at the pulleys and bridge, which reduces the applied tension by an uncertain amount. In this form the sounding box is suspended vertically from a rigid support. The bridges can be shifted on the sounding box and fixed at any desired place on the box. To stretch the wire, the load is directly suspended from the wire. The support is made strong enough not to yield under the tensions applied.

The frequency of the emitted note from a given sonometer wire can be altered by either changing the distance between the two bridges while keeping the tension constant, or by keeping the distance fixed and changing the tension.

In all experiments with the sonometer, the wire is required to vibrate in unison with the frequency of the source to be investigated. To find when resonance occurs, place the sounding body on the sonometer box and a small paper rider on the vibrating length (if horizontal). Alter either the distance between the bridges or the tension on the string till the rider is thrown off. This happens when the natural frequency of vibration of the string agrees with that of the sounding body. At resonance, the string vibrates with a large amplitude and consequently throws off the rider. There may be a slight mistuning which can be detected by the beats that occur between the two sounds. One of the bridges may then be adjusted till the beats disappear. An experimenter with a good musical ear can, without the use of a paper rider, obtain this matching, or "tuning" as it is technically called, by listening.

Note that in the sonometer, as in the violin, the sound comes mainly from the vibration of the air in the wooden box. Holes in the sides of the box enable energy from the vibrating air inside the box to pass into the air outside. The vibrations of the string are communicated through the bridges to the top board of the sonometer box. This is thrown into forced vibration. Its vibration throws the air inside the box into forced vibration. Progressive waves come out of the box through holes in its sides. That is how we hear the sound of the vibrating air. *The string itself is a poor radiator of sound energy.*

Action of a fork. What happens when the stem of a vibrating fork is pressed on the board of a sonometer ? We have noted in Sec. 3-3 that the stem vibrates longitudinally when the prongs are set into (transverse) vibration. The up-and-down movement of the stem throws the board into forced vibration. These are communicated to the string through the bridges. That is how a vibrating fork affects the string.

The forced vibration of the board gets its energy from the fork. So the vibration of the fork lasts for a shorter time than when not pressed on the board. At resonance, the rate of energy transfer is quickest. Hence when resonance occurs, the vibration of the fork diminishes most quickly.

5·5. Worked examples. (i) A wire 50 cm long is under a tension of 25 kg. The mass of the wire is 1·44 g. Find the frequencies of the fundamental and the second harmonic.

[*Solution* : Here $T=25$ kg-wt$=25\times1000\times980$ dyn ; $m=1\cdot44$ g/50 cm. For the fundamental $l=50$ cm. Substituting these values in Eq. 5·2.1, we get fundamental frequency$=291\cdot6$/s. The second harmonic has twice this frequency.]

(ii) Two wires of the same material and diameter have lengths 54 cm and 36 cm and the same frequency. If the tension in the first is 9 kg, what is the tension in the second ?

[*Solution* : Here $n=\frac{1}{2l_1}\sqrt{\frac{T_1}{m}}=\frac{1}{2l_2}\sqrt{\frac{T_2}{m}}$ or, $T_2=\frac{l_2}{l_1}\sqrt{T_1}$.

Putting values, $T_2=4$ kg-wt.]

(iii) The tension in a wire is 10 kg-wt. Another wire of the same material has double the length and diameter of the first. What should be the tension in the second so that its fundamental may be an octave higher than that of the first ?

[*Solution* : Let l_1, d_1 and l_2, d_2 be the lengths and diameters of the two wires respectively. ρ is the same for both. We have $l_2=2l_1$ and $d_2=2d_1$. If n_1 is the frequency of the first, n_2 will be equal to $2n_1$. Therefore, $n_1=\frac{1}{2l_1}\sqrt{\frac{10\text{ kg-wt}}{\pi\rho d_1{}^2}}$ and $n_2=2n_1=\frac{1}{2l_2}\sqrt{\frac{T}{\pi\rho d_2{}^2}}$. From these we get $T=$ 640 kg-wt.

(iv) Find the velocity of transverse waves in a string under a tension of 0·23 kg when its length is 65 cm and the mass 0·52 g. If it emits its fundamental, what is the frequency ? ($g=981$ cm/s^2)

Solution : Tension in *absolute units*$=0\cdot23\times1000\times981$ dyn.

Mass per unit length$=0\cdot52/65=0\cdot008$ g/cm.

$\therefore$ Velocity of the wave $c=\sqrt{T/m}$

$$=\sqrt{\frac{0\cdot23\times1000\times981}{0\cdot008}}=5311\text{ cm/s}.$$

Wavelength of the fundamental$=2\times65$ cm.

$\therefore$ Frequency of the fundamental$=c/\lambda=5311/130=40\cdot85$ Hz.

(v) A steel string 100 cm long is struck at a point 25 cm from one end. What are the possible modes of vibration and which harmonics will be missing? If the vibrations were stopped at 25 cm, which harmonics will be present?

Solution : Those modes of vibration of the string in which it has a node at the point struck, will not be excited. The distance of the point struck is $25/100 = \frac{1}{4}$ fraction of the total length. Hence the 4th, 8th, 12th etc. harmonics will have nodes at that point. They will not be excited.

In the second case, vibrations with a node at 25 cm can be present, *i.e.*, the 4th, 8th, 12th etc. harmonics.

(vi) Two identical wires stretched under the same tension of 5 kg-wt emit notes in unison which are of frequency 300 vibrations per sec. One wire has its tension increased by 100 gm-wt. Find the number of beats heard when they are plucked.

Solution : Original tension of both the wires in absolute units

$$= 5 \times 1000 \times 981 \text{ dynes.}$$

Subsequent increased tension of one of them $= 5{\cdot}1 \times 1000 \times 981$ dynes.

When sounding in unison the frequency of the second wire was 300.

$$\therefore \quad 300 = \frac{1}{2l}\sqrt{\frac{5 \times 1000 \times 981}{m}}$$

At the higher tension the frequency of the second wire is

$$n' = \frac{1}{2l}\sqrt{\frac{5.1 \times 1000 \times 981}{m}}$$

$$\therefore \quad \frac{n'}{300} = \sqrt{\frac{5.1 \times 1000 \times 981}{5 \times 1000 \times 981}} = \sqrt{\frac{5.1}{5}}$$

$$\therefore \quad n' = 300\sqrt{\frac{5.1}{5}} = 303.$$

Hence the number of beats per sec is $n' - 300 = 3$.

(vii) Two wires of the same material have lengths in the ratio 2 : 3. If their diameters are the same what must be the ratio of their tensions for the shorter wire to give a note an octave higher than the longer?

Solution : Let the lengths of the two wires be $2x$ and $3x$, their frequencies n and n', and the tensions T and T' respectively. Since diameters and materials are the same the linear densities are equal.

$$\text{Hence} \quad n = \frac{1}{4x}\sqrt{\frac{T}{m}} \text{ and } n' = \frac{1}{6x}\sqrt{\frac{T'}{m.}}$$

As n is an octave higher than n', we have $n = 2n'$.

$$\therefore \frac{1}{4x}\sqrt{\frac{T}{m}} = \frac{2}{6x}\sqrt{\frac{T''}{m}} \quad \text{or} \quad T/T'' = 16/9.$$

Exercises

1. Write down the equation for the velocity of transverse waves along a stretched string and explain the meanings of the symbols.

2. If a string under tension is fixed at both ends, how will stationary waves be formed in it ?

If such a string vibrates in one or more segments what will be the relation between the frequency, the length of the wire, its tension, etc. ? Write the relation in the form of an equation and explain it.

What is meant by the *fundamental frequency* of a vibrating wire ? What is meant by *harmonics* ?

3. l is the length of a string under tension. m is an integer $= 1, 2, 3$, etc. What harmonics will be present in the vibrating string if l/m is a (i) node, (ii) antinode ?

4. State the laws of transverse vibration of strings.

If the diameter of a vibrating string is doubled, material and length remaining the same, what change should be made in the tension to keep the frequency the same ? [Ans. : To be increased 4 times]

5. Describe a sonometer. What purpose do the holes in its sides serve ? Why is the box made hollow ?

How would you determine the frequency of a fork with a sonometer ?

How are the vibrations of the fork transmitted to the wire ?

6. Illustrate forced vibration and resonance in connection with the above experiment.

7. A 50 cm long wire weighs 1·25 g. Its tension is 25 kg. What will be the frequency of a fork in unison with the wire ? [Ans. : 313/s].

8. A stretched wire is 30 cm long and 0·02 cm in diameter. Its fundamental frequenc is 200/s. Another wire of the same material and tension is 20 cm long and 0·025 cm in diameter. What is the fundamental frequency of the second wire ? [Ans. : 240/s].

9. When the length of a stretched wire under the same tension is 70 or 75 cm, it produces 6 beats per second with a given fork. What is the frequency of the fork ? [Ans. : 174/s]

10. Two tuning forks produce 4 beats per second. They are respectively in unison with stretched wires of lengths 96 and 97 cm, their material, diameters and tensions being the same. What are the frequencies of the forks ? [Ans. : 384/s and 388/s].

CHAPTER 6

VIBRATION OF AIR COLUMNS

6-1. Stationary vibrations in air columns. We have seen in Sec 5-1 how a vibrating string gives rise to stationary waves. The frequency of the vibrations or waves is the frequency of any one of the *natural modes* of vibration of the string (Sec. 5-2). A natural frequency is the frequency of a stationary wave that can persist in a string. Such a wave must satisfy two end conditions (known as *boundary conditions*). The boundary conditions of a stretched string fixed at both ends are that there must be two nodes at the two ends since the ends are rigidly fixed and there can be no motion of the string at either end.

Exactly similar considerations apply to the vibration of an air column contained in a pipe, open at one or both ends. Like the string it has *natural* (also called *characteristic*) modes of vibration. The frequency of vibration of a natural mode is a *natural frequency*. It is the frequency of a stationary wave which can persist in the air column. For persistence, the wave must statisfy the boundary conditions at the two ends of the pipe. So, our arguments in calculating the natural frequencies of an air column will be similar to those in the case of strings.

We shall consider the air column to be contained in (i) a pipe closed at one end and open at the other or (ii) a pipe open at both ends. The former is called a **closed pipe** (or *closed tube*) and the latter, an **open pipe** (or *open tube*). It will be assumed that (a) the diameter of the pipe is small compared with the length of the pipe and the wavelength of sound, and (b) the walls of the pipe are rigid.

6-2. Closed pipe. A vibrating tuning fork, when held over the open end of a pipe, the other end of which is closed, sends a longitudinal wave-train down the pipe. It is reflected from the closed end which we can take as a rigid wall. Superposition of the two identical but oppositely-directed wave-trains produces stationary waves in the air column, generally of small amplitude.

Since the closed end is rigid, the air particles at this end cannot vibrate. Hence there will be a node at the closed end. Again, at the open end the air particles will have the maximum freedom of movement. So the displacement there will be maximum, *i.e.*, there will be an antinode at the open end. Stationary waves which statisfy these two boundary conditions can persist in the tube. Therefore, the natural modes of vibration of an air column in a closed pipe are such that **there is a displacement node at the closed end and an antinode at the open end**. Stationary waves which fulfil this condition can have various wavelengths as shown below.

From Sec. 4-5.1, we find that the separation between a node and an antinode is $\lambda/4$ or an odd multiple thereof. Hence, *the air column in a closed pipe will so vibrate as to produce only those waves that would make the pipelength equal to an odd multiple of* $\lambda/4$. Hence the relation between the pipelength and the wavelength will be

$$l=(2m-1)\lambda_m/4 \text{ or } \lambda_m=4l/(2m-1) \tag{6-2.1}$$

where $m=1,2,3$ etc.

If c be the velocity of the sound waves produced, then *the frequency of the emitted tone* will be

$$n_m=c/\lambda_m=(2m-1)c/4l \tag{6-2.2}$$

6-2.1. Fundamental in a closed pipe. If in Eq. 6-2.2 we put $m=1$ then we have

$$n_1=c/4l \tag{6-2.3}$$

This represents the **fundamental tone** or the **first harmonic**, since this is the lowest frequency that the pipe can emit. Putting $m=1$ in Eq. 6-2.1 we obtain the relation between the length of the pipe and the wavelength produced, which is $\lambda_1=4l$. This is the longest wave produced by the pipe.

Physically it represents the simplest mode of vibration with a node and an antinode at the two ends and no other in the pipe.

The mode of vibration of an air column in a closed pipe emitting the fundamental is shown in Fig. 6.1. The amplitude of vibration of air particles gradually falls off from the open end towards the closed one. The entire column alternately moves towards the node (closed end) and away from it. The pressure variation is a maximum at the displacement node and a minimum at the antinode. Fig. 6.1(b) shows

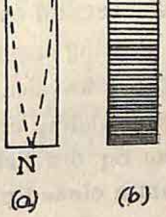


Fig. 6.1

the disposition of air layers throughout the length of the pipe when the compression at the closed end is highest.

[We may imagine the amplitude at any place in the pipe to be represented by a horizontal line drawn at the place and confined between the broken lines in Fig. 6.1(*a*). In this and subsequent figures (Fig.s 6.3, 6.4 and 6.5) though the amplitudes are indicated as if the vibrations are transverse, it must be remembered that they are *longitudinal in nature and paralled to the pipe axis.*]

6-2 2. Higher harmonics of a closed pipe. In the mode of vibration next in simplicity, there is another pair of node (n_1) and antinode (a_1) between the ends of the pipe (Fig.s 6.2*b*). If the wavelength is λ_2 then putting $m=2$ in Eq. 6-2.2 we have, for the corresponding frequency,

$$n_2=c/\lambda_2=3c/4l=3n_1. \qquad (6\text{-}2.4).$$

This represents the **first overtone** or the **third harmonic.** *The frequency is thrice that of the fundamental.* The value of the corresponding wavelength is obtained by putting $m=2$ in Eq. 6-2.1 which gives $\lambda_2=4l/3$.

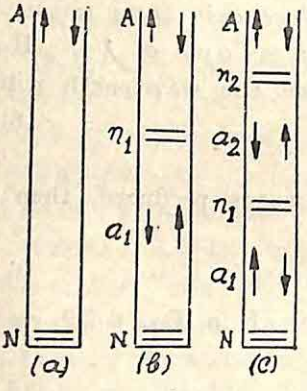


Fig. 6.2

If there be two pairs of nodes and antinodes between the ends of the pipe (Fig. 6.2*c*) then there will be five quarter-wavelengths in the length of the pipe, *i.e.*, $l=5\lambda_3/4$ or $\lambda_3=4l/5$. The corresponding frequency will be

$$n_3=c/\lambda_3=5c/4l=5n_1. \qquad (6\text{-}2.5)$$

This is the same as obtained by putting $m=3$ in Eq. 6-2.2. This is the **second overtone** or the **fifth harmonic.**

Putting $m=4$, 5 etc. in succession in Eq. 6-2.1 and 6-2.2 we can find the wavelengths and frequencies of the higher overtones. The corresponding notes are the 7th, 9th etc. harmonics. This analysis from Eq. 6-2.2 shows that **only the odd harmonics can be obtained from a closed pipe.**

As in the case of a string, it is difficult to excite only one mode of vibration ; more than one are almost always present. The particular overtones excited and their intensity relative to the fundamental depend on the mode of excitation. The quality of the emitted note is governed by the particular overtones present and their intensities relative to that of the fundamental.

6-3. Open pipe. To find the natural modes of vibration and the frequencies of an open pipe, we have first to determine the kinds of stationary waves which satisfy the boundary conditions at the ends of the pipe. Remember that in a stationary wave, the particle displacement is zero at a node. It gradually increases as we move away from the node and reaches a maximum at the nearest antinode. Thereafter, it falls to zero again at the next node.

In an open pipe, the air particles at the ends have the maximum freedom of motion. Hence **only those stationary waves can persist in an open pipe which have maximum displacements, that is, antinodes, at the ends**. This gives us the basis for calculating the natural modes and the corresponding frequencies of vibration of an open pipe. The natural modes of vibration of an open pipe will be such that there are two displacement antinodes at its two ends.

Now, the distance between two antinodes is given by $m\lambda_m/2$ where m is an integer and may have values 1, 2, 3, etc. Hence the relation between the pipe-length and the wavelength will be given by

$$l=m\lambda_m/2, \text{ or } \lambda_m=2l/m \tag{6-3.1}$$

and the frequency of a note will be

$$n=c/\lambda_m=m.c/2l. \tag{6-3.2}$$

As in a closed pipe or vibrating string, it is very difficult to excite one mode only. Depending on the method of excitation, several harmonics, besides the fundamental, are simultaneously present. Quality of a note is determined as in the other related cases.

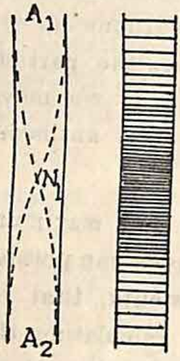
6-3.1. Fundamental and harmonics in an open pipe. The simplest possible mode of vibration of air in an open pipe is that in which there is no antinode in the tube except at the two ends. Denoting the corresponding wavelength by λ_1 we shall have $l=\lambda_1/2$ or $\lambda_1=2l$. The frequency n_1 corresponding to the wavelength is

$$n_1=c/\lambda_1=c/2l. \tag{6-3.3}$$

This is the **fundamental** and can also be obtained by putting $m=1$ in Eq. 6-3.2. There will be a single node and that in the middle of the pipe. The amplitude and disposition of the air layers throughout the length of the pipe are indicated in Fig. 6.3(a) and 6.3(b).

Putting $m=2$ in Eq. 6-3.1 and 6-3.2 we obtain

$$l=\lambda_2 \text{ and } n_2=c/\lambda_2=c/l=2n_1. \qquad (6\text{-}3.4)$$



(a) (b)

Fig. 6.3

This represents the next mode of vibration and the emitted tone is the **first overtone** or the **second harmonic**. Its frequency is twice that of the fundamental. Physically, there are three antinodes and two nodes in the tube as shown in Fig. 6.4(b).

Putting $m=3$ in Eq. 6-3.1 and 6-3.2 we get

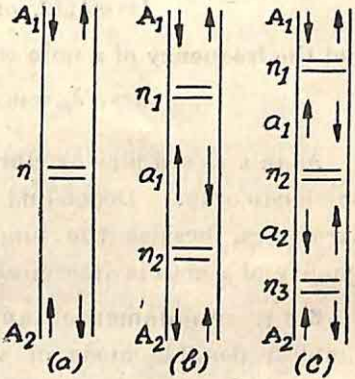
$$l=3\lambda_3/2 \text{ and } n_3=c/\lambda_3=3c/2l=3n_1. \quad (6\text{-}3.5)$$

This is the **second overtone** or the **third harmonic**. It has a frequency thrice that of the fundamental. In this case four antinodes and three nodes occur in the pipe [Fig. 6.4(c)].

Putting $m=4$, 5, 6 etc. in Eq. 6-3.1 and 6-3.2 we get the wavelengths and frequencies of the higher harmonics.

In the open pipe also, a number of overtones are simultaneously present along with the fundamental, whenever it is sounded. In contrast with the closed pipe, **all harmonics, odd as well as even, may be given out by an open pipe.**

In all modes of vibration, the portions of the air column on two sides of a node simultaneously move towards the node or away from it. This is the nature of vibration of air particles in the column. Fig. 6 3(b) represents the relative positions of normally equidistant air layers as they all move towards a node. The arrows in Fig.s 6.4 are meant to represent this kind of motion.



(a) (b) (c)

Fig. 6.4

6-4. Comparison between closed and open pipes.

Closed pipe	Open pipe
1. Must have a displacement antinode at the open end and a displacement node at the closed.	1. Must have displacement antinodes at both ends.

2. Possible wavelengths $\lambda_m=4l/(2m-1)$ where $m=1, 2, 3,$ etc.	2. Possible wavelengths $\lambda_m=2l/m$ where $m=1, 2, 3,$ etc.
3. Possible frequencies $n_m=c/\lambda_m=(2m-1)\,c/4l$.	3. Possible frequencies $n_m=c/\lambda_m=mc/2l$.
4. For the fundamental $m=1$, $\lambda_1=4l$, $n_1=c/4l$.	4. For the fundamental $m=1$, $\lambda_1=2l$, $n_1=c/2l$. Therefore, *for pipes of same length the fundamental of an open pipe is an octave above that of the closed one.* To produce the same fundamental, an open pipe should have double the length of the closed one.
5. The *first overtone* has $m=2$, so that $\lambda_2=4l/3$, $n_2=3n_1$. This is the *third harmonic.*	5. *First overtone* has $m=2$, $\lambda_2=l$ and $n=2n_1$. This is the *second harmonic.*
6. Second overtone has $m=3$, $\lambda_3=4l/5$, $n_3=5n_1$. It is the fifth harmonic.	6. Second overtone has $m=3$, $\lambda_3=2l/3$, $n=3n_1$. It is the third harmonic.
7. **Only odd harmonics can be produced.**	7. **All harmonics, both odd and even, can be produced.**

Fig.s 6.5 and 6.6 show the number, position etc. of the nodes and antinodes in the two types of pipes as each emits its first and second overtones respectively. Every successive overtone has one node and one antinode more than the preceding overtone.

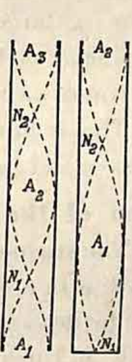


Fig. 6.5

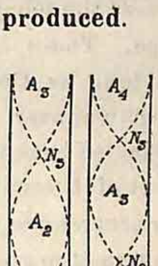


Fig. 6.6

Examples. (1) A closed organ pipe 24.5 cm long is in unison with a tuning fork of frequency 256. If its length is increased by 2 mm, find the number of beats produced when the fork and the pipe are sounded together.

Solution : Assuming that the pipe is sounding its fundamental we have from the given data $256 = c/(4 \times 24.5)$

where c is the velocity of sound. Then $c = 256 \times 5 \times 24.5$

When the length is increased, the frequency becomes

$$n = c/4l = 256 \times 4 \times 24.5/(4 \times 24.7) = 253.9 \text{ or } 254 \text{ (approx.)}$$

$\therefore$ The number of beats $= 246 - 254 = 2$ per sec.

(2) Two organ pipes, one closed and the other open, are respectively 2.5 ft and 5.2 ft long. When sounded together 4 beats are heard per second. Find the velocity of sound.

Solution : If n and n' are the fundamental frequencies of the closed and open organ pipes respectively, then

$$n = c/4l,\ n' = c/2l' \text{ and } n - n' = 4.$$

$$\therefore\ n = c/(4 \times 2.5) = c/10 \text{ and } n' = c/(2 \times 5.2) = c/10.4.$$

$$\therefore\ c\left(\frac{1}{10} - \frac{1}{10.4}\right) = 4 \text{ or } c = 4 \times 10 \times 10.4/0.4 = 1040 \text{ ft/s.}$$

6-5. Vibrating air columns as sources of sound. When we speak of a source of sound, we understand it is a vibrating body which can generate sound waves in air. We have seen that air columns in closed or open pipes can vibrate with characteristic frequencies, and these vibrations are stationary vibrations. How can they serve as sources of sound, giving rise to progressive waves ?

Stationary vibration of air columns in closed or open pipes can be generated in more than one way. A common method is to blow across the mouth of the pipe. This blowing sets up waves within the pipe. These are reflected repeatedly from the two ends of the pipe (so long as the blowing persists), forming stationary waves. Though it may appear surprising, a large part of the wave inside a pipe is reflected back into the pipe at each incidence at an open end. A small part of these incident waves pass throgh the opening and spread out as progressive waves. This is how an open or closed pipe serves as a source of progressive waves and hence is a source of sound.

6-6. Determination of the frequency of a fork by resonace with an air column. The frequency of a fork may be determined by setting up resonance with it in a column of air of adjustable length. The frequency can then be determined from the length of the column if the velocity of sound in the air column is known. An open pipe which has one end immersed in water works as a closed pipe. The length of the enclosed air column can be altered by varying the depth of immersion.

To find the frequency of the fork, the vibrating fork is held at the open mouth of the tube which has been sunk to the maximum depth. The tube is then slowly raised along with the vibrating fork (Fig. 6.7). At a particular position of the tube the air in the tube will be thrown into resonant vibration by the fork and emit a fairly loud sound. Since we started from a very short length the note emitted by the air colum is its fundamental. If c is the velocity of sound in the air column, l the length of the resonant air column and n the frequency, then from Eq. 6-2.3

$$n=c/4l. \qquad (6\text{-}6.1)$$

The velocity so determined is that in moist, saturated air at the temperature of the room. Velocity increases with temperature and moisture content.

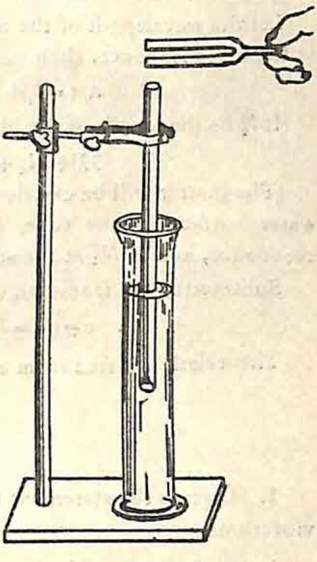


Fig. 6.7

Knowing c and measuring l we can find n, the frequency of the fork. If we know n, we can, from the relation $c=4ln$, find c. By this method the velocity of sound in the most air of the tube can be determined in the laboratory.

If the pipe is filled with a gas other than air, the velocity of sound in the gas can be found by the same method.

End Correction. The famous scientist, Lord Rayleigh, has theoretically shown that if stationary waves are generated in a tube, the antinode is not formed exactly at the open end of the tube but lies a little beyond it. If the radius of the tube is r, the antinode will be at a distance $0.6r$ away from the open end.

Therefore, if the actual length of the tube is l, the distance between the node at the closed end and the antinode at the open end is $(l+0{\cdot}6r)$. To correct for this effect at the end we should write Eq. 6-6.1 as

$$n=c/4(l+0{\cdot}6r) \qquad (6\text{-}6.2)$$

Alternative method. There is an alternative method of finding n or c by eliminating the end effect. If we keep on increasing the length of the air column beyond the first resonance, a second resonance will occur when the length of the tube increases to nearly three times its previous value. In this condition the tube emits its first overtone or third harmonic.

Let the wavelength of the note emitted by the fork be λ. If l_1 is the length at the first resonance, then

$$\lambda/4 = l_1 + 0{\cdot}6r \tag{A}$$

If l_2 be the length of the vibrating column at the second resonance then

$$3\lambda/4 = l_2 + 0.6r \tag{B}$$

[The matter will be clearly understood if we imagine the closed end, *i.e.*, the water surface in the tube, to lie at N_2 in Fig 6.5 (left) at the time of the first resonance, and at N_1 at the second.]

Subtracting (A) from (B), we have $\lambda/2 = l_2 - l_1$

$$\therefore \quad c = n\lambda = 2n(l_2 - l_1) \tag{6-6.3}$$

This relation is free from end correction.

Exercises

1. Discuss the statement that the vibrations of an air column are stationary vibrations.

2. Find the possible frequencies of an air column in a closed pipe. Show that only odd harmonics can be present.

3. Find the possible frequencies of an air column in an open pipe. Show that all harmonics, odd and even, can be present.

4. We speak of wavelengths in connection with the vibration of an air column in a closed or open pipe. Explain which progressive or stationary waves these wavelengths refer to.

How can a vibrating air column be considered a source of sound ?

5. Explain briefly how an open tube can give both progressive and stationary waves.

6. Compare the modes of vibration of air columns in open and closed tubes of the same length.

7. How would you determine the velocity of sound in air with the help of a closed tube and a fork of known frequency. Is it the value in dry air and at 0°C ? If not, what are the conditions of temperature and moisture at which the value has been determined ?

8. Describe the nature of vibration of an air column in a closed and open tube when each is emitting the fundamental.

How will the frequency of the fundamental change when one end of an open tube is suddenly closed ?

9. At the room temperature the air column in a closed brass tube is in unison with a fork. Why should beats appear between the sources as the tube

is heated gradually? How will the number of beats be affected by a greater increase of temperature?

10. How do the air layers vibrate on two sides of (a) a node, (b) an antinode in the air column of an open tube?

11. The air column in an open tube emits a fundamental note of frequency 256/s. If the velocities of sound in air and in coal gas be respectively 350 m/s and 500 m/s, what will be the frequency of the fundamental and its wavelength when the tube is filled with coal gas? [ANS.: 366/s; 1·37 m]

12. A tube, 100 cm long and 2 cm in diameter, is completely full of water. A fork of frequency 510 cps (cycles per second) is sounding near its upper mouth while water is being gradually let out at the bottom. The velocity of sound in the air of the tube is 340 m/s. Explain why there will be resonance when the air column is about 17.3 cm, 50.6 cm and 83.9 cm long. (*Note*: Take end correction into consideration.)

CHAPTER
7

PHYSIOLOGICAL SOUND

7-1. Musical sound and noise. Helmholtz, the nineteenth century scientist (1821-1894), was a physicist and physiologist at the same time. He carried out a vast amount of research on the sensations of light and sound. On the basis of sensations, sound was divided into two classes, namely, (i) musical sound and (ii) noise. *A sensation is not a physical quantity and cannot be measured.* Its study therefore does not properly belong to the realm of physics. Helmholtz sought to associate the various characteristics of different sensations of sound with different physical quantities. The above classification was therefore based on the following definitions :

(i) A sound that is continuous and periodic in nature is called a *musical sound* (or *note*).

(ii) A *noise* is a sound of short duration, sharp and abrupt in nature, or one which changes character continuously if it persists.

The basis for the classification was not pleasantness of hearing in the first case or unpleasantness in the second. A musical sound may be unpleasant to hear ; a noise may in some cases produce a pleasant sensation in the ear. The definition of noise has since been changed. *A noise now means an unwanted sound.* If a good piece of music is diverting your attention from the work you are earnestly engaged in, it will be noise to you.

7-2. Characteristics of musical sound. On the basis of sensation, three characteristics are ascribed to musical sound. They are (i) loudness, (ii) pitch and (iii) quality. All the three are judgements of the brain and none is a physical quantity.

(i) **Loudness** is the intensive attribute of an auditory sensation in terms of which sounds may be ordered on a scale extending from soft to loud. It corresponds to 'brightness' in light.

(ii) **Pitch** is that attribute of auditory sensation in terms of which sounds may be ordered on a scale extending from low to high, such as a musical scale. It corresponds to colour in optics and may be called 'musical colour'. A noise, according to the old definition, is characterized by the lack of a definite pitch.

(iii) **Quality** (or *timbre*) of a musical sound is that attribute of auditory sensation in terms of which a listener can judge that two sounds similarly presented and having the same loudness and pitch are dissimilar. We can easily distinguish a note sounded on a flute from the same note played on a harmonium or sitar. The characteristic of the sound sensation by which we differentiate them is quality. It is by quality that we recognize the voice of a friend or any person we know.

7-3. Physical factors on which loudness, pitch and quality depend.

(i) **Loudness and intensity.** Loudness depends primarily upon intensity ; but the two are not the same. Intensity is the rate of flow of sound energy per unit area. Intensity is a physical quantity and can be measured. But loudness is a physical sensation and cannot be measured like a physical quantity. Intensity is proportional to the square of the amplitude and frequency, and is directly proportional to the density of the medium.

Loudness appears to be approximately proportional to the logarithm of the intensity.

[At low intensities, loudness depends much on frequency. At low brightness, the eye is most sensitive to yellow-green light ($\lambda=5{\cdot}5\times10^{-5}$ cm). Similarly, at low loudness, the ear is most sensitive to frequencies of about 2000 – 2500 Hz(Hz = Hertz, the unit of frequency. 1 Hz is one vibration per second.)]

Intensity is a physical quantity measured in ergs per sq. cm per second in the cgs system. Loudness is a physical sensation, not measurable like a physical quantity. Still, in view of its importance in connection with measurement of noise level, a unit, called the 'phon', has been used to express loudness level. Loudness level is measured in phons by the average sensation a sound produces in many men.

(ii) **Pitch and frequency.** Pitch depends primarily on the frequency of the sound. But it is also dependent to some extent on the intensity and the wave-form. If the intensity of a pure sound (sound of one frequency only ; wave form a sine curve) of fixed frequency is increased, its pitch changes. But if the frequency is greater than 1000 Hz, there is not much change of pitch with intensity.

Pitch and frequency are not the same. Pitch is a sensation while frequency is a measurable physical quantity. It is unfortunate that the word 'pitch' be used in the same sense as 'frequency'. In some elementary books or in some question papers you may find the expression 'determination of pitch' where determination of frequency of a sound is meant. Determination of real 'pitch' is a very complicated affair, and is never undertaken in an ordinary sound laboratory. (Pitch is expressed in a unit called the '*mel*'. It has no connection with the *hertz* : but we cannot go into all these here.)

(iii) **Quality and wave-form**. Quality depends on the nature of vibration of the source, which may be simple or complex. A source executing S.H.M. produces a note of a single frequency, which is called a **tone**. Ordinarily, the vibration of a source is much more complex and has several frequencies at the same time. A musical sound, which we shall call a **note**, generally consists of several tones. The tone of the lowest frequency in a note is called its **fundamental** ; those of higher frequencies are called **overtones** or **upper partials**. Overtones having frequencies that are integral multiples of the fundamental frequency are called **harmonics**. The fundamental and overtones originating from the same source blend together into a single musical note. *The quality of a musical sound is due to the presence of overtones in addition to the fundamental* (Helmholtz). The number, nature and relative intensities of the overtones determine the quality of the musical sound. The wave-form of a sound wave is also determined by exactly these quantities. So, we may say that *quality is determined by the wave-form*. The wave-forms of a note of the same fundamental frequency, but from different instruments, are shown in Fig. 7.1.

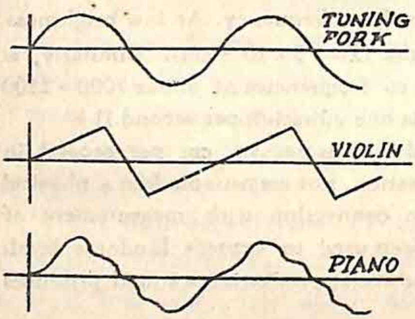


Fig. 7.1

A pure tone corresponds to a single colour in light ; a complex musical note corresponds to a mixture of colours. Since, in light, a mixture of colours is called a 'hue' we may say that 'quality' in sound corresponds to 'hue' in light. By analogy with light, we may say :

Loudness in sound corresponds to brightness in light, pitch to colour and quality to hue.

7-4. Graphical representation of the characteristics of a musical note. Since sound is wave-like in nature, all the characteristics it produces must have their physical counterparts in the characteristics of waves. Now, a regular wave is characterised by (i) an amplitude, (ii) a frequency or wave-length, and (iii) a wave-form or time-displacement curve. Broadly speaking, **the amplitude, frequency and wave-form of a sound wave are associated respectively with the sensations of loudness, pitch and quality**. Simple harmonic wave-forms correspond to pure tones, while mixed tones, *i.e.*, notes, have more complex wave-forms.

A. Difference in loudness. In Fig. 7.2(*a*), both curves are sine curves of the same frequency. They have the same pitch and quality, but their amplitudes differ. They will be found to differ in loudness. Other things remaining the same, the larger amplitude will produce the louder sound.

B. Difference in pitch. In Fig. 7.2 (*b*), we have two sine curves which differ only in frequency but not in amplitude. The tones they represent differ in pitch. The curve with the higher frequency has the higher pitch.

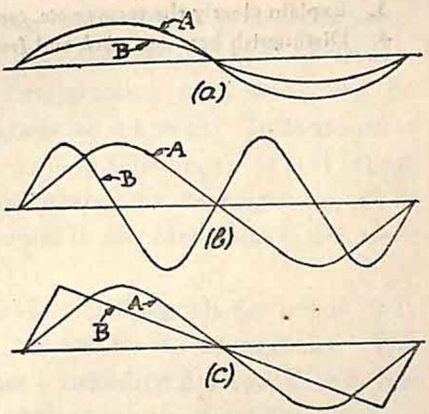


Fig. 7.2

C. Difference in quality. In Fig. 7·2 (*c*), the amplitude and frequency of the curves *A* and *B* are the same; but their wave-forms are different. The sounds they represent will have about the same loudness and pitch but their qualities will be different. The sine curve *A* represents a pure tone; it has a single frequency. The curve *B* has a fundamental frequency equal to the frequency of *A*; but it also contains overtones. The wave form depends upon the frequencies and relative amplitudes of the fundamental and the overtones present.

7-5. Noise. Noise has been defined as 'sound not desired by the person receiving it.' The definition is clearly subjective. What is music to one man may be noise to another. A musical sound is *periodic* in nature, but a noise is *not*. Some noises seem to have a periodicity; but this is often due to some secondary effect connected with the noise.

The most disturbing fact about noise is its loudness. Noise diverts attention and causes annoyance. In a factory, noise reduces the efficiency of a worker. Noise also affects adversely the human nervous system. Reduction of noise is becoming a more and more important problem of modern civilization.

Exercises

1. Distinguish between *musical sound* and *noise.* What are the characteritics of musical sound ? Define them. Are they physical quantities ?

2. State with what physical characteristics of a sound wave the loudness, pitch and quality of a musical note are principally connected ?

3. Explain clearly the terms *note*, *tone*, *fundamental*, *overtone* and *harmonic.*

4. Distinguish between *pitch* and *frequency* of a note.

CHAPTER 8

RECORDING AND REPRODUCTION OF SOUND

8-1. Broad principle of recording and reproduction of sound. A train of sound waves falling on a thin diaphragm sets it into vibration according to the pressure variations in the incident wave. If that or any other diaphragm can be made to vibrate exactly the same way, it will produce the same variation in air presure as the original sound waves did. Thus the same sound as the original one will be reproduced.

The original sound has a characteristic wave-form of its own. In sound recording, we have to catch and, if necessary, store this wave-form in whatever way we can. Reproduction of sound may be (i) *instantaneous* or (ii) or *subsequent* as we need. Instantaneous reproduction may be (a) long distance, as in telephony, or (b) short distance, as in a public address. For subsequent reproduction, as in a gramophone, tape or film, the sound is recorded, that is, *its wave form is stored up in a device.*

In all modern cases of recording, the sound signals are in the first instance converted into electric signals by a microphone. The amplified microphone current operates a recording device. This device makes a permanent record in which *some characteristic of the recording material is changed according to the wave form* of the microphone current (that is, practically of the incoming sound). In reproduction, electrical signals (or mechanical, as in a gramophone) having the same wave-form are generated from this record, and are finally converted into sound.

Microphones form the first element in these series of operations, and loudspeakers the last. (In between there is a complex network of amplifiers, electric filters, transmission lines, etc.)

8-2. The phonograph. The phonograph, invented by Thomas Alva Edison in 1878, was the first device which successfully recorded and reproduced the sounds of speech and music. Its construction and action are as follows :

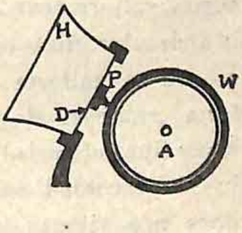


Fig. 8.1

The sound to be recorded is produced in front of a horn (H ; Fig. 8.1). The narrow end of the horn is closed by a circular diaphragm D. A light style P is fixed at right angles to D at its centre.

The sound waves, collected by the horn, cause the diaphragm to vibrate according to the variations of pressure in these waves. This causes the style to vibrate at right angles to the diaphragm.

The style is in contact with a layer *W* of special wax covering the drum *A*. This drum can rotate about its axis and advance as it rotates. If no sound waves impinge on *H* as the drum rotates and advances, the style traces a groove of uniform depth in the wax. But when sound waves fall on *D*, the style vibrates and traces *a groove of varying depth*. This is the *record* of the sound.

During this process the wax is soft. But after the impression has been taken it is allowed to harden.

To reproduce the sound from this record, the style is placed at the starting point and the drum is set in motion. It is made to move at the same constant speed as when taking the record. The point of the style follows the groove and moves up and down. This makes the diaphragm vibrate in the same way as it did at the time of recording. The air is set into similar vibration, reproducing the sound.

Since the invention of the phonograph there has been much improvement both in recording and reproduction of sound. The gramophone with disc records, the tape-recorder and sound tracks on talkie films are examples. The modern *dictaphone* has preserved some features of the phonograph.

The main defects of the phonograph are the following :

(1) the record on the wax deteriorates very rapidly ;

(2) the reproduced sound is much distorted due to the mechanical resonance of the horn and the membrane.

8-3. The Gramophone. The defects of the phonograph have been largly removed in a gramophone. Here, the record (Sec. 8-3.1) is a circular disc of a hard material. The sound track is a spiral groove of uniform depth on the surface of the disc with the waveform represented as transverse undulations. These *side-cuts* are wavy lines of varied but small amplitude, which have been produced by the recorded sound and are characteristic of it. Hence, the pin does not vibrate up and down as in the phonograph, but moves horizontally in a wavy track. This and the hardness of the material of which the record is made reduce the wear and give the record a long life. Further, by improvement in the designs of the horn and the vibrating membrane, undesirable resonance is kept at a minimum.

About three decades ago the most important part of a gramophone was the sound box (Fig 8.2). It had a thin circular metal diaphragm *D* corrugated in a special way to improve the reproduction of notes of low pitch. It was held around its outer edge between rubber washers. One end of a metal lever *L* was attached to the middle of the diaphragm. The needle *N* lay at the other end of the lever which had its fulcrum at *P*.

The diaphragm closed the aperture of a cylindrical box which communicated with a horn, part of which formed the *tone-arm*. The tone-arm could rotate about a point at some distance from the revolving record. This enabled the needle to follow the groove with little side thrust.

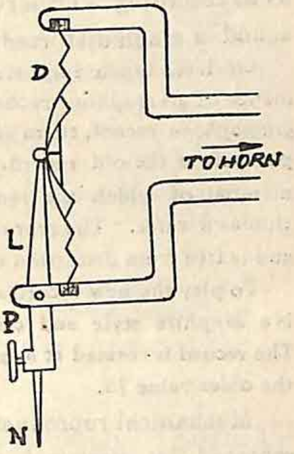


Fig. 8.2

When the record rotated with the needle in the groove, the needle moved to and fro laterally along the wavy track at the bottom of the groove. These movements were transmitted by the lever to the diaphragm, which was made to vibrate. These vibrations generated vibrations in the air of the tone arm, which were transmitted into the open through the horn mouth. The purpose of the horn was chiefly to enable the diaphragm to disturb a larger mass of the air and thus to increase the intensity. The quality of the sound depended to some extent on the shape of the horn.

8-3.1 The Record. In modern disc-recording the sound is collected by a microphone which converts the pressure variations of the sound waves into an alternating electric current. This is amplified and made to move the point of the cutting stylus laterally. The disc rotates at a uniform speed, while the shaft carrying the stylus moves slowly inwards from the edge of the disc. The stylus thus traces on the disc the wave-form of the sound along a gradually narrowing spiral.

The original record is taken on a wax disc and a reversed copy of it is made on copper by electro-deposition. The thin copper is backed by a steel plate to give it strength. The discs are now generally made of a vinyl plastic material. While hot it is pressed on to the copper negative in a hydraulic press. It thus becomes a true copy of the orginal record.

During reproduction the disc must be turned at the same speed as at recording. Otherwise the frequencies are all changed and the sound is much distorted.

Of late, much improvement has been made in the construction and performance of gramophone records. In the new record, known as the *micro-groove gramophone record*, there are 300 grooves per inch in place of the 100 grooves per inch of the old record. This has much increased the playing time. The material of which the record is made is a vinyl plastic ; the record itself is as thin as a wafer. The reproduction is characterised by remarkable clarity of tone and is free from distortion even to the trained musical ear.

To play the new record a much sharper needle is required. The new needle is a sapphire style and can be used 15,000 times before needing replacement. The record is rotated at a speed of 45 or 33⅓ revolutions per minute in place of the older value 78.

Mechanical reproduction, as described above, has now largely been replaced by *electrical reproduction* from the same disc records. The *pick-up* replaces the sound box, and the *loud-speaker* replaces the horn. These two, together with an amplifier, form the essential parts of the new gramophone.

There are various kinds of pick-up. In each kind, the transverse movement of the pin generates a varying electic current, potential difference or magnetic flux. The varying electric or magnetic signals are converted into appropriate electric signals, properly amplified and made to actuate a loud-speaker.

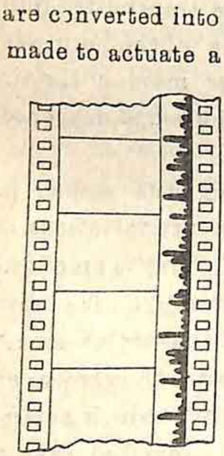


Fig. 8.3

8-4. Other methods of recording and reproduction.

Film recording and reproduction. There are several other methods of recording sound. In talkies, the sound is recorded photographically on one side of the film which records the picture. The pressure variations of the sound waves are converted into electric currents of varying magnitude by a microphone. In one method the amplified microphone current passes through a loop of wire in a magnetic field and causes the loop to vibrate. A mirror is attached to the loop from which a reflected beam of light falls on the sound track of the film and blackens regions of variable width. This is the sound record. It looks somewhat like that shown in Fig. 8·3.

In reproduction, light from a narrow source is focussed on the sound track. The quantity of the transmitted light depends on the width of the transparent portion. This light of varying intensity falls on a photo-electric cell (P ; Fig. 8.4) and produces a minute electric current proportional to the intensity. This current is amplified and made to actuate a loudspeaker.

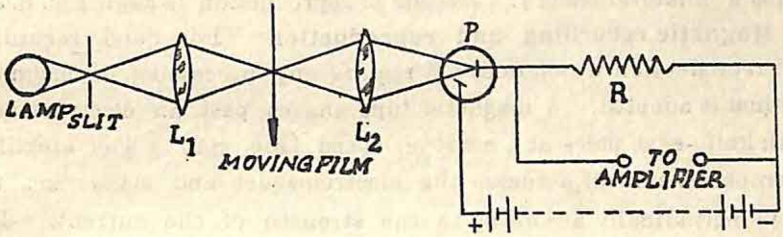


Fig. 8.4

There has since been much improvement in recording. A relatively modern method is illustrated in Fig. 8.5. The vibrator is a moving-iron armature type of galvanometer carrying a mirror M about $\frac{1}{3}$ mm

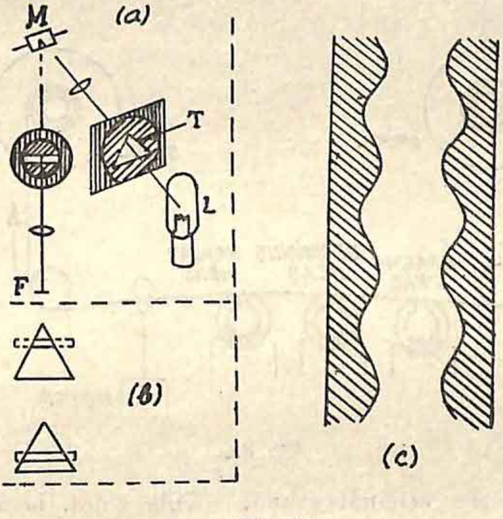


Fig. 8.5

square. The mirror throws the image of a triangular aperture T on a horizontal slit S. When an electric signal is applied, this mirror material of which the record is made is a vinyl plastic ; the record itself is as thin as a wafer. The reproduction is characterised by remarkable clarity of tone and is free from distortion even to the trained musical ear.

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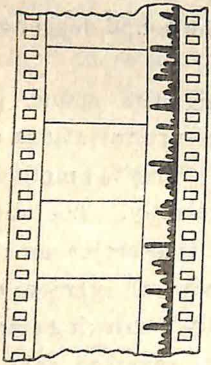

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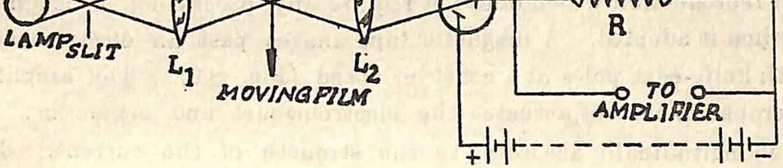

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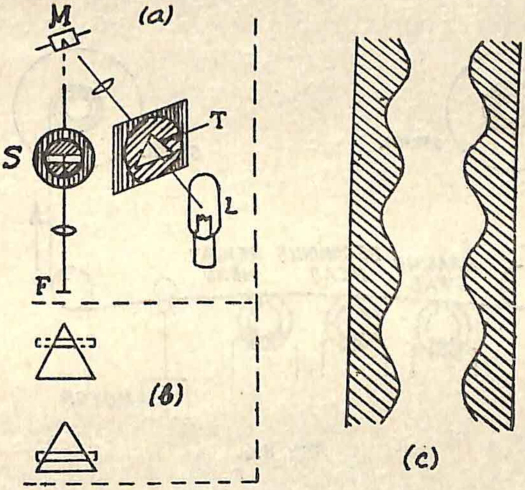

Fig. 8.5

square. The mirror throws the image of a triangular aperture T on a horizontal slit S. When an electric signal is applied, this mirror turns about a central axis in such a way that the triangular image which falls on the slit moves at right angles to the slit. The image of the slit is always focussed on the film. The portion of the slit which is illuminated depends on the current through the galvanometer. The width of the exposed portion of the film (Fig. 8.5*c*) is proportional to the rotation of the mirror (as may be understood from Fig. 8.5(*b*). This rotation is proportional to the electric current caused by the variation of sound pressure in the microphone. [Fig. 8.3 represents what is called a 'unilateral track'; Fig. 8.5 (*c*) is called a 'bilateral track']. Method of reproduction is as in Fig. 8.4.

Magnetic recording and reproduction. For quick recording and reproduction which does not require any processing, a magnetic method is adopted. A magnetic tape moves past an electromagnet with knife-edge poles at a constant speed (Fig. 8.6). The amplified microphone current actuates the electromagnet and magnetizes the tape longitudinally according to the strength of the current. The wave-form of the sound is thus stored as a varying magnetization of the tape. The tape then passes the *reproducing* or *play-back-head* (marked 'replay head' in the figure) which has a coil wound round it. The variations in the magnetic flux through this coil as the tape passes this head induces in the coil a varying e.m.f. corresponding in wave-form to the original sound. This e.m.f. is amplified and delivered to a loudspeaker.

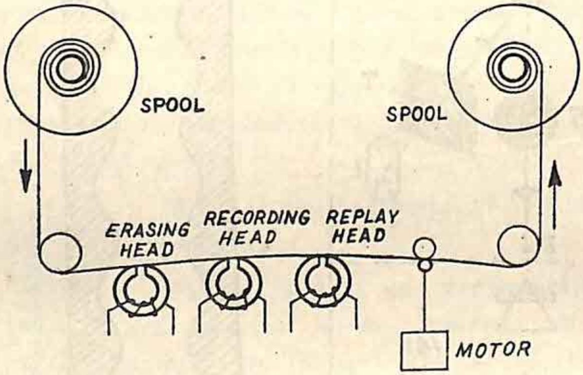

Fig. 8.6

The recorded sound may be reproduced instantaneously in this way. If subsequent reproduction is wanted, the tape is wound into a spool and the spool preserved until needed. It is then run at the same speed in the same machine. The machine has an *erasing head* to demagnetize the tape, so that the same tape may he used for recording of other sounds.

The tape-recorder is the most versatile of all the recording and reproducing devices.

Exercises

1. Briefly outline the principle on which recording and reproduction of sound is based. Illustrate the principle with reference to any machine that you know.

2. What is a phonograph ? How does it work ?

3. Give an outline of how the record of a sound is made on a gramophone disk.

How is the sound reproduced from the disk ?

4. Write a short note on magnetic recording and reproduction.

5. Write a short note on the recording of sound in a talkie and its reproduction.

CHAPTER

9

LIGHT AS A WAVE PHENOMENON

9-1. Light is a wave phenomenon. The reasons why light is considered as a wave phenomenon are that light shows all the properties of waves. These are (i) reflection, (ii) refraction, (iii) interference, (iv) diffraction, (v) scattering and (vi) polarization. It also shows Doppler effect, though it requires delicate apparatus to detect it. Moreover, light moves with a definite speed in a given medium. Light also shows the effects of superposition, and stationary waves can be formed with it.

You are already familiar with reflection and refraction of light. Interference has been discussed in Sec 9-6, and diffraction in Sec. 9-4. Polarization (Sec. 9-3) of light shows that light waves are *transverse* in nature.

Between the 16th and 19th centuries there was no agreement among scientists as to the nature of light. Newton considered light as particles, while Huyghens suggested that they were waves. The question was settled by Maxwell (1831-1879). From theoretical considerations he came to the conclusion that light was a wave phenomenon. His conclusion was confirmed experimentally by Hertz (1857-1894).

Medium for light waves. Light reaches us from the sun and stars, which are millions of kilometres away from us. Between them and the earth there is no material medium. The region is a void (vacuum). So light waves do not require any material medium for transmission. They are *non-material waves*.

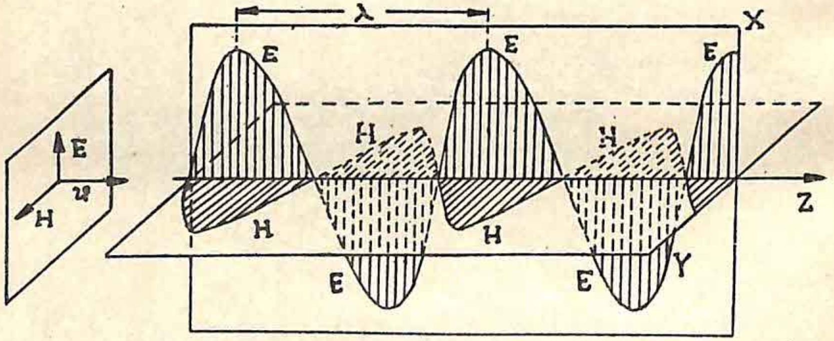

Fig. 9.1

What is it that vibrates in light? This was answered by Maxwell. The vibrations are those of an electric vector **E** and a magnetic vector H. They are respectively called electric intensity and magnetic intensity. E and H are at right angles to each other. Both lie in a plane perpendicular to the direction of propagation of light waves (Fig. 9.1). Hence the transverse character of light. Light waves are known as *electromagnetic waves.*

Early in the history of light an imaginary medium, called the *Aether* (Ether), was called in to provide a medium for light waves. This has been found to be unnecessary. The existence of aether can neither be proved nor disproved. It is better not to think of it.

When we speak of light vibrations, we are really speaking of the vibration of the electric intensity vector E.

9·2. Velocity of light. Light has been found to have the tremendous velocity of about 3×10^{10}cm/s $(=3\times10^8\text{m/s}=3\times10^5$ km/s) in vacuum. In a material medium such as glass, water, air, etc. it travels with a lower speed. This speed depends on the colour of light. If the material medium has a refractive index μ for light of a particular colour, the speed c of light of that colour is c/μ.

A Danish astronomer named Romer was the first to determine the velocity of light. He did so by making observations on the eclipses of one of Jupiter's satellites during two halves of a year. The first successful experiment from observations on the earth was done by Fizeau (1849). Later Foucault (pronounced Foo-ko), Michelson and others devised different methods for measuring the velocity of light (in vacuum) more and more accurately. Michelson (1852-1931), an American physicist, spent about fifty years of his life on such experiments. He was awarded the Nobel prize for his work. The best value he got for the velocity of light in vacuum was

$$c=299,796\pm4 \text{ km/s}.$$

The error is a little over one part in hundred thousand.

After the Second World War, great inprovements were made in the methods of measuring length and short time intervals very accurately. Also very strong beams of light, visible and invisible, could be produced. (The former is known as 'laser' and the latter as 'microwaves'.) With their help many scientists determined c more and more accurately. Considering all accurate measurements upto 1964, scientists decided that the best value is

$$c=299,792{\cdot}5\pm0{\cdot}3 \text{ km/s}$$

The error here is one part in a million (a value 10 times better than Michelson's.

The advanced nations did not stop there. Their national laboratories are still carrying on the work. In 1972, the scientists of the National Bureau of Standards (U.S.A.) found

$$c = 299{,}792{\cdot}4562 \text{ km/s}.$$

They claim that the error in the value is about one hundredth of the previous lowest error.

9-3. Polarization of light. Let us first see what polarization means and how it distinguishes transverse waves from longitudinal waves.

Refer to Fig. 9.2. AD is a stout string, or better, a fine spiral spring. Whichever we use, we shall call it a string. Its D end is fixed. The A end can be vibrated in any direction in a plane perpendicular to AD. The string passes through two narrow slits B and C in two parallel metal sheets. Either slit can be rotated in the plane of the sheet in which it lies. Note that these planes and slits are perpendicular to the string AD.

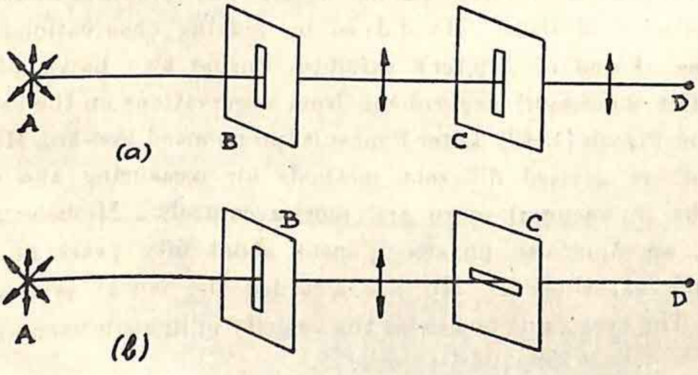

Fig. 9.2

First, let us have the slits parallel to each other. Vibrate the end A parallel to the slit B. These vibrations will pass through both B and C and reach D (Fig. 9.2 *a*). But if you now turn slit C in its own plane till it is perpendicular to B, no vibrations will pass through C. D will not feel any effect due to vibration of A. There will be vibrations between B and C, but none beyond C.

Now, periodically pull the end A longitudinally (parallel to AD). Longitudinal vibrations will pass along AD and reach D. Turning of the slits B and C will not stop the longitudinal vibrations from reaching D.

We thus find that propagation of transverse vibrations can be stopped by two slits at light angles. But such arrangement will not stop the propagation of longitudinal vibrations.

If the end A were vibrated at an angle to the length of the slit B, the component of the vibrations in the direction of the slit will pass through B. If the direction of vibration of A is rapidly changed, B will allow only the component parallel to it to pass through. On passing B, transverse vibrations are all confined to one plane. This effect is called **plane polarization** of transverse vibrations. The slit C allows the component of such vibrations in its own direction to pass through. By turning C gradually this component is reduced in strength. When C is perpendicular B, no vibration passes C. Thus B produces plane polarization while C detects it. We may call B the **polarizer** and C the **analyser.**

In the case of *light waves*, certain crystals play the same part as the slits in the above case of mechanical waves. Tourmaline is such a crystal. Take a thin plate of tourmaline and allow a light beam to pass through. Light is slightly reduced in intensity in passing through the crystal. But this may go unnoticed. The light beyond the tourmaline plate is plane polarized. This can be detected by another similar tourmaline plate. A naturally occurring tourmaline crystal has a certain clearly defined direction which we call its *axis*. When we place a second tourmaline crystal beyond the first, the first one acts as polarizer and the second as analyser of light. With the axes of both parallel, light passes through. As you rotate the second in its own plane, the light that passes it falls in intensity. When the axes of the two are perpendicular to each other, no light passes the second.

This phenomenon which occurs in the case of light is known as *polarization of light*. It establishes the fact that light waves are transverse in nature.

There are many other ways in which polarization of light can be demonstrated. In the case of sound waves we cannot produce polarization. Sound waves are longitudinal.

9-4. Light rays and diffraction. We imagine a light ray as a line (without breadth) which marks the path of travel of a wave from one point to another. In a homogeneous medium rays are straight lines. Light waves are said to travel in straight lines in such media (rectilinear propagation of light).

Let us see what we find when we try to realize a ray of light in practice. Refer to Fig. 9.3. S is a small source of light, A is a diaphragm whose aperture can be gradually reduced and B is a receiving screen. Let the aperture in A be large. Join S to the points marking the boundary of the aperture in A and produce the lines to meet B. The illuminated region on B will, *for all practical purposes*, be confined to the region so marked (Fig. 9.3a). But as the aperture is gradually reduced in size, the light on B will gradually spread beyond the geometrical limit marked in the same way as before (Fig. 9.3b). When the aperture is very much reduced the spreading of light on B becomes much greater (Fig. 9.3c). The narrower we make a beam, the more it spreads beyond the aperture. This phenomenon is known as **diffraction**, and is a property of waves of all kinds.

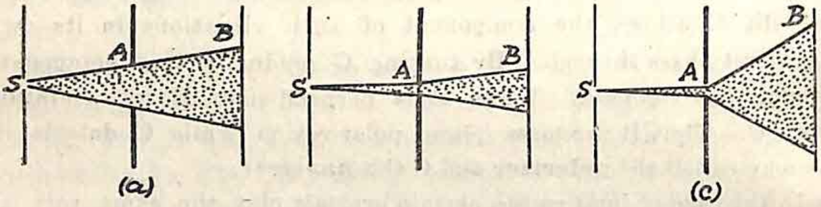

Fig. 9.3

When the width of the beam equals the wavelength of light the spreading beyond the aperture covers a hemisphere.

9.5. Validity of geometrical optics. If a ray of light cannot be realized in practice, how can we then accept the results of geometrical optics which is based on the ray concept? We have seen in the last section that *when the light beam has a width large enough compared with the wavelength of light, the ray concept is good enough as an approximation* (see Fig. 9.3a). The mean wavelength of visible light may be taken as 6×10^{-5} cm. So a lens or mirror 1 cm wide is large enough to apply the ray concept. Even then, if you examine small regions close to the geometrical boundary of a shadow, or a focus, you will find small departures from simple geometrical laws. (These are due to diffraction.)

In optical instruments the mirrors, prisms and lenses used are very large compared with the wavelength of light. Hence their action may be described in terms of rays to a good approximation.

9-6. Interferference of light. The phenomenon of interference, as we shall illustrate here, is a case of diffraction and superposition. Briefly, we can say that when two identical wave trains, proceeding in about the same direction, *continue* to be so superposed that at some places they are always in phase and at some other places they are always out of phase, there will be no vibration where the waves are not of phase and maximum vibration where they are in phase (Refer to Fig.s 1.10 and 1.11). This phenomenon is called *interference*. It is a property of all waves.

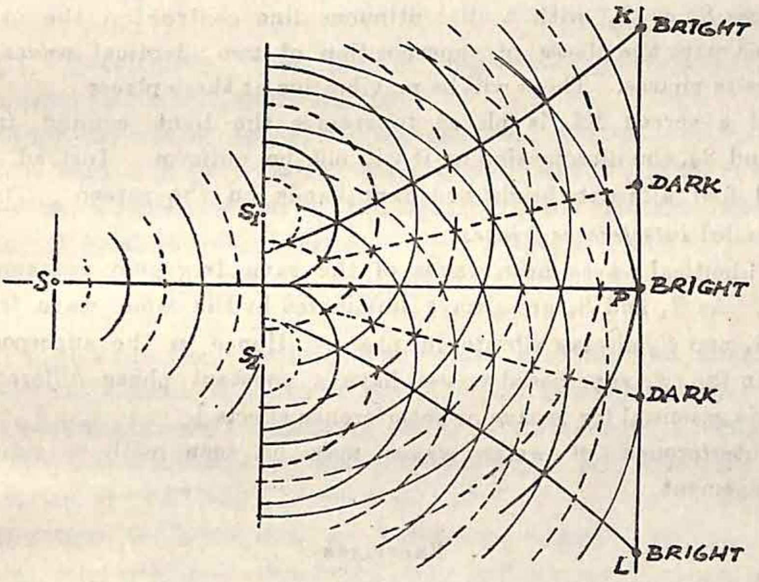

Fig. 9.4

Fig. 9.4 shows an arrangement for producing interference of light. A strong beam of monochromatic light (say, sunlight filtered through red glass) falls on a narrow aperture S. Due to diffraction the beam passing through S spreads out and falls on two other small apertures S_1, S_2. The latter are so small that the waves passing through them spread out almost in the form of hemispherical waves and are superposed on each other over a region.

In the figure, the continuous lines represent surfaces (or wave-fronts) on which the displacements are maximum. The broken lines represent surfaces (or wave-fronts) of minimum displacement—both at a given instant. The intersections of the continuous lines and of the discontinuous lines centred on S_1 and S_2 represent superposition of vibrations in the same phase. These points of intersection mark paths along which vibrations are strongest.

The intersections of a continuous line centred on one of the sources S_1 or S_2 with a discontinuous line centred on the other source mark the places of superposition of two identical waves in opposite phases. There will be no vibration at these places.

If a screen KL is placed to receive the light coming from S_1 and S_2, the illumination on it will not be uniform. Instead, we shall find alternate bright and dark bands on the screen. These are called *interference fringes.*

[Identical waves mean waves of the same frequency and amplitude. As S_1 and S_2 are always illuminated by the same wave from S, S_1 and S_2 always vibrate in phase. Hence in the superposed region the two superposed waves have a constant phase difference. This is essential for producing interference effects.]

Interference of water waves may be seen with a similar arrangement.

Exercises

1. What reasons can you give to show that light is a kind of wave? Give a rough value and, as far as possible, an accurate value of the velocity of light in vacuum.

2. What vibrates in light waves? Does light require any material medium for propagation?

3. What is meant by *polarization* of waves? Briefly explain how polarization decides whether a wave is longitudinal or transverse. To which of these kinds do light waves belong? Why do you think so?

4. Illustrate with a simple arrangement the phenomenon of *interference* of light. State how your arrangement satisfies the conditions of interference.

5. If light is a kind of wave, and waves have the general property of bending round corners, how can we accept the results of geometrical optics? Geometrical optics is based on the assumption that light travels in straight lines in a homogeneous medium.

PROPERTIES OF MATTER

CHAPTER 1

GRAVITATION

1-1. Gravitation. The law of universal gravitation, first enunciated by Newton, states that :—

Every particle of matter in the universe attracts every other particle with a force F which varies directly as the product of their masses (m, m') and inversely as the square of the distance between them. In symbols, we may write

$$F = \frac{mm'}{r^2} \quad \text{or} \quad F = G\frac{mm'}{r^2} \qquad (1\text{-}1.1)$$

G in this equation is called the **universal gravitational constant.** In the cgs system its value is 6.67×10^{-8}. This means that two point particles of mass 1 g each, placed at a distance of 1 cm from each other, attract each other with a force of 6.67×10^{-8} dyn. (In the mks system $G = 6.67 \times 10^{-11}$ newton m^2/kg^2.)

Gravitational attraction of extended bodies : Homogeneous spheres, solid or hollow, attract each other as if the entire mass of each were concentrated at its geometrical centre. This property of spheres enables G to be determined rather easily, for they behave like particles with masses concentrated at the centres. For bodies of other shapes, it is practically impossible to calculate the gravitational attraction between any two. This is so because in extended bodies each particle of one attracts each particle of the other along lines joining the two particles. Thus the total attraction between two such bodies is made up of a very large number of forces of different magnitudes and directions. To find their resultant is an impossible task.

Take two homogeneous spheres of masses M_1 and M_2. Let the distance between their centres be R. Then the gravitational attraction between them is $F = GM_1M_2/R^2$. But if the masses had other shapes, such calculation is not possible. If, however, the distance R

is very very large compared with the sizes of the bodies, the last relation can be used.

Universality of the law of gravitation. Newton satisfied himself of the universality of the law by showing that the gravitational force of the earth upon the moon was equal to the centripetal force necessary to hold the moon in its nearly circular orbit around the earth. Astronomers found that it accounted satisfactorily for the motions of the planets around the sun. Leverrier and Adams predicted and located the planet Neptune with the help of this law.

Laboratory experiments show that the law is not modified by the presence of any material between the attracting bodies, *i.e.*, by the medium in which the bodies are placed. It is independent of the nature of the molecules which make up the bodies, nor does it depend on the state of aggregation of attracting matter (*i.e.*, whether it is solid, liquid or gaseous). It is not affected by temperature, pressure or any other physical factor. We are thus led to believe that gravitational attraction depends only on the masses of the attracting bodies and their distance apart.

1-2. Gravitational attraction of the earth. In accordance with the law of universal gravitation the earth attracts all bodies on its surface with a force proportional to the mass m of the body. Taking the earth to be a homogeneous sphere of radius R and mass M, the gravitational attraction, W, which it exerts on a body on its surface, is given by

$$W=G\frac{Mm}{R^2} \tag{1-2.1}$$

This force, called the **force of gravity**, is directed towards the centre of the earth. *This is also the weight of the body.* We know that when a body falls freely under gravity it does so with a constant acceleration which we denote by g. Hence by Newton's second law of motion the weight W of a body of mass m is given by $W=mg$.

$$\therefore\ mg=GMm/R^2 \quad \text{or,} \quad g=GM/R^2, \tag{1-2.2}$$

Mass of the earth. Since we know the values of g, G and R, we can calculate M, the mass of the earth. Let $g=9{\cdot}8$ m/s^2, $G=6{\cdot}67\times10^{-11}$ mks unit and $R=6{\cdot}367\times10^6$m. Substituting values, we get $M=5{\cdot}96\times10^{24}$kg. If ρ is the *mean density of the earth* then $M=\frac{4}{3}\pi R^3\rho$. Substituting values, we get $\rho=5{\cdot}5\times10^3$kg/m^3 or $5{\cdot}5$g/cm^3.

The density of rocks in the earth's outer crust is about 3·5 g/cm^3. Hence the interior of the earth must be made of heavier stuff. The earth is not really a homogeneous sphere.

1-3. Motion under gravity.

By *gravity* is meant the attraction due to the earth. The force of gravity always acts towards the centre of the earth.

In olden times it was thought that heavier bodies fell towards the earth more rapidly than lighter ones. This was disproved by Galileo. He dropped stones of different weights from the top of the Leaning Tower at Pisa (Italy) and showed that they fell together when released at the same instant.

Galileo's work was further extended by Newton. He devised the well-known guinea-and-feather experiment. It shows that if the motion of a falling body is not slowed down by air, all bodies, however light or heavy, will fall together under gravity. In this experiment, a guinea and a piece of feather were kept in a long glass tube from which air could be pumped out. When there was no air inside, the guinea and the feather moved together as the tube was inverted. But the motion of the feather was slowed down on admitting air into the tube.

1-3.1. Laws of falling bodies.

From a study of falling bodies Galileo drew some conclusions regarding the nature of their motion. They are known as the **laws of falling bodies.** These may be stated as follows :

When a body falls *freely* from *rest* under gravity

(*i*) *all bodies travel equal distances in equal times,*

(*ii*) *the velocity of a body is proportional to its time of motion,* and

(*iii*) *the distance travelled in a given time is proportional to the square of the time.*

1-3.2. Freely falling bodies have the same constant acceleration.

We can easily show that a motion having the above properties is one of constant acceleration. From law (*ii*) above, for a given body $v \propto t$. Therefore v/t is constant. But v/t is the rate of change of velocity, i.e., the *acceleration.* Thus we conclude from law (*ii*) that the acceleration of a freely falling body is constant. We have now to show that this constant has the same value for all bodies.

Since the body starts from *rest* and moves with a constant acceleration we can put $u=0$ in the equation $s=ut+\frac{1}{2}ft^2$ and get $s=\frac{1}{2}ft^2$. This shows that the distance traversed by the body in a given time is proportional to the square of time. This is law (*iii*).

From law (*i*) we know that for a given s, t is the same for all bodies. Hence from the relation $s=\frac{1}{2}ft^2$ of the previous paragraph, f will be the same for all bodies, *i.e. all bodies falling freely under gravity have the* **same** *constant acceleration* (g).

1-4. Variation of g. g varies from place to place on the surface of the earth as the earth is neither a true sphere nor is it homogeneous. g increases in value with latitude, as the polar radius is a little shorter than the equatorial radius. It has the lowest value at the equator and the highest value at the poles.

Another cause of variation with latitude is the *diurnal rotation of the earth*. As the earth rotates about its axis, a body on its surface moves in a circle about the earth's rotation axis. The centripetal force for this rotation in a circle comes from the earth's gravitational pull on the body. The centripetal force ($m\omega^2 r$) is largest at the equator and smallest at the poles, because ω is the same at all latitudes, while r diminishes with increasing latitude. (r is the distance of m from its rotation axis, which is the north-south axis of the earth.)

Besides, g diminishes away from the surface of the earth, both above and below. At the centre of the earth $g=0$.

For places above the earth, we can take its mass to be concentrated at its centre. In such a case we can write,

$$g=GM/(R+h)^2=GM/\{R^2\ (1+h/R)^2\}=GM(1-2h/R)/R^2 \qquad (1\text{-}4.1)$$

(expanding binomially and retaining only the first power of h/R). Here h is the height above the earth's surface. For places below the earth's surface, as in a mine, the gravitational attraction is due not to the entire earth. It is due only to the sphere which lies between the body and the centre of the earth. At a depth d below the surface, we shall therefore have

$$g=G\ \tfrac{4}{3}\pi\ (R-d)^3\rho'/(R-d)^2=\tfrac{4}{3}\pi G\rho'(R-d) \qquad (1\text{-}4.2)$$

in which ρ' is the mean density of the smaller earth-sphere.

When $d=R$, $g=0$.

1-5. Motion of planets. The gravitational attraction of the sun causes the planets to move round it. Though the orbits are elliptical, we can get interesting results by treating the distance between the sun and a planet as constant, and equating the gravitational pull to the centripetal force required to make the planet move in a circular orbit. Let M = mass of the sun, m = mass of a planet, v = speed of the planet in its orbit and r = the distance between them. Then, if T is the periodic time of the planet.

$$GMm/r^2 = mv^2/r \text{ or } GM/r = v^2 = (2\pi r/T)^2$$

$$\therefore \; T^2/r^3 = 4\pi^2/GM \qquad (1\text{-}5.1)$$

This relation is independent of the mass of the planet, and hence is the *same for all planets*. It is known as Kepler's third law.

Example. Calculate the mass of the sun, given that the radius of the earth's orbit is $1{\cdot}5 \times 10^{13}$ cm and $G = 6{\cdot}67 \times 10^{-8}$ cgs unit.

[**Hint** : Take $T = 1$ year $= 365$ days $= 365 \times 86,400$ seconds and apply Eq. 1-5.1. Ans : 2×10^{33} g nearly.]

1-6. Motion of satellites. The gravitational attraction of a planet on its satellite makes the satellite move round the planet. Take the orbit of the satellite to be circular. The gravitational attraction supplies the necessary centripetal force. This applies equally to natural and artificial satellites.

Let M = the mass of the planet, m = the mass of the satellite, r = the radius of the circular orbit in which the satellite moves, T = the time which the satellite takes in going once round the planet (that is, its periodic time), v = the speed of the satellite in its orbit. Then, since gravitational attraction = centripetal force, we have

$$GMm/r^2 = mv^2/r \text{ or } GM/r = v^2 = (2\pi r/T)^2$$

$$\text{or} \quad T^2/r^3 = 4\pi^2/GM \qquad (1\text{-}6.1)$$

You will note that this is the same equation as for the motion of planets round the sun. T^2/r^3 will have the same value for all satellites, real or artificial, of a given planet.

Examples. (1) An artificial satellite circles the earth near its surface. Find its periodic time, given that radius of the earth $= 6{\cdot}4 \times 10^8$ cm and $g = 980$ cm/s^2.

[**Hint** : $g = GM/r^2$. Hence $T^2 = 4\pi^2 r.\,(r^2/GM) = 4\pi^2(r/g)$ or $T = 2\pi\sqrt{g/r}$.

Ans : 1 hour 25 minutes nearly.]

(2) At what distance from the centre of the earth should an artificial satellite move in a circle so that it may appear stationary to an observer on the earth ?

[**Hint** : The period T of the satellite should be 24 hrs = 86,400 s. In Eq. 1-6 1. give T this value. Put down known values of G and M and calculate r. r is the required value. Ans : About 36,000 km.]

1-6.1. Weightlessness in artificial earth satellites. In an orbiting earth satellite the whole of the earth's gravitational pull is spent in giving the satellite the necessary centripetal pull. It is actually falling towards the centre of the earth with the acceleration due to gravity. But its orbital speed prevents it from crashing on the earth. A man seated in such a satellite does not therefore experience any force pressing him to his seat, as he does when seated on a stationary body on the earth. He feels weightless. The situation is very much the same when one descends in a lift. As the lift accelerates downwards the reaction of the floor on a person standing in the lift diminishes. The person feels lighter. If the lift descended with the acceleration of a free fall under gravity, the reaction of the lift on the person will be zero. The person will feel weightless. It is this kind of thing that happens in an orbiting earth satellite.

We have defined the weight of a body as the pull of the earth on it. But our sense of weight comes from the reaction to this pull. When we hold a book on our palm, the weight of the book presses on the palm. The reaction which the palm exerts on the book gives us a sense of the weight of the book. As we stand on the floor, the reaction which the floor exerts on our feet gives us the sense of our weight. In an artificial earth satellite, the floor does not exert any reaction on one standing on the floor.

(In water, a man feels almost weightless. This gives you some idea of what weightlessness is like.)

1-7. Escape velocity. The gravitational pull of a larger body can hold a smaller body captive in its gravitational field if the speed of the latter is not too large. The minimum velocity which a smaller body must have to escape from the gravitational attraction of a larger body is called the **escape velocity.**

If the larger body is spherical and has a mass M and a radius R, the escape velocity v_e from its surface is given by $v_e = \sqrt{2GM/R}$. G is the universal gravitational constant. Note that the escape velocity does not depend on the mass of the body which is to escape. The escape velocity from the surface of the earth is 11·2 km/s (about 7 miles per second). It is about 30 times the speed of sound in air. The escape velocity from the planet Mars is 5·0 km/s ; from Mercury it is 3·8 km/s and from the moon it is 2·4 km/s.

If you can give a body on the surface of the earth a velocity of 11·2 km/s it will leave the earth for good. It does not matter in which direction you throw it.

For journey to another planet, a space ship should be given an initial speed greater than its escape velocity from the earth to save time of travel. An artificial earth satellite has a smaller velocity than the escape velocity. The escape velocity diminishes as the distance of the smaller body from the centre of the earth increases.

Exercises

1. State the law of universal gravitation and express it in symbols. What is meant by the *universal gravitational constant* ? What is its value in cgs units ? Why is it called a *universal* constant ?

2. Two spheres have masses M_1 and M_2 and radii r_1 and r_2. What will be the gravitational attraction between them if their centres are at a distance d apart ? Will the formula apply if the bodies, instead of being spherical, had some other shape ? Explain your answer.

3. What part does gravitation play in the motion of planets ? Assuming that the orbit of a planet is circular, derive a relation connecting the periodic time of the planet and the radius of its orbit.

4. What is meant by *acceleration due to gravity* ? How is it related to the mass of the earth ? If the earth's mass $= 5{\cdot}96 \times 10^{27}$ g, its radius $= 6{\cdot}367 \times 10^{8}$ cm and gravitational constant $= 6{\cdot}67 \times 10^{-8}$ cgs unit, what will be the value of g on the surface of the earth ?

5. Discuss why g changes with latitude. At a given place why does g have the greatest value on the surface of the earth than above or below it ?

6. Distinguish between *mass* and *weight*. Under what circumstances will a body have no weight ? Will it have no mass at the same time ? Why ?

A spring balance can detect differences of weight at different places on the earth ; but a common balance cannot. Explain why.

7. Why does a body appear weightless in an artificial satellite ? Will it require a force to move a body in such a satellite ? Explain.

8. State the laws of falling bodies. How can you deduce from them that all freely falling bodies move with the same constant acceleration ?

9. What is meant by *escape velocity* ? What is its value on the earth's surface ? Is it the same on all planets ? If a body is to leave a planet not to return, in what direction should it be given the necessary velocity ?

10. Calculate the value of the acceleration due to the moon's gravity on its own surface, given that the radius of the moon is 0·27 earth radius and its mass is 1/82 of the earth's mass. [ANS. About 1/6 the earth's gravity.]

How heavy will a boy weighing 60 kg on the earth feel on the moon ? [ANS. Like 10 kg on the earth.]

CHAPTER

2 ELASTIC PROPERTIES OF MATTER

2-1. Elasticity—Stress, Strain and Hooke's Law. While unbalanced forces produce motion, *balanced forces acting on a body produce deformation.* Consider a spiral spring (Fig. 2.1*a*) supported rigidly at its upper end and weighted at the lower. If the lower end is pulled downwards the spring is acted on by two forces which balance, namely, (*i*) the applied downward pull, and (*ii*) an equal and opposite pull exerted by the support on the spring. The result of these two forces is an elongation—a deformation—of the spring. Very often we leave out of our consideration the force exerted on the spring by the support and speak of the elongation as being due to the pull exerted on it.

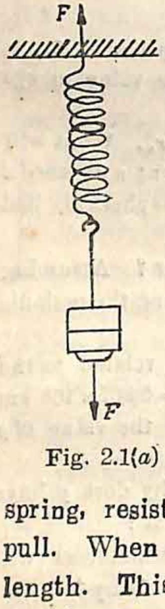

Fig. 2.1(*a*)

In the above example, the hand that pulls the spring experiences an upward force. This upward force, brought into play by the deformation of the spring, resists the deformation. It is larger the greater the applied pull. When the pull is removed, the spring acquires its original length. This simple observation shows that—

(*i*) When a body (here the spring) is deformed, the body resists the deformation, and

(*ii*) When the deforming force is removed, it tends to go back to its original condition.

Elasticity. The property of matter by virtue of which matter resists the action of a force tending to change its shape or size and returns to its original shape or size on removal of the deforming force, is called **elasticity.**

If the body entirely regains its shape and size, it is said to be *perfectly elastic.* If it fully retains its altered shape or size it is called *perfectly plastic.* If it does not undergo any change under the action of a deforming force, it is said to be *perfectly rigid.*

Strain. If the shape or size of a body changes under the action of a force, the body is said to be *strained.* The change in shape or size that it undergoes is its *deformation.* The fractional deformation or the relative change so produced is called its *strain.*

Stress. When a body in equilibrium is strained, forces applied to it are transmitted through the body, causing reactional forces to be set up within the body. As a result, *if we consider an imaginary surface within the body, the part of the body on one side of the plane will exert a force across the surface on the other part. Such a force per unit area of the surface is called* **stress.** Stress is measured by the force per unit area. The force can be divided into two components: one perpendicular to the surface and the other parallel to the surface. The perpendicular component is called the **normal stress**, and the parallel component is called the **tangential stress.**

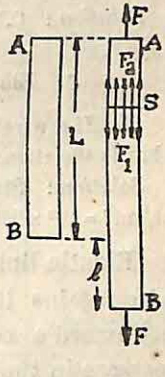

Fig. 2.1(b)

Fig. 2.1(b) shows a rod AB of original length L. A longitudinal pull due to a force F has increased its length by l. If we consider any imaginary plane S anywhere perpendicular to F, the portions of the rod both above and below S are in equilibrium. Consider the equilibrium of the upper portion. The upward force F on it must be balanced by equal and opposite force (F_1) acting across its lower surface. Similarly, the lower portion must be acted on by a force F_2 upward across S. As there is equilibrium, we must have $F_1=F_2=F$. The lower portion exerts the force F_1 across S on the upper portion. The upper portion exerts the force F_2 on the lower portion, both across S. F/S is the longitudinal stress. Had we taken S parallel to F, the stress across it would be zero.

Units of strain and stress. Consider a piece of wire, rigidly supported at its upper end, being stretched by a load at its lower end. As the load increases, so does the elongation. If

e = the elongation for a load F,
l = the initial length of the wire,
a = area of cross section of the wire,

then strain $=e/l$ and stress $=F/a$. Strain, being a ratio of two similar quantities, is expressed as a *pure number*. F is in reality a force. Hence stress is to be expressed as a *force per unit area*. Note that this is also the way we express a pressure. Often F is expressed in mass units.

Examples. (1) If a wire, 2 metres long, elongates by 1 mm under a load, what is the strain ?

Solution ; Strain = elongation/length = 1 mm/2 metres = 1/2000 = 0.0005.

(2) What must be the elongation of a wire 5 metres long so that the strain is 0.1 of 1% ?

Solution : $0.1 \text{ of } 1\% = \frac{1}{10} \text{ of } \frac{1}{100} = \frac{1}{1000}$.

$\therefore$ Elongation = strain × length $= \frac{1}{1000} \times 5$ metres = 5 mm.

(3) If a wire has a cross-section of 1 mm^2 and is stretched by a load of 10 kg, what is the stress ?

Solution : Stress = Load/area = 10 kg-wt/1 mm^2 = 1000 kg-wt/cm^2 or 10^6 gm-wt/cm^2 = $10^6 \times 980$ dynes/cm^2.

Elastic limit. In the case of a stretched wire, we find that the wire regains its initial length on removal of the load if the load does not exceed a certain limit. If the limit is exceeded a permanent increase in the length occurs. This limit is known as the *elastic limit.* Such a limit exists for all kinds of strain.

If we plot the extension of the wire under different loads, we get a curve as shown in Fig. 2.2. The graph is a straight line upto a point A. The load corresponding to this point divided by the cross section of the wire gives the elastic limit. Upto the elastic limit, the extension is proportional to the load, or the stress is proportional to the strain.

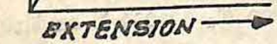

Fig. 2.2

Hooke's Law. The fundamental law relating to elasticity was discovered by Robert Hooke (1635-1703), an English physicist. It states that **so long as the elastic limit is not exceeded the stress is proportional to the strain.**

$\therefore$ Stress = a constant (say, C) × strain.

The constant C has a value characteristic of the material of the elastic body, and is called the *modulus of elasticity*. There are different moduli of elasticity depending on the nature of the strain.

Verification of Hooke's law. To verify Hooke's law we can use a spiral spring. We take its elongation under a load as the strain, and the load itself as the stress.

Clamp a metre scale vertically with the zero mark uppermost (Fig. 2.3). Suspend the spring from a rigid clamp. Attach a long pin to its lower end so that the point of the pin moves over the vertical scale. Attach a weight to the bottom of the spring so that any kinks in the spring are removed. Note the scale reading (l_0). Gradually increase the load W at the end of the spring. (The kink-removing load will always be there; but it should not be included in W.) For each load, note the length l of the spring from the scale.

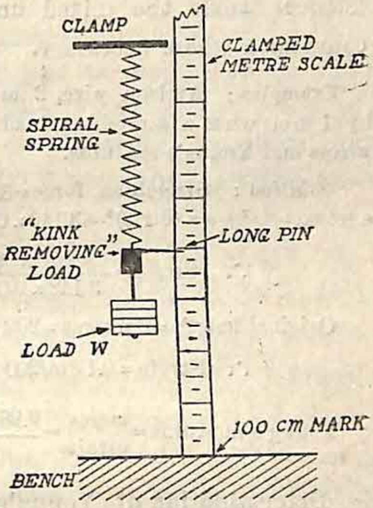

Fig. 2.3

Plot graphically the load W along the x-axis and the corresponding elongation $(l-l_0)$ along the y-axis. The graph may be a straight line all along. This shows that stress is proportional to strain and the elastic limit has not been exceeded. Or, you may find that the graph, after running straight upto a certain distance, slowly bends. The point of bending marks the elastic limit of the spring. The load W' corresponding to this point or the corresponding elongation $(l-l_0)$ may be taken as the elastic limit of the spring.

2-2. Moduli of elasticity. The most important moduli of elasticity are (1) Young's modulus, (2) bulk modulus and (3) modulus of rigidity (or shear modulus).

(1) **Young's modulus.** If e is the elongation produced by a load F when applied to a wire of original length l and cross section a, then the stress $=F/\alpha$ and the strain $=e/l$. The ratio of this stress to the strain is called Young's modulus. Denoting the latter by E (internationally recommended symbol; many authors use Y)

$$\text{we have } E=\frac{F/\alpha}{e/l}=\frac{Fl}{\alpha e} \qquad (2\text{-}2.1)$$

Since strain is a number, Young's modulus is expressed in the same units as a stress, e.g., in absolute units such as dyn/cm^2 in the cgs system, or N/m^2 in the mks system. Often it is given in gravitational units per unit area, or in mixed units, such as lb-wt/in^2, kg-wt/cm^2, etc. In working out numerical problems remember to convert all quantities in absolute units of the same system, or, if you have confidence, tackle the mixed unit symbols algebraically with an eye to conversion where necessary.

Examples ; An iron wire, 2 metres long and of diameter 1 mm, stretches by 1 mm when a stretching load of 8 kg is applied to it. Find the strain, stress and Young's modulus.

Solution : Stretching force = 8 kg-wt = 8000×980 dynes. Area of cross-section $= \pi r^2 = \pi \times (0.1/2)^2 = 3.142 \times 0.0025$ cm^2.

$$\therefore \quad \text{Stress} = \frac{8000 \times 980 \text{ dyn}}{3.142 \times 0.0025 \text{ cm}^2} = 9.984 \times 10^8 \text{ dyn/cm}^2.$$

Original length = 2 metres = 200 cm. Increase in length = 1 mm = 0.1 cm.

$$\therefore \quad \text{Strain} = 0.1 \text{ cm}/200 \text{ cm} = 5 \times 10^{-4}.$$

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}} = \frac{9.984 \times 10^8 \text{ dyn/cm}^2}{5 \times 10^{-4}} = 1.997 \times 10^{12} \text{ dyn/cm}^2.$$

Determination of Young's modulus. Suppose the experimental material is available in the form of a uniform wire. We take two pieces of it, each about 3 metres long and suspend them from a common support (C, Fig. 2.4). One of the wires (A) carries a finely etched scale (S) and is kept free of kinks by a load D at its lower end. The other wire B carries a vernier V which can slide along S. It is also kept free of kinks by a suitable load attached to its lower end (E). The vernier reading is taken with no other load. Additional known loads are then placed on E, and the vernier reading taken for each additional load. After getting about 5 or 6 such readings we calculate the extension for each load. Let the mean value of all the extensions be e and the mean value of the additional loads be W. Measuring the diameter of the wire at several places we can get its mean diameter and hence its cross-section α. The length l of the wire B from its point of attachment at the top to that with the vernier is measured. We now have the stretching force $F = Wg$ (where g is the acceleration due to gravity), the cross-section α, the length l and

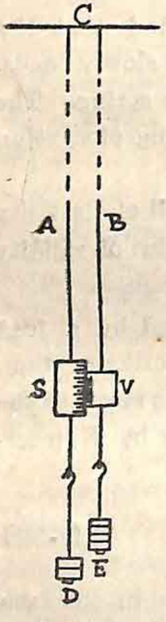

Fig. 2.4

the elongation e for the force F. Application of formula 2-2.1 will give us the value of E, the Young's modulus.

Two wires are required to neutralize the effect of change of length due to change of temperature, as also of the sag of the support C due to loading.

Tensile and compressive strain. The extension of a wire under a load represents the kind of strain known as *tensile strain.* The corresponding stress is called *tensile stress.* If the force is one of compression along a direction instead of extension as above, the corresponding strains and stresses are called *compressive strains* and *compressive stresses.*

When a beam supported at the ends is loaded in the middle, the depression is proportional to the load so long as a certain limiting load is not exceeded. A portion of the beam is compressed and a portion elongated. Both tensile and compressive strains and stresses appear in the beam.

(2) Volume stress: Bulk modulus. The volume of a body, whether solid, liquid or gaseous, can be reduced by applying a uniform pressure. If an *increase of pressure* p reduces the volume from V to $V-v$, v/V is the *volume strain.* The *volume stress* is p. The ratio of volume stress to volume strain is called *bulk modulus.*

$$\text{Bulk modulus } (K) = \frac{\text{volume stress}}{\text{volume strain}} = -\frac{p}{v/V} \qquad (2\text{-}2.2)$$

p is measured by the force applied per unit area ; v/V is a pure number. The negative sign comes out of the fact that an increase of p diminishes V. Hence K is expressed in *force per unit area.*

The reciprocal of bulk modulus is called the *compressibility* of the substance. Gases are highly compressible and have a small value of K. In the case of a gas the bulk modulus may be shown to be equal to its pressure. Both solids and liquids have high values of K, of the order of 10^5 kg-wt per cm^2.

(3) Shearing stress and strain : Modulus of rigidity.

To understand what a shear is, we may think of a pack of cards or a thick book lying on a table. Laying the palm flat upon the pack of cards or the book we may apply a horizontal force so that the rectangular pile is changed into a parallelepiped, each card or the page sliding over the one beneath. The shape changes without change of volume. When an elastic body is changed in this way, its *shape being changed without change of volume*, it is said to undergo a *shear*. A shear requires the application of a tangential force. Fig. 2.5 illustrates a shear. The base Q of the rectangular block is kept fixed and a tangential force F is applied to the top P. As a result the body is deformed from the shape $ABCD$ to the shape $AB'C'D$. The angle $BAB'=\theta$ is the angle shear.

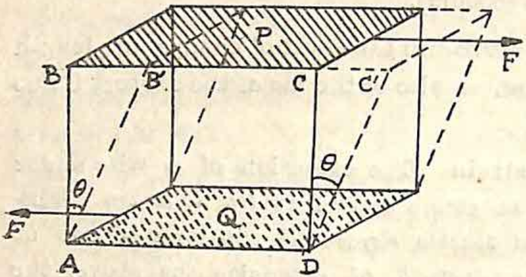

Fig. 2.5

When a body is *sheared*, the ratio of the shearing stress to the shearing strain is called the *modulus of rigidity* (*or shear modulus*). If a tangential force F applied over an area α produces the shear, the shearing stress is F/α. If due to the shear a straight line perpendicular to the plane of the shear becomes inclined to its original direction by an angle θ, the shearing strain is taken as $\tan\theta$ ($=\theta$ when θ is small, as it generally is). Thus the modulus of rigidity may be written as

$$G=\frac{F/\alpha}{\theta}=\frac{F}{\alpha\theta} \tag{2-2.3}$$

In cgs units it is measured in *dynes per sq. cm* (dyn/cm^2) like the other two elastic moduli. (G is the internationally recommended symbol for modulus of rigidity. Many authors use n for it.)

When two plates held by a rivet (Fig. 2.6) are pulled away from each other parallel to the common surface the rivet sustains a shear, its lower part tending to slide off the upper. The name shear is derived from the fact that when a piece of paper is severed by a pair of shears (scissors) precisely this kind of stress is applied.

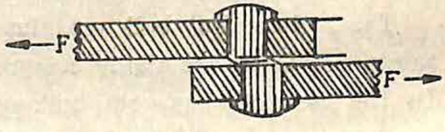

Fig. 2.6

The twist in a wire or rod is proportional to the couple producing the twist provided the latter does not exceed a certain value. The behaviour of a wire or rod under the action of a twisting couple is determined by its modulus of rigidity.

Liquids and gases do not have any modulus of rigidity as the smallest shearing stress goes on increasing the shearing strain in them. They cannot be in equilibrium under a shearing stress.

A spring balance measures force. When a force is applied along the axis of a spiral spring, the elongation of the spring will be proportional to the force if the latter is not too large. This property of a spring may be utilized for measuring a force. Known forces may be applied to a spring and its elongations graduated in terms of the force.

When a spring balance is used to measure the mass of a body, what we actually measure is the weight of the body. But weight depends on g, the acceleration due to gravity, and g varies from place to place. Hence if the same mass is weighed by the same spring balance at different places, it will indicate different values due to variation of g. The difference, however, is very small.

2-3. Elastic constants. By elastic constants are meant the *three elastic moduli* and *Poisson's ratio*. The three elastic moduli are Young's modulus, bulk modulus and modulus of rigidity.

Poisson's ratio. When a body elongates freely in the direction of a tensile force, it contracts laterally, *i.e.*, in a direction perpendicular to the force. When a compressive force is applied, it expands laterally. When there is no bar to lateral contraction or expansion, *the ratio which the lateral change in length per unit length bears to the longitudinal change in length per unit length is called Poisson's ratio.* It is a *pure number*, being the ratio of two strains.

$$\text{Poisson's ratio} = \frac{\text{lateral strain}}{\text{longitudinal strain}} \qquad (2\text{-}3.1)$$

The elastic constants determine the behaviour of homogeneous, isotropic elastic bodies under the different stresses. They are very important in engineering practice. Elongation and contraction of beams and struts as well as bending of beams under load are determined by Young's modulus, and twist of wires and rods by the modulus of rigidity.

Table : Moduli of Elasticity

Substance	Young's modulus		Bulk modulus		Modulus of rigidity	
	$\frac{\text{dyn}}{\text{cm}^2}$	$\frac{\text{lb-wt}}{\text{in}^2}$	$\frac{\text{dyn}}{\text{cm}^2}$	$\frac{\text{lb-wt}}{\text{in}^2}$	$\frac{\text{dyn}}{\text{cm}^2}$	$\frac{\text{lb-wt}}{\text{in}^2}$
Aluminium	7×10^{11}	10×10^6	7×10^{11}	10×10^6	$2{\cdot}5 \times 10^{11}$	$3{\cdot}6 \times 10^6$
Copper	10 ,,	14 ,,	12 ,,	17 ,,	4·2 ,,	6·1 ,,
Iron (drawn)	20 ,,	29 ,,	9·6 ,,	14 ,,	5·1 ,,	7·4 ,,
Iron (cast)	11·5 ,,	16·8 ,,	—	—	—	—
Steel (mild)	20 ,,	32 ,,	16 ,,	23 ,,	8 ,,	11·6 ,,
Water	—	—	0·2 ,,	0·3 ,,	—	—
Mercury	—	—	2·6 ,,	3·7 ,,	—	—

Examples : (1) A steel wire 7 metres long and 1 mm in diameter is subjected to a tension of 30 kg, and its elongation is observed to be 1.21 cm. Find the strain, the stress, and Young's modulus for the specimen.

Solution :

$$\text{Elongational strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{1{\cdot}21 \text{ cm}}{700 \text{ cm}} = 0{\cdot}00173.$$

$$\text{Stress} = \text{Applied force per unit area} = \frac{30 \text{ kg}}{\pi.\ (0{\cdot}5 \text{ mm})^2} = 38{\cdot}2 \text{ kg/mm}^2.$$

$$\therefore \text{ Young's modulus} = \frac{\text{Stress}}{\text{Strain}} = \frac{38{\cdot}2}{0{\cdot}00173} \text{ kg/mm}^2 = 2{\cdot}208 \times 10^4 \text{ kg/cm}^2.$$

(2) A load of 60 tons is carried by a steel column having a length of 24 ft and a sectional area of 10·8 in^2. What decrease in length will this load produce if the elastic modulus is 30×10^6 lb/in^2 ?

Solution : Let the decrease in length be x in.

$$\text{Strain} = \frac{x \text{ in}}{24 \times 12 \text{ in}}. \quad \text{Stress} = \frac{60 \text{ ton}}{10{\cdot}8 \text{ in}^2} = \frac{60 \times 2240}{10{\cdot}8} \text{ lb/in}^2.$$

$$\text{Now, elastic modulus} = \frac{\text{Stress}}{\text{Strain}}. \text{ Hence } 30 \times 10^6 \text{ lb/in}^2 = \frac{60 \times 2240}{10{\cdot}8} \text{ lb/in}^2 \div \frac{x}{24 \times 12}$$

$$\therefore x = \frac{60 \times 2240 \times 24 \times 12 \text{ in}}{10{\cdot}8 \times 30 \times 10^6} = 0{\cdot}112 \text{ in.}$$

(3) A steel wire of length 2 metres and diameter 0·8 mm is stretched horizontally between rigid supports attached at its ends. When a load is hung from the mid-point of the wire a depression of 1 cm is produced. Calculate the load, given that $E = 2 \times 10^{12}$ dyn/cm^2.

Solution : (Draw the necessary diagram.) Half-length of stretched wire $= \sqrt{(100^2 + 1)} = 100{\cdot}005$ cm. Its extension $= 0{\cdot}005$ cm. If θ is the angle the stretched wire makes with the vertical line through the point of attachment of the load, $2T \cos \theta = \text{load}$, where T is the tension in the wire. $\cos \theta =$ depression/half-length of stretched wire $= 1/100$ very nearly.

∴ Tension $T = 50 \times$ load. The stretching force per unit area or stress $= T/$area of cross-section $= 50 \times \text{load}/(\pi \times 0{\cdot}04^2\ \text{cm}^2)$. Strain $= 0{\cdot}005/100$.

$$\therefore\ 2 \times 10^{12}\ \text{dyn/cm}^2 = \frac{50 \times \text{load}}{\pi \times 0\ 04^2\text{cm}^2} \div \frac{0{\cdot}005}{100} \text{ whence load} = 10{\cdot}3 \text{ g-wt.}$$

(4) A wire of diameter 1 mm supports a load which keeps it straight. If the temperature falls by 20°C what additional load will be required to keep the length of the wire unchanged? ($E = 2 \times 10^{12}$ cgs units; coefficient of linear expansion $= 1 \times 10^{-5}$ per °C.)

Solution: Let l be the initial length of the wire. Its contraction due to fall in temperature $= l \times 10^{-5} \times 20$. The extra load must produce this much of elongation. Hence the strain should be $l \times 10^{-5} \times 20 \div l = 20.10^{-5}$.

The cross-section of the wire $= \pi \times (0{\cdot}05)^2 = 0{\cdot}0025\ \pi\ \text{cm}^2$. If the extra load is W g-wt, the stress is $980\ W/0{\cdot}0025\pi\ \text{dyn/cm}^2$.

$$\therefore\ 2 \times 10^{12} = \frac{980W/0{\cdot}0025\pi}{20 \times 10^{-5}}, \text{ whence } W = 3{\cdot}2 \times 10^{3} \text{ (g-wt).}$$

(5) Calculate the bulk modulus of glycerine, if a litre of this liquid contracts 0·21 cm³ when subjected to a pressure of 10,000 g/cm².

Solution: Volume strain $= \dfrac{\text{change in volume}}{\text{original volume}} = \dfrac{0{\cdot}21\ \text{cm}^3}{1000\ \text{cm}^3} = 0{\cdot}21 \times 10^{-3}$

Stress $= 10{,}000$ g/cm².

$$\therefore\ \text{Bulk modulus} = \frac{\text{stress}}{\text{strain}} = \frac{10{,}000}{0{\cdot}21 \times 10^{-3}}\ \text{g/cm}^2 = 4{\cdot}76 \times 10^7\text{g/cm}^2$$

(6) An aluminium rod is 2 metres long and 2 cm. in diameter. A load of 70 kg elongates it by 30 parts in a million. If Poisson's ratio for aluminium is 0·33, find the contraction in diameter due to the load.

Solution: Longitudinal strain $= 30 \times 10^{-6}$, Lateral strain = longitudinal strain × Poisson's ratio $= 30 \times 10^{-6} \times 0{\cdot}33 = 10^{-5}$.

∴ Contraction in diameter = diameter × lateral strain $= 2 \times 10^{-5}$ cm.

2-4. Distinction between solids, liquids and gases on the basis of the three elastic moduli. Solids have all the three moduli of elasticity—Young's modulus, bulk modulus and shear modulus. Liquids and gases have bulk modulus only. The difference between liquids and gases lies in the large value of the bulk modulus in the case of the former, and small value in the case of the latter. The reciprocal of bulk modulus is called *compressibility*. Liquids have very little compressibility (an ideal liquid is treated as incompressible). Gases are highly compressible. The bulk modulus of a gas at a fixed temperature comes out to be equal to its pressure.

The fact that liquids and gases have no shear modulus is also expressed by saying that *they cannot resist a shearing force*. Any tangential force, however small, causes them to flow. Hence liquids and gases together are known as *fluids*.

Exercises

1. Explain the terms *elasticity*, *strain* and *stress*. What do you understand by *elastic limit* and *Hooke's law* ? What is meant by *modulus of elasticity* ?

2. Explain the terms *tensile stress and strain*, *compressive stress and strain*, and *shearing stress and strain*. Define *Young's modulus*, *bulk modulus* and *modulus of rigidity*. Distinguish between solids, liquids and gases on the basis of these three moduli.

3. What is Poisson's ratio ? In what unit is it expressed ? What are the cgs units for the three elastic moduli ? What cgs units do you use for stress and strain ?

4. A piece of copper wire is 2 m long and 0·5 mm in diameter. A load of 10 kg increases its length by 2·38 mm. What is the Young's modulus for copper ? [Ans : $4{\cdot}28 \times 10^6$ kg-wt/cm^2]

5. A 10 kg weight suspended from 5 m long wire causes a strain of 0·1%. If the cross section of the wire is 1 mm^2, what are its elongation, stress and Young's modulus ? [Ans : 5 mm , $9{\cdot}8 \times 10^8$ dyn/cm^2 ; $9{\cdot}8 \times 10^{11}$ dyn/cm^2]

6. A piece of wire, 600·5 cm long and 1 mm^2 in cross section, has a load of 20 kg suspended from one end. If the load is removed the length diminishes by 0·5 cm. What is the value of Young's modulus ?

[Ans : $2{\cdot}35 \times 10^{12}$ dyn/cm^2]

7. (*a*) A horizontal plank supported at two ends is depressed in the middle by 2 cm when a load of 5 kg is placed there. How much will a load of 7·5 kg depress it in the middle ? What load will be required for a depression of 3·75 cm ? [Ans : 3 cm ; 8·75 kg]

(*b*) If a torque of 1000 dyn. cm is applied at the lower end of a suspended wire it is twisted by 90°. What torque will be required to produce a torque of one radian ? [Ans : $2000/\pi$ dyn. cm.]

(*c*) A 2 kg load elongates a spring by 2·5 cm. What will be the elongation for a load of 1·5 kg ? What load will be required for an elongation of 1·75 cm. ? [Ans : 1·875 cm , 1·4 kg.]

How can you convert such a spring into a spring balance ? What precaution should you observe to keep the scale linear ?

8. A rod of length 250 cm and radius 2·5 cm has Young's modulus $= 2 \times 10^{12}$ dyn/cm^2 and Poisson's ratio $= 0{\cdot}3$. What strain in the diameter will be caused by a load of 1000 kg ? [Ans : $1{\cdot}47 \times 10^{-4}$]

9. 200 cm^3 of air is at a pressure of 760 mm of mercury. If the pressure is increased by 1 mm of mercury and the temperature remains constant, the volume of air increases by 0·263 cm^3. What is the bulk modulus of air ?

[Ans : 760 mm of mercury]

CHAPTER

3

HYDROSTATICS

3-1. Density. By *density* of a substance is meant its *mass per unit volume.* In the mks system, the unit of density is 1 kg/m^3 ; in the cgs system it is 1 g/cm^3 and in the fps system it is 1 lb/ft^3. Measurement of density of a substance involves finding the mass and the volume of a body made of the substance. These can be done in various ways as we shall see.

When densities of substances are expressed in different units, we cannot immediately say which of them is denser (*i.e.*, contains more mass per unit volume) without converting the values to the same system of units. But if we know the ratio of a density to the density of some standard substance, we can immediately understand from two such ratios which of the two substances is denser without any reference to any system of units. (Remember that a ratio of two quantities of the same kind is independent of units and is a pure number.) Such a ratio is called the *specific gravity* of the substance. Generally, water at its maximum density (about 4°C) is taken as the standard substance with which to compare the densities of solids and liquids. We leave out gases for the present.

Notes : In modern scientific writing the term 'specific gravity' is no longer preferred. The term used in its place is **relative density.** The word 'specific' in modern scientific writing is used to mean 'per unit mass'. Examples are 'specific volume', 'specific latent heat', 'specific heat capacity', etc. However, as the syllabus mentions 'specific gravity', we shall be using it.]

3-2. Specific gravity. *Specific gravity of a substance is defined as the ratio of the mass (or weight) of a volume of the substance to the mass (or weight) of an equal volume of water at its maximum density.* This is the same thing as the ratio of the density ρ_s of the substance to the density ρ_w of water at the temperature at which it is densest (*i.e.*, about 4°C). For, let V be the volume of the substance. Then the specific gravity S of the substance is given by

$$S=\frac{\text{Mass of volume } V \text{ of the substance}}{\text{Mass of volume } V \text{ of water (at } 4°\text{C)}}=\frac{V\rho_s}{V\rho_w}=\frac{\rho_s}{\rho_w} \qquad (3\text{-}2.1)$$

In the mks system $\rho_w = 1000$ kg/m^3.

In the cgs system $\rho_w = 1$ g/cm^3.

In the fps system $\rho_w = 62{\cdot}4$ lb/ft^3.

It appears from these relations that in the

$$\left.\begin{aligned} &\text{mks system, } \rho_s = S \times 1000 \text{ kg/m}^3 ;\\ &\text{cgs system, } \rho_s = S \times 1 \text{ g/cm}^3 ;\\ &\text{fps system, } \rho_s = S \times 62{\cdot}4 \text{ lb/ft}^3 . \end{aligned}\right\} \qquad (3\text{-}2.2)$$

Only in the cgs system, the numerical values of ρ_s and S *are the same.*

From the definitions of density and specific gravity you must have understood the difference between the two quantities. Densities are expressed in mass units per unit volume, while specific gravity is a pure number.

3-3. Archimedes' principle. All of us know the story how Archimedes found that King Hiero was cheated by a goldsmith whom the king had asked to make a crown of pure gold. He had found that a body immersed in water displaces its own volume. Later, he also noticed that the body feels lighter when immersed. By careful reasoning he reached the conclusion that an upward force equal to the weight of the displaced liquid acts on the body immersed. This is now known as the Principle of Archimedes. We may state it as follows :

A body wholly (or partly) immersed in a liquid appears to lose a part of its weight, which is equal to the weight of the displaced liquid.

If the weight of the body in air $= W$ g*, and

its weight in the liquid $= w$ g,

then the apparent loss in weight $= (W - w)$ g.

By Archimedes' principle, $(W - w)$ g $=$ the weight of the liquid displaced by the solid.

In Sec 3-8.3, we shall see that a liquid exerts an upward force on a body partly or fully immersed in it. This upward force, or *upthrust*, is equal to the weight of the displaced liquid. The apparent loss of weight of a body immersed in a liquid is due to this upthrust, also called the *force of buoyancy*. By **buoyancy** is generally meant this apparent loss of weight. The term buoyancy is also used to mean the force of buoyancy.

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* Here W g means W g-wt. There should be no confusion as a weight is clearly implied. Here and later, the terms mass and weight will be indifferently used if there is no confusion as to whether a mass or a weight is implied.

Archimedes' principle applies not only to liquids but also to gases. Liquids and gases together are known as 'fluids'. (A fluid means any thing that flows.) A more general way of stating Archimedes' principle will be to say

"*When a body is immersed in a fluid, it is acted on by an upward force equal to the weight of the displaced fluid.*"

The ascent of balloons is governed by Archimedes' principle.

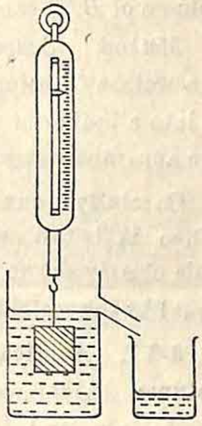

Fig. 3.1

Demonstration. *To verify Archimedes principle.*

(i) *Apparatus* : Solid (a lump of glass, stone or metal), spring balance, overflow can, beaker, measuring cylinder, thread and water (Fig. 3.1).

Method : Suspend the solid by a piece of thread from a spring balance, and record its weight. Fill the overflow can, and after the overflow has stopped, place an empty beaker under its side-tube. (The first overflow should be caught in a vessel and thrown away.) Slowly lower the solid, still suspended from the spring balance, into the overflow can till it is fully immersed. Record the apparent weight of the solid as given by the spring balance. It will be found that the weight of the water collected in the empty beaker is equal to the difference of the weights o the solid in air and in water.

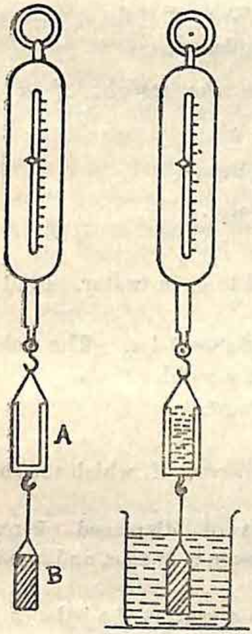

Fig. 3·2

(ii) *Apparatus* : Solid, spring balance, beaker, water, thread, a metal cylinder exactly fitting into another (Fig. 3.2).

The cylinder B is closed. It fits exactly into A and fills A completely. The inner volume of A is thus equal to the outer volume of B. B can be withdrawn from A and attached below it.

Method : Suspend the cylinders from the spring balance and record the weight, B being attached to the bottom of A as in Fig. 3.2. Lower B into a beaker of water so as to be fully surrounded by water. Record the apparent weight as given by the spring balance.

Carefully pour water into A (from a pipette or a wash-bottle). When it is full note that the initial reading of the balance is restored. This clearly shows that the apparent loss in weight on immersion is equal to the weight of the displaced liquid.

3-3.1. Application of Archimedes' principle to determination of volume. Archimedes' principle states that when a body is weighed (i) in air, and (ii) when *fully immersed* in water, the difference between the observed weights is equal to the weight of the water displaced by the body.

If the weight of the body in air $= W$ g,

its apparent weight when fully immersed in water $= w$ g,

then $(W-w)$ g $=$ weight of the water displaced.

Now, the *weight of* 1 cm^3 *of water is equal to the weight of a one gram mass.*

$\therefore$ Volume of water displaced $=(W-w)$ cm^3.

But this is also the volume of the solid.

$\therefore$ Volume of solid $=(W-w)$ cm^3.

Examples. (1) A body weighs 20.52 g in air and 12.48 g in water. Find its volume, density and specific gravity.

Solution : Weight of displaced water $= 20.52 - 12.48 = 8.04$ g. The volume of this water is 8.04 cm^3. This is also the volume of the soild.

Hence density of solid $= 20.52$ g $\div 8.04$ cm$^3 = 2.55$ g/cm^3.

Specific gravity $= 2.55$.

(2) What will the above solid weigh in kerosene, of which the sp. gr. is 0.8 ?

Solution : Volume of solid $= 8.04$ cm^3. Weight of displaced kerosene $= 8.04 \times 0.8 = 6.43$ g. Hence loss of weight in kerosene is 6.43 g, and apparent weight in kerosene is $20.52 - 6.43 = 14.09$ g.

Problem. Two pieces of metal are suspended from the arms of a balance and are found to be in equilibrium when kept immersed in water. The mass of one piece is 82 g and its density is 8 gm/cm^3. The density of other is 5 g/cm^3. What is its mass ? [Ans : 35 g.]

3-3.2. Specific gravity of a substance relative to any liquid and to water.

Let

the weight of a body in air $= W$ g,
its weight in water $= w$ g, and
its weight in a liquid $= w'$ g.

Let the density of the liquid be ρ' g/cm^3. Then ρ' is also the specific gravity of the liquid relative to water.

$$\text{The sp. gr. of the body relative to water} = s = \frac{W}{W-w}.$$

$$\text{The sp.gr. of the body relative to the liquid} = s' = \frac{W}{W-w'}.$$

$(W-w)$ gram is the mass of the displaced water

$(W-w')$ gram is the mass of the displaced liquid. Both masses have the same volume which is the volume of the body. Then $(W-w')/(W-w)=\rho'$, the specific gravity of the liquid relative to water.

$$\therefore\quad s = \frac{W}{W-w} = \frac{W}{W-w'} \times \frac{W-w'}{W-w} = s' \times \rho'$$

or Specific gravity relative to water (s) = sp. gr. relative to a liquid $(s') \times$ sp. gr. of the liquid (ρ').

Examples : (1) A body weighs 100 g in air and 60 g in water. What will be its weight in kerosene of sp. gr. 0.8 ? What is the sp. gr. of the body relative to kerosene ?

[*Solution* : Volume of displaced water = 100 - 60 = 40 cm^3. This is also the volume of displaced kerosene. Weight of this volume of kerosene is $40 \times 0.8 = 32$ g. Hence weight of the body in kerosene is 100 - 32 = 68 g. Sp. gr. relative to kerosene is 100/32 = 3.125]

(2) A body weighs 300 g in air and 270 g in a liquid of sp. gr. 0.9. How much will the body weigh in water ? What are its volume and sp. gr. ?

[*Solution* : Volume of the body in cm^3 = Volume of the displaced liquid = (300 − 270)cm^3/0.9 g/cm^3 = 33.3 cm^3. In water it will weigh 33.3 g less than in air. Its sp. gr. = its mass in air/mass of displaced water = 300/33.3 = 9]

Exercises

(*On Sections 3-1 to 3-3.2*)

1. Distinguish between *density* and *specific gravity*. In what sense can we say specific gravity is relative density ?

State and explain Archimedes' principle. Describe an experiment in support of the principle.

What is meant by *upthrust* on an immersed body ?

2. How can you find the volume of a body with the help of Archimedes' principle ?

3. A body weighs 50 g in air and 40 g in water. Find its volume, density and specific gravity. [ANS : 10 cm^3 ; 5 g/cm^3 ; 5]

How much will the body weigh in a liquid of specific gravity 1·25 ? [ANS. : 37·5g]

4. A piece of brick weighs 2·5 kg in air. Sp. gr. of brick is 1·5. If half the piece is immersed in water, how much will it weigh ?

[**Hint** : Find the volume and weight of the displaced water. This weight is equal to the upthrust on the brick. [ANS. : 1·67 kg].

5. In the last problem, if half the brick is immersed in a liquid of density 3 g/cm^3, what will be its apparent weight ? [ANS. : 0]

6. A string can support a maximum load of 2 kg. A piece of brick weighing 2·5 kg in air is suspended by the string under water. As the string is pulled up, more and more of the brick comes out of water until the string snaps. What fraction of the brick will be outside the water when this happens ? Specific gravity of the brick = 1·5.

[**Hint** : If the string snaps when a fraction x is out of the water, then the weight of this portion is 2·5 x kg. Within the water the volume of the brick is $(1-x)$ of its total volume. Apparent weight of this portion is 2·5 $(1-x)-2·5$ $(1-x)/1·5$ kg. The sum of these two quantities will be 2 kg. [ANS. : 0·7]

7. A glass ball weighs 188 g in air, 116 g in water and 125 g in turpentine oil. Find the density of glass and the sp.gr. of turpentine oil. [ANS. 2·61 g/cm^3 ; 0·875].

8. A piece of sodium weighs 0·34 g in an oil of sp. gr. 0·8, and 0·54 g in another oil of sp. gr. 0·7. Find the density of sodium. [ANS. 0·97 g/cm^3].

9. A body weighs 7·55 g in air, 5·17 g in water and 6·35 g in another liquid. Find the densities of the body and the liquid. [ANS. 3·17 g/cm^3 and 0·504 g/cm^3.]

10. A piece of an alloy of gold and silver weighs 20·0 g in air and 18·7 in water. If the sp. gr. of gold and silver are respectively 19·3 and 10·5, how much gold does the piece of alloy contain ? [ANS. : 13·9 g]

11. You are asked to find the density of a metal coin by weighing it in air and water. Explain how you will proceed.

12. A hollow glass stopper weighs 23.4 g. Sp. gr. of glass is 2·50. The apparent weight of the stopper when fully immersed in water is 3·90 g. What is the volume of the hollow of the stopper ?] ANS.: 10·1 cm^3]

13. A gold ring set with stone weighs 4·000 g in air and 3·720 g in water. Density of gold is 19·3 g/cm^3 and that of the stone is 3·50 g/cm^3. How much does the stone in the ring weigh ? [ANS. : 0·311 g]

14. A piece of lead and a piece of sulphur weigh the same in water. If their sp. gravities are respectively 11·4 and 2, what is the ratio of their volumes ? [ANS. : 1 : 10·4]

15. A hollow sphere made of copper weighs 523 g in air and 447 g in water. If the density of copper is 8·9 g/cm^3, what is the volume of the hollow ? [ANS. : 17·24 cm^3]

16. A piece of steel of sp.gr. 7·80 weighs 0·50 kg in air. If it is immersed by a thread in methylated spirit of sp. gr. 0·83, what will be the tension in the string ? [ANS. : 0·448 kg.]

3·4. Floating bodies. According to Archimedes' principle a body, even if *partly* immersed in water, will experience an upward buoyant force equal to the weight of the liquid displaced. The downward force on the body is its weight. When the two forces are equal, the body will float. We may say that *a body floats when it displaces its own weight of the liquid.* When a floating body is placed in a liquid it will sink until it displaces its own weight of the liquid. If pushed further into the liquid, it will rise when released. When a body is unable to displace its own weight of liquid, it sinks.

A. Let V be the volume of a floating body of density (or specific gravity) ρ. *Let n be the fraction of its volume immersed in water when the body floats.* Then its weight is $V\rho$ and the weight of the displaced water is nV, *all in consistent units.*

$$\therefore \quad V\rho = nV \qquad \text{or,} \quad \rho = n. \tag{3·4.1}$$

If the liquid in which it floats has a density ρ', we shall have

$$V\rho = nV\rho' \qquad \text{or,} \quad \rho = n\rho' \tag{3·4.2}$$

The result may be expressed in words as follows :

If a solid floats in a liquid with a fraction n of its volume immersed, then the density (or the specific gravity) of the solid is n times that of the liquid.

Examples. (1) Ice has a specific gravity of 0.917. What fraction of its volume will emerge out of water ?

Solution : If V is the volume and a fraction n is submerged, then $V \times 0.917 = nV$ or $n = 0.917$. The emergent fraction is $1 - 0.917 = 0.083$ (nearly one-twelfth).

(2) If the emergent volume of an iceberg (sp. gr. = 0.917) is 1000 m³, how much of it is below sea-water (sp. gr. = 1.028) ?

Solution : If V m³ is the volume below the surface, then

$$V \times 1.028 = (V + 1000) \times 0.917, \text{ whence } V = 8261 \text{ m}^3.$$

Problems. (1) A piece of wax of volume 22 cm³ floats in water with 2 cm³ above the surface. Find the weight and specific gravity of the wax.

[Ans. : 20 g, 10/11]

(2) A block of ice weighing 1000 kg is thrown into the sea. Determine the volume of ice submerged. The density of ice is 0.917 g/cm³ and density of sea water is 1.03 g/cm³.

[Ans. : $9·71 \times 10^5$ cm³]

B. Let l_1 and l_2 be the distances to which a cylinder of cross-section A sinks in two liquids while floating in them. Let ρ_1 and ρ_2 be the respective densities of the liquids. By Archimedes' principle the weight w of the cylinder

$$= Al_1\rho_1 = Al_2\rho_2 \qquad \text{or,} \quad \frac{\rho_2}{\rho_1} = \frac{l_1}{l_2}. \tag{3·4.3}$$

Now, if ρ_1 and l_1 refer to values for water, ρ_2/ρ_1 is the specific gravity of the other liquid. Hence we may say that

The specific gravity of a liquid is equal to the ratio of the depth l_1, which the cylinder sinks in water, to the depth l_2, which it sinks in the liquid.

Examples. (1) A wooden cylinder of uniform cross-section is 10 cm long. It floats in water with 2 cm above the surface. In a salt solution it floats with 3 cm above the liquid surface. Find the density of the salt solution.

Solution : Let A cm² be the area of cross-section of the cylinder, and ρ, the density of the solution in g/cm³. Then the weight of the cylinder = weight of displaced water = weight of displaced liquid.

Now, weight of displaced water $= 8A$ g-wt.

Weight of displaced liquid $= 7A\rho$ g-wt.

$\therefore \quad 7A\rho = 8A$ or $\rho = 8/7$.

(2) In the above problem find how much of the cylinder will be above the liquid surface when floating in a liquid of specific gravity 1.25.

Solution : If x cm is the length sought, then $(10-x)$ cm will be below the liquid surface. Hence $8A = (10-x)A \times 1.25$ or $x = 3.6$ cm.

Problem. Sea water is 1.03 times as dense as fresh water. How many cubic metres of sea water will be displaced by a ship of total weight 5,000 tonne? (1 tonne = 1000 kg). [ANS : 4874 m³]

The average density of the human body is very close to that of water. When we breathe out air our body becomes denser than water and will sink. But when air is inhaled the body is lighter than water.

The water of the dead sea contains much salt in solution and is so dense that one does not sink in it.

Problem. (1) A cubical block of wood of sp. gr. 0.7 floats in water, just completely immersed, when a body of unknown weight is placed in it. Find the weight of this body, if the volume of the block of wood is 100 cm³.

[ANS. : 30 g.]

(2) A piece of iron is placed on a piece of cork and the two together float on water in a tumbler. If now the piece of iron is taken off the cork and dropped into the the water, will the level of water in the tumbler rise or fall ?

[**Hint** : While on cork, the iron displaces its own weight of water. While in water, it displaces its own volume of water. The former is larger than the latter. Smaller displacement causes smaller rise in water level.]

3-4.1. Some special examples of flotation. (*i*) When ice floats in a tumbler, full to the brim with water, will the melting of ice cause water to overflow ? When the ice floats it displaces its own weight of water. Let V be the volume of water displaced. On melting the ice will form exactly this volume V of water and just fill up the space which the ice displaced. So when ice melts there will be no change in the level of water in the tumbler.

(*ii*) A ship may be made of steel and yet float. It is given such a shape that, as it sinks into water, it displaces a large volume and soon the weight of the displaced water equals its own weight. The factor which makes it possible for a ship to float, is the shape. The density of the material of which it is built is greater than that of sea-water. Buoys marking the channel in a river are hollow iron spheres. They float because each one can displace a weight of water greater than its own weight.

When a ship sinks does it reach the bottom of the sea ? The density of water at great depths differs but little from the value at the surface. Even at a depth of 5 miles the density does not exceed the value at the surface by more than 5%. An object of density very slightly greater than that of sea-water, but not exceeding the latter by more than the above value (*i.e.*, lying within the approximate range of densities from 1.03 to 1.08 gm/cm^3). may be supported at a suitable level in sea-water. But a ship has a higher density and sinks to the bottom.

(*iii*) **Submarines :** The submarine is so built that it can float like an ordinary ship. It has two shells, one inside the other. The inner shell is much stronger than the outer. The space between the two shells is divided into chambers. When they are full of air, the submarine floats. When the chambers are filled with water, the submarine sinks. When it is required to rise to the surface, water is expelled from the chambers by means of pumps driven by compressed air. This lightens the ship, and it floats up.

Exercises

(*On floating bodies*)

1. Find the condition that a solid will float or sink in a liquid. Why does a balloon filled with hydrogen or helium ascend ? Though steel is heavier than water why does a ship made of steel float ?

2. Sp. gr. of ice is 0.917. What fraction of the volume of a block of ice will stick out of water ? [ANS. 0.083]

3. (*a*) A straight stick of uniform section floats in water with 0.9 of its volume immersed. In another liquid, it floats with 0.8 of its volume immersed. What is the density of the liquid ? [ANS. 1.125 g/cm^3]

(*b*) A straight stick of 10 cm² section is immersed 1.2 m in water. What is the upthrust on it? If the stick weighs 1 kg what force will be required to keep the stick in the above position?

[ANS. 1200 g-wt. 200 g-wt downward]

4. A body floats with 1/6 of its volume sticking out of water. If it is floated in a liquid of density 1.2 g/cm³ what fraction of its volume will lie above the liquid? [ANS. 11/36]

5. Sp. gr. of ice is 0·916 and that of sea-water is 1.025. If 1000 cubic metres of an iceberg remain above the sea-water, what volume of the iceberg is under water? [ANS. 8404 m³]

6. How much of a cube of side 5 cm and sp. gr. 0.4 will sink in water? What is the upthrust on it? If the cube is floated in kerosene of sp. gr. 0·8, what will be the upthrust? [ANS. 2 cm ; 50 g-wt.]

7. When an elephant is landed on a boat, the boat sinks 4 cm in water. If the average cross-section of the boat at the water-line is 125 m², what is the weight of the elephant? [ANS. 5000 kg.]

8. The hollow of a metal globe has a volume of 1720 cm³. The total volume of the globe is 2000 cm³. To immerse the globe fully into water requires a force of 2 g-wt. What is the density of the metal? [ANS. 7.13 g/cm³]

9. The inner and outer radii of a hollow sphere are 9 and 10 cm. If floated in a liquid of sp. gr. 0.8 just half of the sphere is submerged. What is the density of the material of the sphere? What is the density of the liquid in which the sphere will just sink? [ANS. 1.4 g/cm³.]

10. (*a*) Explain in which of these cases the greater part of your body will remain under water—(*i*) breath fully inhaled, (*ii*) breath fully exhaled.

(*b*) When a ship sinks in an ocean does it go down to the bottom or remain suspended somewhere? Give the reason for your answer.

(*c*) The top of the tower of a submarine was just submerged when it was in a river. Explain how the position will change when the submarine reaches the sea.

3-5. Hydrostatics. Liquids and gases cannot resist the action of any tangential stress however small (For definition of tangential stress, see Sec. 2-1). They flow under the action of such a force, and hence are called *fluids*. This is their fundamental difference with solids. *Hydrostatics* is the study of fluids at rest under gravity.

There are **two fundamental laws** governing the properties of fluids at rest. Both follow from the property that fluids cannot resist tangential stresses. The laws are:

(*i*) *The force exerted by a fluid on any surface with which it is in contact, is perpendicular to that surface.* (Archimedes' principle follows from it.)

(ii) Any pressure communicated to a confined volume of fluid is transmitted undiminished through the whole fluid in every direction. This statement is known as **Pascal's law.**

The concept of *pressure at a point* plays a very important part in hydrostatics. We shall first of all try to make its meaning clear.

3·6. Pressure—What it means. Hold a pencil vertically with its flat end resting on your palm and load the pencil by putting a book of moderate weight on its top. The weight of the book exerts a thrust on your palm. Now invert the pencil so that its sharp end rests on your palm while the flat end supports the book. The weight of the book exerts the same thrust as before, but the sensation to which it gives rise is more acute and may even be painful if the pencil is very sharp.

What is the reason for the difference in sensation ? The force with which the palm was pressed, was the same in both cases. But the areas over which this force acted were different. With the flat end of the pencil on your palm, the weight of the book was spread over a wider area than when the sharp end pressed you. What mattered was not the force alone, but also the area over which it acted.

It is not difficult to get other examples of the kind. If you wrap round a heavy parcel with a thin string and hold it for some time by the string, you will soon feel pain. But if the string is attached to a wide handle, and the parcel is held by this handle, it could be carried with less discomfort. In the former case the force exerted per unit area on your fingers by the load was larger than in the latter case.

When you walk bare-footed on plane ground the weight of your body acts over that area of your foot which is in contact with the ground. But if you walk on a bed of pebbles, particularly with sharp edges you feel pain because the weight of your body now acts over much smaller areas.

Pressure and thrust. *Pressure is defined as the force per unit area.* If a total force F acts perpendicularly over an area A, the pressure P is given by

$$P=\frac{\text{Force}}{\text{Area}}=\frac{F}{A} \qquad (3\cdot6.1)$$

The force F is often called a **thrust.** It acts perpendicular to the area A and gives rise to the pressure.

Units of pressure. It is clear from the above that 'pressure' is not the same as 'force'. Force is expressed in the usual units, such as the newton, dyne or poundal, or kg-wt, g-wt, lb-wt, etc. Pressure involves the unit of force in the numerator and a unit of area in the denominator. Pressure can therefore be expressed in such absolute units as newton/metre^2, dyn/cm^2, etc., or in gravitational or mixed units such as kg-wt/m^2, lb-wt/m^2, g-wt/cm^2, etc.

Pressure at a point. We often speak of 'pressure at a point', particularly when discussing liquids. Since a point has no area, the question naturally arises as to the meaning of the term. If F is the force exerted over an area A surrounding the point in question, *A being so small that the pressure over it may be taken as uniform*, then the ratio $P = F/A$ is called the pressure at the point.

Problems. (1) Express a pressure of 1 tonne per square metre in dyn/cm^2. (1 tonne = 1000 kg.)

Solution: 1 tonne here means 1 tonne weight = 1000 kg-wt $= 1000 \times 1000$ g-wt $= 10^6 \times 980$ dynes. $1 \text{ m}^2 = (100 \text{ cm})^2 = 10^4 \text{ cm}^2$.

So, the required pressure is $\dfrac{10^6 \times 980 \text{ dyn}}{10^4 \text{ cm}^2} = 9{\cdot}8 \times 10^4 \dfrac{\text{dyn}}{\text{cm}^2}$.

(2) Show that a pressure of 1 ton/ft^2 equals $10{\cdot}72 \times 10^4$ N/m^2.

(3) How much is 1 dyn/cm^2 equal to in N/m^2 ?

3-7. A liquid at rest exerts a normal thrust on any surface with which it is in contact. We can prove it theoretically.

In Fig. 3.3 let ACB be a portion of the surface in contact with a liquid *at rest under gravity*. If possible, let the force F which the liquid exerts on a small portion of the surface near C, be inclined to the surface in the direction DC. The reactional force which the surface exerts on the liquid is F acting along CD. Resolve the force at right angles to the surface into a normal thrust N on the liquid and a tangential force T parallel to the surface. The component T constitutes a shearing strees and tends to make the fluid slide over the surface. Since a liquid cannot resist a shear, it should move in the direction of the shear. But the liquid is at rest. Hence there can be no tangential component, *i.e.*, the force is entirely at right angles to the surface.

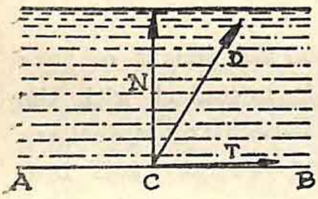

Fig. 3.3

If we take an imaginary surface drawn anywhere in a liquid, the force exerted by one portion on the other must on the same ground be perpendicular to the surface

3-8. Pressure at a depth inside a liquid.

Having seen that a liquid exerts a normal thrust on any surface with which it is in contact and that *the surface may even be an imaginary one in the liquid*, we proceed to find the magnitude of the pressure at a point on such a surface.

To determine the pressure at a point A, distant h below the free surface of a liquid at rest under gravity, imagine a small horizontal area a surrounding the point A. Consider the liquid contained in the vertical cylinder which has a as its base (Fig. 3.4). The vertical forces on the cylinder are (i) the upthrust on the base and (ii) the weight of the liquid in the cylinder. Since the cylinder is in equilibrium, the two forces are equal.

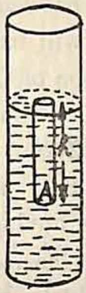

Fig. 3.4

The weight of the liquid in the cylinder = mass of the liquid cylinder × acceleration g due to gravity

$$= \text{volume} \times \text{density} \times g$$

$= ha\rho g$, where ρ = density of the liquid (assumed constant).

This is the total thrust on the base.

∴ The pressure P at any point of the base

$$= \frac{\text{Normal thrust on the base}}{\text{area of the base}}$$

$$\text{or } P = \frac{ha\rho g}{a} = h\rho g \qquad (3\cdot8.1)$$

***i.e.*, pressure = depth × density × acceleration due to gravity.**

If the quantities h, ρ, g are expressed in absolute cgs units, P comes out in dyn/cm^2. If in fps units, P will be in poundals/ft^2. In mks units, P will be in newtons/m^2.

If we divide P by g in the corresponding unit, we get pressure (P') = depth × density = $h\rho$. So written, the pressure is in *gravitational units of force* per unit area. This gives rise to an **indirect way of stating pressure** by mentioning only the liquid and the depth. The expression 'a pressure of 76 cm of mercury' means the hydrostatic pressure due to a column of mercury 76 cm high, *i.e.*, the pressure at a depth of 76 cm in mercury. Mercury has a specific gravity of 13.6. Hence its density is 13.6 g/cm^3. The pressure is therefore 76 cm × 13.6 g/cm^3 × 980 $cm/s^2 = 1.013 \times 10^6$ dyn/cm^2.

A pressure of 100 m of water will be $= 10^4$ $g\text{-}wt/cm^2$. In statements of pressure of this kind we often write kg or g for kg-wt or g-wt.

3-8.1. Fluid pressure at a point acts equally in all directions. Note carefully that the fluid pressure at a point (within it acts equally in all directions. Consider a very small area A surrounding the point. The pressure at the point is $p = FA$ where F is the thrust on the surface. In whatever direction the surface is turned about the point, the ratio F/A will remain the same. But this pressure will be exerted by one portion of the liquid on the other portion across A and perpendicular to A. Thus fluid pressure at a point acts equally in all direction, but on different planes of the liquid.

3-8.2. Pressure of a fluid at rest is the same at all points in the same horizontal plane. This requires first of all a definition of the word 'horizontal'. The plane in which the surface of a liquid in a relatively wide vessel lies when the liquid is in equilibrium under gravity, is called the **horizontal plane** at the place. The vessel should be at least a few decimetres in width. Near the boundary, the liquid surface is curved. The plane of the central part is the horizontal plane. (The 'vertical' is perpendicular to this plane.)

Obviously, the pressure at all points at the same depth below the free surface, will be the same, since pressure depends on depth alone. But even if the liquid surface at the top is curved (as in a liquid drop), the pressure at all points on the same horizontal plane within the liquid will be the same.

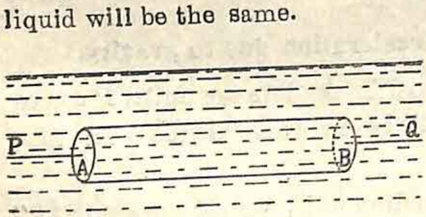

Fig. 3.5

To prove it, consider a small uniform horizontal cylinder (Fig. 3.5) with AB as the axis, and the ends at A and B perpendicular to the axis. Let P and Q be the pressures at A and B respectively. They act perpendicular to the end sections each of which has an area α. The forces on the curved walls due to fluid pressure are perpendicular to the walls, *i.e.*, to the line AB. Hence these forces have no component along AB.

Therefore the only forces parallel to AB are $P\alpha$ and $Q\alpha$ acting in opposite directions. Since the cylinder is in equilibrium

$$P\alpha - Q\alpha = 0 \quad \text{or } P = Q. \tag{3-8.2}$$

Thus we find that *in a fluid at rest in equilibrium the pressure is the same at all points in any horizontal plane.* This result is put in other ways too. We say

The pressure at two points in the same horizontal plane connected together by a liquid at rest is the same.

Or, *In open vessels connected together by a liquid at rest the liquid stands at the same height in all.* Thus in a kettle, the level of the liquid is the same in the nozzle as in the pot.

3-8.3. Buoyancy : Upthrust on a body immersed in a liquid. When a body is immersed in a liquid it experiences an upward force equal to the weight of the displaced liquid. It therefore suffers an apparent loss of weight. This is due to the property of a liquid that it exerts a normal pressure proportional to the depth on any surface in contact. The upthrust is known also as the *force of buoyancy*. This force acts at the centre of gravity of the displaced liquid. This point is known as the *centre of buoyancy*.

To see how this upthrust arises, consider for simplicity a cube, of sides l, immersed in a liquid of density ρ, with one face horizontal (Fig. 3.6). Let the upper face of the cube be at a depth h below the free surface of the liquid. The thrusts on an opposite pair of vertical faces of the cube balance each other. The downward thrust on the upper face is $l^2 h\rho$. The lower face experiences an upward force equal to $l^2(h+l)\rho$. Hence the net force acting on the cube is $l^3\rho$ directed upward. But $l^3\rho$ is the weight of the displaced liquid, *i.e., the upthrust on the immersed body is equal to the weight of the displaced liquid.* The same applies if the solid is partially immersed.

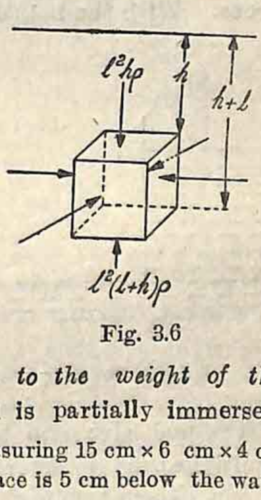

Fig. 3.6

Problems. (1) A rectangular block of metal measuring 15 cm × 6 cm × 4 cm is suspended vertically in water so that its top surface is 5 cm below the water surface. Calculate the downthrust on the top, the upthrust on the base, and the weight of water displaced. [ANS. 120 g-wt ; 480 g-wt ; 360 g-wt.]

(2) A brick measures 10 in × 5 in × 3 in. What will be the upthrust when it is suspended (*a*) in fresh water, (*b*) in sea water of sp. gr. 1.03 ? (Density of fresh water = 62·4 lb/ft^3). [ANS. 5·42 lb , 5.58 lb.]

(3) A thread which will break when the pull on it exceeds 2 kg-wt, is attached to a piece of iron weighing 2.1 kg which is under water. What fraction of the volume of the piece of iron must remain under water in order that the thread may just be able to support it ? Sp. gr. of iron is 7.2. [ANS. 0.343]

3·9. Pascal's law of transmission of pressure in liquids. From the fact that pressure within a free liquid depends only upon the depth and density of the liquid and that it is exerted in all directions, Pascal, a French mathematician (1623-1662), drew a very important conclusion. It is known as Pascal's law and may be stated as follows :

Pressure applied anywhere to a body of confined liquid is transmitted by the liquid and acts undiminished at every point of the liquid and on the walls of the container. It can be deduced theoretically by considering the equilibrium of a *fluid* confined in a closed space. For if the pressure at any point be increased, the pressure at all other points in the fluid must increase by the same amount. Otherwise the fluid will flow from places of high to places of low pressure and lose its equilibrium. The law of transmission of pressure applies equally to liquids and gases.

Principle of application of Pascal's law to the hydraulic press. With the help of Pascal's law *a small force can be transformed into a large one.* Consider the arrangement illustrated in Fig. 3.7. It is a *confined* body of liquid connecting two cylinders fitted with pistons of areas a and A respectively. If a force f is applied to the small piston, the increase of pressure on the liquid will be f/a. This pressure, transmitted undiminished by the liquid, will act on the larger piston and give rise to a force $F=A\times fa$. The force f is thus multiplied by a factor A/a. If $a=1$ cm^2 and $A=1000$ cm^2 a force of 1 kg applied to the small piston will cause a force of 1000 kg to be applied to the large piston.

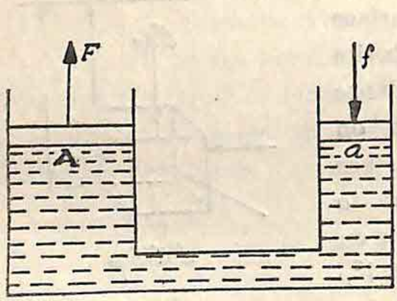

Fig. 3.7

Magnification of the force does not violate the principle of conservation of energy. The liquid may be treated as incompressible. If f is moved through a distance s_1, the work done on it is fs_1. The volume which the piston a sweeps out is as_1. If F moves through s_2, it does Fs_2 amount of work, and sweeps out a volume As_2. Since $as_1=As_2$, and $f/a=F/A$, we get, by multiplication, $(as_1)(f/a)=(As_2)(F/A)$ or $fs_1=Fs_2$. Hence the work done on the liquid is equal to that done by the liquid.

3-9.1. Hydraulic press. The above principle is employed in the hydraulic press. It is a machine used for many purposes, such as compressing bales of cotton, jute etc., punching holes through metal plates, pressing metal sheets into shape, testing strength of iron beams, extracting oil from seeds etc.

Fig. 3.8 represents diagrammatically a hydraulic press in section. As the small piston p is raised, oil from the cistern C enters the piston chamber through the valve v. As soon as the downstroke begins the valve v closes and the valve v' opens. The pressure applied on p is transmitted through the tube K to the large reservoir. There it acts on the large cylinder P. If the areas of cross section of p and P are a and A respectively, and the force on p is f, then the force F acting on P is $f\times A/a$.

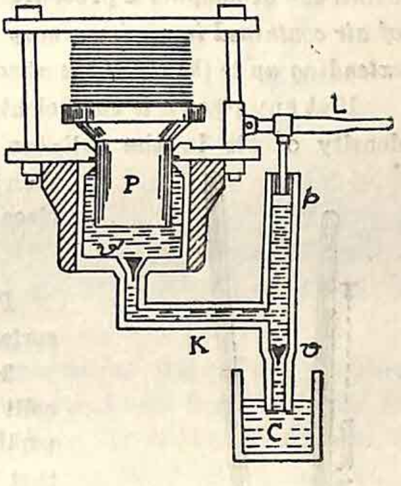

Fig. 3.8

[Hydraulic press. P = pressure piston or ram ; p = plunger ; v, v' = valves ; C = oil tank. A tube (not shown) with a drain cock (or release valve) connects the large reservoir with the oil tank. When a compression is over, the drain cock is opened and drains the oil from the reservoir.]

The lever L also helps in multiplying the force. For if f' is the force applied to the lever, and m is the ratio of its longer arm to the shorter arm, then $f=mf'$. So, by exerting a force f' we get a force $F=mf'\times A/a$. The ratio $F/f'=mA/a$ is called the *mechanical advantage* of the machine.

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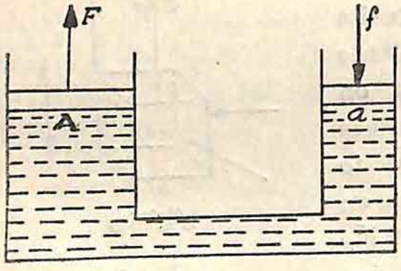

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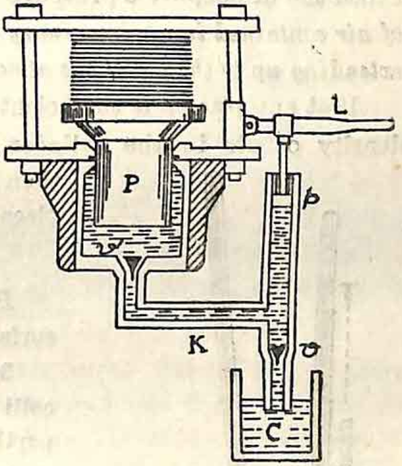

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Hydraulic brakes used in automobiles, the dentist's chair, the hydraulic jack which raises heavy loads or the garage-lift which lifts automobiles provide further examples of application of Pascal's law.

Problem : The cylinders of a hydraulic press have radii 1 cm and 10 cm respectively. The piston is attached to a handle 1 m long at a point 10 cm from the pivot, which is at one end. What force must be applied to the end of the handle for the press to exert a force of 1000 kg ? [ANS. 1 kg-wt.]

3-10. Atmospheric pressure. The gaseous envelope surrounding the earth is called the *atmosphere.* It is the only known earthly example of a gas at rest in equilibrium under gravity. (In considering the atmosphere, we shall here ignore the causes which make air masses move.) Like a liquid at rest under gravity, the atmosphere exerts a hydrostatic pressure on all bodies immersed in it. This pressure is called the **atmospheric pressure.** *It is equal to the weight of a column of air contained in an imaginary vertical cylinder of unit cross-section extending up to the top of the atmosphere.*

If at any place h is the height of such a cylinder, ρ the average density of air in the cylinder and g the acceleration due to gravity, then the atmospheric pressure P at the place is given by

$$P = h\rho g \qquad (3\text{-}10.1)$$

P diminishes with height above the surface of the earth.

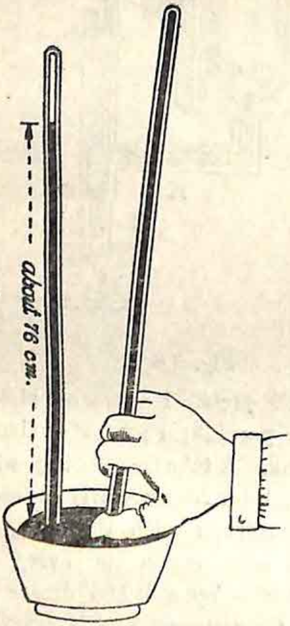

Fig. 3.9

3-10.1. Torricelli's experiment. Torricelli (1662), an Italian scientist and a pupil of Galileo, was the first to conclude that it was atmospheric pressure which supported a column of mercury in an evacuated tube. He completely filled a tube about a metre long with mercury and inverted it in a vessel of mercury (Fig. 3.9). The mercury in the tube came down to a height of about 76 cm above the mercury in the vessel. It fell no further because atmospheric pressure, pressing on the free surface of mercury in the wider vessel, was able to support the weight of the column of mercury. The condition for the balance is

Pressure exerted by the atmosphere = Hydrostatic pressure exerted by the column of liquid.

In order to test Torricelli's theory that the mercury in the tube was supported by atmospheric pressure, Pascal had the experiment performed at a height of about 1700 m above sea level. If atmospheric pressure at a place is due to the weight of the air above the place, the pressure should decrease as we go higher. The height of mercury supported in the tube should therefore be less the higher we go. Pascal found this to be true.

The space above the mercury column (in Torricelli's experiment) is a vacuum since air has been excluded from the tube.* It is known as *Torricellian vacuum*. If the tube is gradually inclined, more mercury goes into it, but mercury in the tube remains at the same vertical height above the mercury outside (Fig. 3.10). If the tube is tilted enough to make the mercury strike the end of the tube, it does so with a sharp click, showing that there is no air inside.

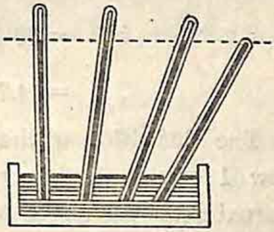

Fig. 3.10

The height to which the mercury rises is independent of the diameter of the tube provided it is not very narrow. In very narrow tubes another effect, due to what is called *surface tension* (Sec. 4-1), slightly reduces the height.

3·11. Measure of atmospheric pressure.

Atmospheric pressure is measured by the height of the mercury column it can support. It varies with the height of a place above the sea-level. Even at a given place it varies slightly according to the weather condition. For convenience of reference a standard value for atmospheric pressure has been defined as follows :

A pressure of one **standard atmosphere** *is the hydrostatic pressure exerted by a column of mercury 76 cm high at 0°C, 45° latitude and mean sea-level.** (Also see the portion marked 'An indirect way of stating pressure' under Sec. 3-8).

---

*There will, however, be a little mercury vapour in the space.

* Mean sea-level is defined by an Act of the British Parliament as the halfway level between the average high and low tides at Newlyn in Cornwall.

A pressure of one atmosphere may be expressed in other units as follows :

1 atmospheric pressure = hydrostatic pressure exerted by 76 cm of mercury

$$= 76 \times 13.6 = 1034 \text{ g-wt per cm}^2 \text{ (Sec. 3·8)}$$

$$= 1034 \times 980 = 1.013 \times 10^6 \text{ dyn/cm}^2 \text{ (absolute cgs units)}$$

If we take atmospheric pressure to be equal to 30 inches of mercury, which is the value we accept while using the British system of units, we shall have

Pressure of 1 atmosphere = hydrostatic pressure due to 30 inches of mercury

$$= 30 \text{ in} \times \frac{13.6 \times 62.4 \text{ lb-wt.}}{12 \times 12 \times 12 \text{ in}^3}$$

$$= 14.7 \text{ lb-wt per sq. inch.}$$

The definition of the standard atmosphere has since been changed. Now 1 standard atmosphere = 1013250 dyn/cm^2. This value closely approximates the above definition, but is more precise.

Problem. Express in atmospheres the pressure at a depth of 300 feet of water, given that the density of water is 62.4 lb per cubic foot, and that a pressure of one atmosphere equals 14.7 lb per square inch. [ANS. 8.8]

The millibar. Meteorologists usually express atmospheric pressure in *millibars*. As one millibar is equal to a pressure of 1000 dynes per sq. cm, a pressure of one atmosphere is equal to 1013 millibars (*i.e.* 1.013 bars).

The mmHg. When pressure is expressed in millimetres of mercury we write mmHg for the unit of pressure. One atmosphere is a pressure of 760 mmHg.

3·11.1. Height of the water barometer. The hydrostatic pressure due to 30 inches of mercury is equal to the pressure of 30×13.6 inches. *i.e.*, 34 feet of water. Atmospheric pressure can therefore support a column of water 34 ft high. This is known as the height of the *water barometer*. In metres, the height of a water barometer is $0.76 \times 13.6 = 10.34$. But if you really construct a water barometer whose top is closed, you cannot get the water to rise to 10.34 m. In the closed space above the water there will be water vapour which will press down on the water with a pressure dependent on temperature (about 3.2 cm of mercury around 30°C).

3-11.2. Height of the homogeneous atmosphere. The height of a column of air at S. T. P. which would exert the same pressure as the standard atmosphere is known as the height of the homogeneous atmosphere. If h cm is this height, the weight of a column of air at S. T. P. and of height h and cross section unity should be equal to the weight of a 76 cm tall column of mercury of unit cross-section. Since density of air at S. T. P. $= 0.001293$ g/cm^3 we shall have

$h \times 0.001293 = 76 \times 13.6$ whence $h = 8$ kilometres (5 miles) nearly. The atmosphere would extend to this height if the density of air remained as at S. T. P. throughout this height. But we know air thins out as we go up.

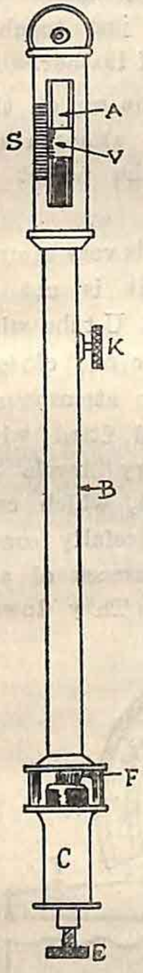

Fig. 3.11

3-12. The barometer. A barometer is a device for measuring atmospheric pressure. We have already seen that atmospheric pressure can support a column of liquid. So the measurement of atmospheric pressure consists in measuring the height of a liquid column supported by it. The liquid chosen is mercury. The chief advantages of so doing are

(i) The high density of mercury makes the height of the column reasonable ;

(ii) It can be obtained with high purity and has a low vapour pressure ;

(iii) Its change of density with temperature, and other relevant properties can be accurately determined.

3-12.1. Fortin's barometer. The most reliable barometer is the one devised by Fortin. It acts on the principle of Torricelli's experiment. In Fig. 3.11, C is a mercury cistern in communication with the atmosphere, over which a glass tube A is inverted. The two are enclosed in a metal tube B on which a scale S is etched. There is also a vernier V which can slide in a slot made in the metal tube.

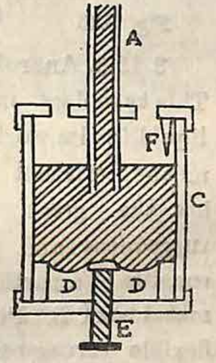

Fig. 3.12

The tube is so slotted as to make the top of the mercury column visible. The zero of the scale starts from the tip of an ivory pointer *F* which projects downwards from the ceiling of the cistern. The lower part of the instrument is shown magnified in Fig. 3.12. The surface of the mercury in the cistern can be adjusted by the screw *E* so that the tip of the ivory pointer *F* just touches it. The bottom of the cistern is made of leather.

The scale reading corresponding to the top of the mercury column gives the value of atmospheric pressure at the moment in terms of the height of the column.

3-12.2. The siphon barometer. It is very simple in construction and is portable, but it is not so reliable as Fortin's. It is (Fig. 3.13) a U-tube with one arm about 90 cm long with the upper end closed. The other arm is short and open to the atmosphere. The tube is set up on a wooden board fitted with a scale. The difference of the mercury levels in the two arms gives the barometric height, which can be read from the scale. Unless very carefully constructed a siphon barometer may have traces of air and vapour in its Torricellian space. They lower the reading.

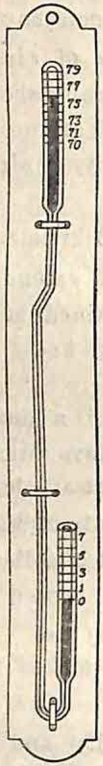

Fig. 3.13

3-12.3. Aneroid Barometer. This type does not require any liquid for its working ('aneroid' means without liquid) and has the great advantage of ruggedness and portability. It consists of a partially evacuated metal box (*B* : Fig. 3.14) with a flexible corrugated top. A box of this type will collapse under pressure of air. This is prevented by a stiff steel spring *S* which pulls the top upwards.

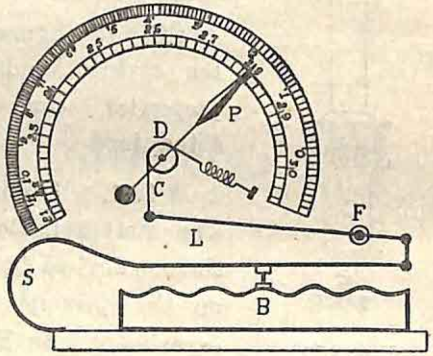

Fig. 3.14

The free end of the spring *S* is connected toa lever *L* which has its fulcrum at F. The other end of L is connected to a small chain C which is wrapped round a spindle D carrying a pointer P which moves over a dial. The movement of the corrugated top due to changes in the atmospheric pressure is magnified by the lever L and transmitted to the pointer P. The dial of the aneroid is calibrated by comparison with a standard mercury barometer.

Altimeters. Since the atmospheric pressure diminishes as we go up to higher altitudes, the barometer may conveniently be used to determine elevation. Altimeters are simply aneroid barometers with an altitude scale attached. Altimeters are very handy and may be made small enough to be carried in a pocket.

3-13. Change of pressure with height. At sea-level the atmospheric pressure changes approximately by 1 mm of mercury for every 11-metre increase of height or 0.1 inch per 90 ft of ascent. The difference of pressure between the two levels is equal to the weight of a column of air of unit cross-section and height equal to the difference in the levels.

Examples. (1) The atmospheric pressure at the ground floor of a building is 75·85 cm of mercury and that on the top of the building is 75·63 cm. If the density of air outside is 0·00125 g/cm^3, how tall is the building ?

Solution : The difference of pressure (0·22 cm of mercury) is due to the weight of a column of air of height h equal to that of the building and of cross-section 1 cm^2 .

$\therefore\ 0{\cdot}22 \times 13{\cdot}6 = h \times 0{\cdot}00125$ whence $h = 23{\cdot}9$ metres.

(2) The atmospheric pressure at the top of a mountain is 4·5 inches of mercury less than the value at the base. If the mean density of air is 0·075 lb/ft^3, find the height of the mountain.

Solution : Let h be the required height in feet. Then the pressure due to a column of air h ft in height and of density 0·075 lb/ft^3 is equal to 4·5 in of mercury.

$\therefore\ h \times {\cdot}075 \times g = (4{\cdot}5/12) \times 13{\cdot}6 \times 62{\cdot}5 \times g$, since the specific gravity of mercury is 13·6 and 1 cu. ft of water weighs 62·5 lb.

$$\therefore\ h = \frac{4{\cdot}5 \times 13{\cdot}6 \times 62{\cdot}5}{12 \times 0{\cdot}075} = 4249 \text{ ft.}$$

Exercises

On Hydrostatics ; (*Sections 3-4 to 3-13*)

1. Distinguish between *pressure* and *thrust*. If pressure involves an area, what is meant by *pressure at a point* ?

A 20 kg table stands evenly on four legs each 5 centimetre square. What are the values of thrust on each leg and the pressure at a point of the leg ?

[Ans : 5 kg ; pressure = 0·2 kg/cm^2]

2. What is the characteristic property of a fluid ? How can you prove that a liquid exerts a normal pressure on any surface with which it is in contact ? Explain if this will apply also to an imaginary surface within the liquid.

3. Derive an expression for the pressure at a depth h in a liquid of density ρ.

What do we understand when we say that fluid pressure at a point acts equally in all directions ?

What is meant by the statement that a pressure is equal to 76 cm of mercury ?

4. (*a*) How can you prove that the pressure of a fluid at rest is the same at all points in the same horizontal plane ?

What do we mean by 'horizontal' and 'vertical' ?

(*b*) If there be several open vessels connected together by a liquid, why should the liquid levels be the same in all ?

What is the meaning of the statement 'a liquid finds its own level' or 'a liquid stands at the same height in communicating vessels'.

5. What is meant by the *force of buoyancy* ? Where does it act ?

Show theoretically with the help of a simple example that the upthrust on an immersed body is equal to the weight of the displaced liquid.

What relation has the apparent loss of weight of an immersed body with the fact that a liquid exerts a normal pressure on any surface with which it is in contact ?

6. State Pascal's law of transmission of fluid pressure.

Explain how the law can be applied to magnify a force. Show how the principle of conservation of energy is not violated in such magnification.

7. Briefly discuss the construction of a hydraulic press, and explain the principle on which it acts. Derive an expression for the mechanical advantage of the machine.

8. What is meant by *atmospheric pressure* ? How do we express it ? What is meant by *standard atmospheric pressure* ?

9. How did Torricelli prove that it is atmospheric pressure which supports a column of mercury in an evacuated tube ? What is a Torricellian vacuum ?

A glass tube, 1 metre long, is closed at one end and completely filled with mercury. It is then inverted over a trough of mercury. Explain what happens. If the tube is then gradually tilted what change occurs in the length of the mercury column.

10. What is a *barometer* ? Describe with a simple diagram any barometer that you know.

If the height of a mercury barometer is 75 cm of mercury, what will be the height of a water barometer ? Density of mercury = 13·6 g/cm^3.

11. (a) A U-tube has its limbs vertical. Some mercury is poured into it so as to reach straight parts of the limbs. 10 cm of water is now poured into one limb. What happens to the mercury levels in the limbs ?

[Hint : Consider the horizontal level at the surface of separation of mercury and water. At this level the pressure in both limbs will be equal. In the limb with the water, the pressure at this level is due to 10 cm of water, which is equal to 10 × 1 × g. In the other limb the mercury level will be higher by an amount x (say), which exerts the same pressure. So $10\,g = x\rho g$ where ρ = density of mercury.]

(b) If in the above problem, kerosene of sp. gr. 0·8 is poured so as to bring the mercury heads to the same level in both limbs, what height of kerosene should be added ?

[**Hint** : If h is the height of the kerosene column, the pressure due to it should be equal to that of the water column. Therefore $h \times 0{\cdot}8\,g = 10\,g$.]

12. A U-tube is partly filled with mercury of sp. gr. 13·6. Salt solution of sp. gr. 1·10 is poured into one limb so as to make the difference of levels of mercury in the two limbs equal to 1 cm. What is the height of the column of solution ? [Ans : 12·4 cm]

13. The water supply tank in a household is 30 m above ground. When a tap is opened 10 m of water head is lost for overcoming friction. What is the pressure of water in a tap 8 m above the ground ?

[Ans. : 12 m of water, that is, 1200 g-wt/cm^2.]

If the tap has a diameter of 1·2 cm what force should be applied to its mouth so as to stop flow of water ? [Ans : About 1560 g-wt]

Why should we get a stronger flow of water from a tap lower down than another, but having the same diameter of the mouth ?

3·14. The siphon.

A siphon is a simple device for transferring a liquid from one vessel to another *at a lower level* without tilting the vessel. When it is not possible or convenient to lift a vessel of liquid, such as a petrol tank, to empty it, we often use a siphon. We also use it to draw off the upper layers of a liquid without disturbing the lower layers.

In its simplest form a siphon (Fig. 3.15) consists of an inverted U-tube (A′B′ ; Fig. 3.15) of unequal limbs, *completely filled with liquid.* The shorter limb dips into the liquid to be transferred ; the longer limb leads outside and extends below the level of the liquid to be drained.

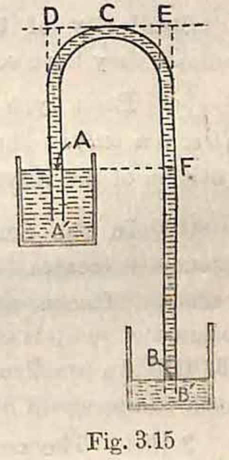

Fig. 3.15

The action of the siphon may be understood as follows. Since the siphon tube is full of liquid, the liquid must flow through the tube if the force pushing on the liquid at one end of the tube is greater than that at the other end. In Fig. 3.15, the upward pressure P_A at A, a point inside the tube at the same level as the liquid outside, is equal to the atmospheric pressure P *minus* the downward pressure due to the column of liquid DA. If $DA=h_1$ and density of the liquid in the siphon is ρ, then we have $P_A=P_o-h_1\rho g$. The upward pressure P_B at a similar point B inside the tube at the lower end is $P_B=P-h_2\rho g$, where $h_2=BE$. Hence the pressure at A is greater than the pressure at B by an amount equal to

$$P_A-P_B=(P_o-h_1\rho g)-(P_o-h_2\rho g)=(h_2-h_1)\rho g.$$

This is the pressure due to the liquid column $FB=EB-DA$.

The siphon will cease to act when the liquid is at the same level in the two vessels, for then $FB=0$ and the forces acting at the two ends of the siphon are equal and opposite. If the liquid is water it will also cease to act if the bend C is more than 34 ft above the surface of water in the higher vessel, for then the atmospheric pressure will be unable to lift the water up to the bend C. A siphon will not work in vacuum, for it is the atmospheric pressure which pushes the liquid through.

3-15. Water pumps. Water pumps are devices for raising water from a lower to a higher level. Since liquids have a natural tendency to flow from a higher to a lower level, the lifting is possible only at the expense of energy supplied from outside.

Similarity of the pumps. Though there are different types of pumps, they have some similarity in construction and action :

(a) Each type has valves which allow flow of water *in one direction only*. The difference between the types lies mainly in the position of the valves.

(b) In the first stage of action, each pump sucks water by the vacuum it creates. Atmospheric pressure forces the water into this vacuum. Hence none of these pumps can work if the level of water below the pump is greater than the height of the water barometer (34 ft). In practice, however, due to leakage and to vapour pressure such pumps would not work if the height is greater than 30 ft or so.

3-15.1. The common or the suction pump. The construction and the action of the pump will be clear from Fig 3.16. A piston

B, with a valve C in its head, fits airtight into the cylinder A. The valve C opens upward and so allows flow only in the upward direction through the piston. From the bottom of the cylinder a pipe goes down into the reservoir from which water is to be pumped up. At the junction of the pipe and the cylinder there is another valve D which also opens upwards.

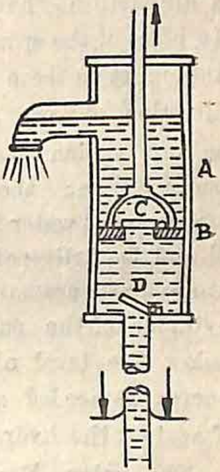

Fig. 3.16

Suppose the piston occupies the lowermost position at the start. During the upstroke, a partial vacuum is created in A between the piston head and the valve D. The higher pressure from below lifts the valve D and air from inside the pipe enters the cylinder. As a result a partial vacuum is created in the pipe also.

Atmospheric pressure now forces some water up the pipe. As the pump is gradually worked water rises more and more into the pipe and ultimately enters the cylinder. In this condition each downstroke will cause some water to pass through C and collect above the piston head. During the next upstroke this water will flow out through the spout, while more water will flow from the pipe into the cylinder. This intermittent outflow of water at each upstroke will continue so long as the pump is worked.

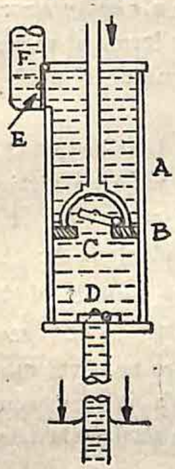

Fig. 3.17

Priming. The valve C is generally not airtight, particularly when the pump has been in use for some time. As a result the degree of vacuum within the cylinder is not sufficient to make the valve D open. To avoid this difficulty a part of the cylinder above C is filled with water from outside to make the valve C airtight. Suction is now effected easily. This operation is called 'priming'.

3-15.2. The lift[*] pump. A suction pump, as described above, cannot raise water more than

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[*] There is no uniformity in the use of the word 'lift' pump. Some authors use it to mean the 'common pump described in Sec. 3-15.1. Some use it to mean pumps' which can lift water to heights above the height of the water barometer.

30 ft or so for reasons stated earlier. To lift water to greater heights a lift pump may be used. The mechanism will be clear from Fig. 3.17. In place of the spout a delivery tube F is attached to the cylinder. At the junction there is a valve E which opens outwards, i.e.. in the direction of flow. The other valves C and D are in the same position as in a suction pump. The principle of action is as in the suction pump. When the cylinder is filled with water, every upstroke forces some water into F through E. The water gradually rises into F and is delivered at the desired height. Note that here also atmospheric pressure forces the water from the reservoir into the cylinder of the pump. Hence the level of water in the reservoir below the level of the pump should not exceed about 30 ft. Extra energy is needed at each upstroke to push the water into the pipe F against the hydrostatic pressure of the liquid column in F.

The valve E may also be placed near the lower end of the cylinder. In that case, water is forced up F during the down stroke of the piston.

3-16. Air pumps. Air pumps are used to pump air *out* of a vessel or *into* a vessel. When used to pump air out of a vessel, an air pump is generally called a ***vacuum pump***. A pump that forces air into a vessel is generally called a ***compression pump*** or ***air compressor***.

3-16.1. Vacuum pumps. Though there are various forms of vacuum pumps, we shall consider first the *piston pump*, commonly used in beginners' laboratories.

A. The piston pump. A simple piston pump is illustrated in Fig. 3.18. It consists of a metal cylinder A fitted with an airtight piston B which has a *light* valve C on it opening outwards. A is connected through the tube E to the vessel G to be evacuated. Another *light* valve D, which opens into the cylinder, closes the mouth of the tube E. It will be clear that the arrangement of valves is the same as in a suction water pump. The valves C and D are however *much lighter* than in the suction pump, and open or close with a very small difference of pressure between the two sides. To make good seals there are layers of oil on C and D.

The action is the same as that of a suction pump. When the piston rises, the volume of the air below B increases and its pressure falls. The valve C closes due to the pressure above it being greater than that below. As the pressure in A is less than atmospheric, the air in G

forces the valve *D* open and some air from *G* enters *A*. When the piston descends, air in *A* is compressed. This increased pressure in *A* closes *D*. When pressure in *A* rises above atmospheric, the valve *C* opens and air from *A* passes out. Thus at each upstroke some air from *G* enters *A*, and during each downstroke it leaves *A*. The vessel *G* is exhausted of air in this way.

The action of the pump ceases when the pressure in *G* fails to lift the valve *D*. The valve is generally made of oiled silk. The pressure in *G* may thus be reduced to about 1 or 2 mm of mercury.

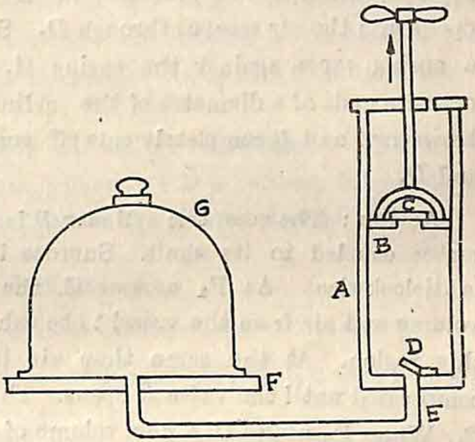

Fig. 3.18

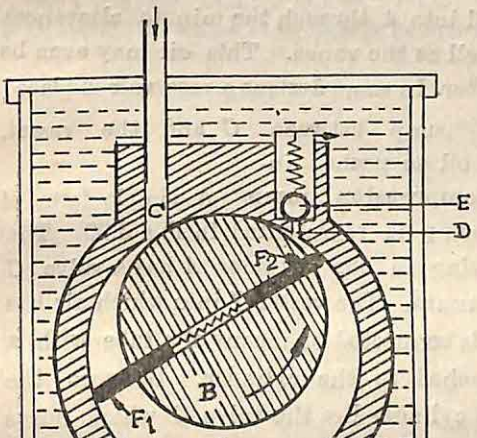

Fig. 3.19

B. Rotary pump. This pump is a modern device, very rapid in action, and produces vacua as high as 10^{-3} mm of mercury or even better. It is largely used for laboratory and industrial work.

Construction. Fig. 3.19 represents diagrammatically a rotary pump. *A* is a hollow steel cylinder inside which another cylinder *B* (called the *rotor*) can rotate eccentrically touching the casing *A* (called the *stator*) as it rotates. *C* and *D* are two openings in *A* through which air enters and leaves it. The vessel to be exhausted is connected with the pump through *C* by means of a thick rubber tube. Air from this vessel can enter *A* through *C*. The outlet opening *D* can be closed by a valve *E* operated by a spring. If the pressure in *A* exceeds a certain value, *E* opens and the air escapes through *D*. Steel vanes F_1, F_2, separated by a spring press against the casing *A*. They lie in slots cut at the opposite ends of a diameter of the cylinder *B*. The place of contact between *A* and *B* completely cuts off connection between the parts *C* and *D*.

Action : The eccentric cylinder *B* is rotated by means of an electric motor coupled to its shaft. Suppose it turns as shown in the figure (anticlockwise). As F_1 crosses C, the region behind F_1 increases in volume and air from the vessel to be exhausted enters through *C* into this region. At the same time air in the region in front of F_2 is compressed until the valve *E* opens. This air then passes out through *D*. When F_2 passes *C*, a new volume of air is drawn in behind it. The air in front of F_1 is compressed and expelled as before.

The entire system is kept immersed in oil. This prevents the leakage of air into *A*. When the pump is shut down it should be disconnected from the vessel to be evacuated. Otherwise, atmospheric pressure will slowly force the oil into *A* through the minute clearances between the casing and *B* as well as the vanes. This oil may even be forced into the evacuated chamber. In some designs a reservoir is placed within the pump between *C* and the vessel. It holds the oil so pushed in.

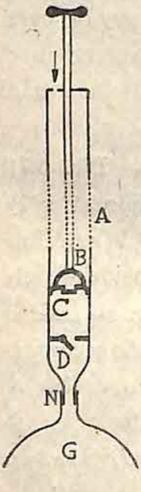

Fig. 3.20

3-16.2. Compression pump. A simple form of compression pump is represented in Fig. 3.20. The piston *B* moving in the cylinder *A* has a valve *C* which opens *inward*. The vessel *G* into which air has to be pumped is connected by means of a tube with a nozzle *N* attached to the cylinder. Between the nozzle and the cylinder lies the valve *D* which opens towards *G*.

Action. Suppose *G* and *A* contain air at atmospheric pressure. When the piston is drawn upward the pressure of air between *C* and *D* is reduced. The valve *C* opens and atmospheric pressure pushes some air into the space between *C* and *D*. During the down stroke, known as the *compression stroke*, air between *C* and

D is compressed. The valve *D* opens and allows some air to enter *G*. In this way, some air enters the cylinder at each suction stroke, and is then forced into the vessel *G* at each compression stroke. The valve *D* remains closed during each suction stroke, since the pressure on the lower side of *D* is greater than that on the upper during suction.

The rotary pump may serve as an air compressor. The end *C* (Fig. 3.19) is kept open to the atmosphere while a delivery tube is fitted to the outlet port *D*. As the pump runs, it forces air through *D*.

Exercises

(*Sec. 3-14 to 3-16.2*)

1. Explain the action of a siphon. For what purpose is it used? Under what conditions does a siphon fail to act?

2. Describe the action of a piston pump that can draw water from a well. Why is there a limit to the depth of the water level up to which the pump will act?

How would you alter the construction of the pump so that it can lift water to heights greater than the above limit?

3. Describe the actions of a vacuum air pump and an air compression pump. Why should the valves of air pumps be *light*?

CHAPTER

4

SURFACE TENSION AND VISCOSITY

4·1. Surface tension of a Liquid. The cohesive force between molecules of a liquid gives the liquid surface a property known as *surface tension*. **Surface tension** is that property of liquids by virtue of which *a liquid surface always behaves like a stretched membrane and tends to contract to the smallest area possible.* Because of this tendency drops of fog or rain assume a special shape (almost spherical) while falling through air, and a small quantity of mercury or water gathers up into globules instead of spreading into a thin film on a level surface. For a given volume, a sphere has the smallest surface. Therefore the drops become spherical, except for a slight flattening due to gravity.

Lead shots are made by allowing molten lead to fall into a pool of water through a sieve at the top of a tower. They become spherical due to surface tension.

If a needle or a razor blade be placed on a small piece of tissue paper and floated in water, the tissue paper will soon become waterlogged and sink, leaving the needle or razor blade floating. It is supported by surface tension of water. Some insects can walk on the surface of water, being supported by surface tension.

4-1.1. Capillarity. The ascent of water or the descent of mercury (Fig. 4.1) in a capillary tube is due to the difference in the magnitudes of the cohesive force of the liquid and the adhesive force between the liquid and the material of the tube. When a capillary glass tube is dipped in water, the glass wall immediately above the edge of the water attracts water molecules to it by adhesion. These molecules in turn attract nearby water molecules, pulling them up by cohesion. Water continues to rise in the tube until the upward pull is equalised by the weight of the

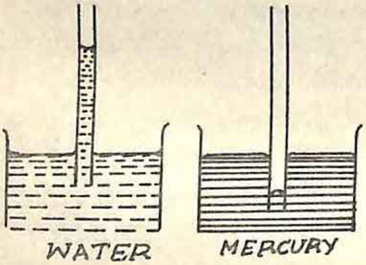

Fig. 4.1

liquid pulled up. The narrower the tube, the higher the ascent of water. It is in this way that liquids are soaked up by wicks, blotting paper, sponges and the like. Water is drawn from the wet subsoil into the top layers through the fine pores of soil. In dry weather, gardeners break up the top soil. This breaks the capillaries. Hence water from the deeper soil cannot come up to the surface and be lost by evaporation. Sandy soil is dry because subsoil water cannot rise far through the large interspaces between sand particles. In mercury cohesion is stronger than adhesion. The cohesive force pulls down mercury molecules from the vicinity of the walls. Contamination greatly alters the forces.

The phenomenon of descent or ascent of liquids in narrow tubes is known as *capillarity*.

4·1.2. Molecular theory of surface tension.—It should be remembered that the surface layer of a liquid is not a skin in the true sense of the term, but *behaves like* a skin under tension. To understand the contractile tendency, imagine a molecule of the liquid at *A* (Fig. 4.2) well below the surface. This molecule is attracted by all other molecules around it. But these attractive forces balance one another and the molecule at *A* does not experience any resultant force. Now imagine another molecule at *B* on the surface of the liquid. The downward pull on it is not balanced by any upward pull. This shows that molecules in or near the surface of a liquid are pulled towards the interior and hence, the surface tends to decrease. The surface of a liquid thus acquires a contractile property like a stretched membrane.

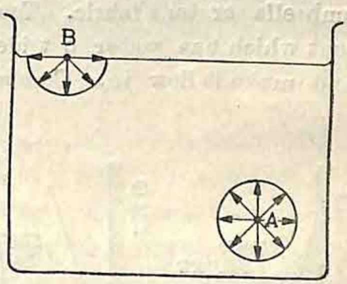

Fig. 4.2

If you make an incision in a stretched rubber membrane, the portions on the two sides of the incision will be drawn away from each other (Fig. 4.3 ; right). This shows that there are forces acting at right angles to the line of incision. In the case of a liquid surface, we imagine, by analogy with a stretched membrane, the existence of forces acting across any line on the surface (Fig. 4.3 ; left). The **surface tension** of a liquid is measured by the tangential force acting across a line of unit length on the surface of the liquid.

4-1.3. Some practical aspects of surface tension.—Sharp edges of glass rods and tubes are rounded merely by heating them until they soften, for surface tension then pulls the edges into a circular shape. Since a liquid of lower surface tension spreads more readily, soap is added to spraying mixtures. Soap solution has a surface tension smaller than that of pure water. The spreading quality of paint or solder depends on its surface tension. The surface tension of water prevents its passing through the pores in umbrella or tent fabric. Touching the inside of an umbrella or a tent which has water outside, will break the surface of the water and make it flow in. Campers should not touch the inside of their tent canvas during rain. Water can be carried in a wire tray of narrow mesh particularly when treated with wax.

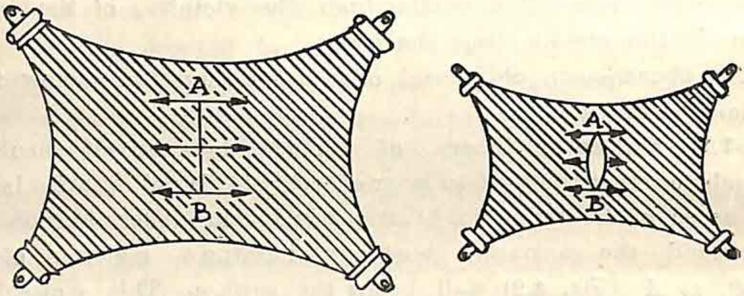

Fig 4.3

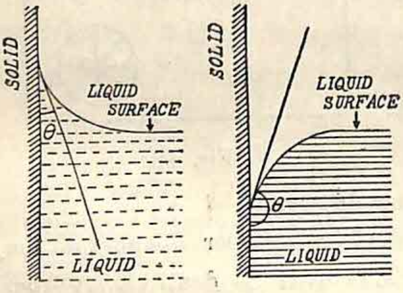

Fig. 4.4

Tiny bits of camphor, scraped into a dish of clean water, dart about in all directions. The camphor dissolves rapidly at the sharp corners and reduces the surface tension more at that point. The higher tension on the opposite side pulls it away. If a light powder is sprinkled over the surface of water and then a drop of alcohol added, the powder is pulled away from the area covered by the alcohol. This is due to lowering of the surface tension of water by contamination with alcohol. Oil or grease also lowers the surface tension of water.

4-1.4. Angle of contact. When a liquid surface meets a solid it does so at an angle whose value depends on the nature of the liquid and the solid. The angle between the solid and the tangent to the liquid surface measured within the liquid in a vertical section is called *angle of contact* between the liquid and the solid. Fig. 4.4 shows two cases, one in which the angle of contact θ is smaller than 90° and another in which it is bigger.

If a small quantity of water or benzene is placed on the surface of a *clean* glass plate, it is found to spread over the whole surface. The angle of contact is very small, nearly zero. Ethyl alcohol and methyl alcohol behave in the same way. Many other liquids, such as mercury, paraffin, turpentine, do not spread, but form drops or pools with definite boundaries, meeting the glass surface at a definite angle. For mercury the angle is about 140°, for paraffin 26°, for turpentine 17°.

Impurities and adulterants in the liquid may alter the contact angle considerably. In recent years a number of chemicals have been developed which are very potent as *wetting agents* or *detergents*. These compounds change the contact angle from a large value, greater than 90°, to a value much smaller than 90°.

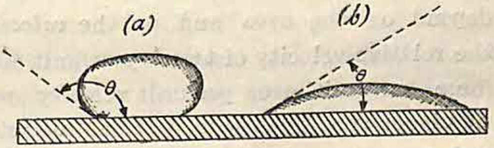

Fig. 4·5

The angle of contact between water and paraffin is about 107°. Water on paraffin will therefore gather up in the form of a drop as shown in Fig. 4.5. Addition of a detergent to water causes the water to spread as shown, the angle of contact having been much reduced.

Addition of soap to water lowers the surface tension and reduces the angle of contact. This causes the water to spread well on unclean (greasy) fabric. The soap can then act on the grease and remove it. Gardeners use soap in spraying solutions for the same reason.

Waterproofing agents applied to a cloth cause the contact angle of water in contact with the cloth to be larger than 90°. The adhesive force between these agents and water is small. Water does not therefore pass through the meshes of waterproofed fabric, while air can.

4-2. Viscosity: The property of a liquid by virtue of which it offers a resistance to flow is called **viscosity**. When two beakers, one containing alcohol and the other oil, are tilted from side to side, much less mobility is observed in oil than in alcohol. Oil offers more resistance to its motion and is said to be the more viscous of the two liquids.

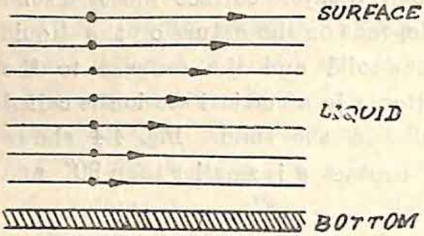

Fig. 4.6

When a liquid flows over a flat surface, the layer of liquid in contact with the surface remains stationary because of adhesion. The next upper layer moves slowly over the first (Fig. 4.6), The third layer over the second and so on. The speed of each layer increases with its distance from the solid surface. Between two adjacent layers tangential forces come into play, the faster layer tending to accelerate the slower and the slower layer tending to retard the faster. These tangential forces come into play due to the viscosity of the liquid. They depend on the area and on the *velocity gradient* of the layers, *i.e.*, the relative velocity of two layers unit distance apart. The tangential force per unit area per unit velocity gradient is called the **coefficient of viscosity**. The cgs unit of the coefficient of viscosity is called the **poise**.

The viscous forces do not act as long as the liquid is at rest. They come into play only when there is a relative motion between the layers. The force is thus of the nature of friction, and viscosity is sometimes called *internal friction*. When water in a bowl has been stirred, it is brought to rest after a time by the internal friction between the layers of water.

Gases also exhibit internal friction or viscosity. A raindrop falling through air is retarded by the viscosity of air. This viscous "drag" increases with the velocity of the drop. After a time the drag reaches a value equal to the force of gravity on the drop. When this condition is reached the velocity of the drop does not increase further. The rest of its path it continues to fall with a constant velocity, called the **terminal velocity**, The terminal velocity is proportional to the radius of the drop.

Ships moving through water and aeroplanes moving through air are subjected to very large drags due to the viscosity of the medium through which they move. By properly designing the shape of the moving body the drag can be reduced to a minimum. This is known as "streamlining".

When a liquid moves through a narrow tube, the inner layers move faster than the outer layers. Viscous forces therefore come into play. Work must be done to overcome this internal friction. Let just enough force be applied to make a liquid move through a tube without appreciable gain of kinetic energy. Then the volume flowing out per second will be proportional to the pressure gradient and to the fourth power of the radius, and inversely proportional to the coefficient of viscosity of the moving liquid. (This is known as Poiseuille's law.)

4-3. Motion in fluids : Streamline and Turbulent flow. Study of the motion of fluids (that is, liquids and gases) is much more difficult than the study of the motion of solids. Though fluids are discontinuous in structure, we treat them as continuous in discussing motion. A *particle* of a fluid is a very small volume of it ; it is supposed to retain its mass and volume unchanged during the motion. As in the case of a solid, a fluid particle is also represented by a point. It obeys Newton's laws.

The simplest kind of fluid motion is known as *steady flow*. In it, the velocity, pressure, density, etc. at any point of the fluid remain constant in time, though they may differ from point to point. The path followed by a fluid particle in motion is called a *flowline*, In a field of flow, a ***streamline*** is such a line that the tangent to it at any point gives the direction of flow of the fluid at that point. (Note the similarity of this definition with the definition of a line of force in an electrostatic or magnetic field.) In steady flow a flowline is also a streamline.

The word 'streamline flow' is somewhat misleading. A 'steady' fluid motion is called 'streamline motion' by some writers. But when conditions in the field of flow of the fluid are changing slowly, we can still imagine streamlines in them. These are no longer the path of the same particle ; but the lines satisfy the definition. This kind of fluid motion is *non-steady* motion, but still can be described in terms of streamlines.

Steady motion occurs when the fluid flows through a pipe or channel slowly. As the velocity of motion increases the motion gradually becomes unsteady, but can still be thought of in terms of streamlines. With further increase in velocity the pattern of flow becomes more complicated. Parts of the fluid acquire an increasingly rotational motion, called vortices. Such motion cannot be described in terms of streamlines, and is called **turbulent motion.** With increasing speed, the flow pattern of a fluid gradually changes from steady to turbulent. It is a gradual transition. But still we say that the transition from streamline flow (that is, steady flow) to turbulent flow occurs above a certain speed, called the *critical speed.*

The distinction between steady flow and turbulent flow can be demonstrated by means of a simple experiment designed by Osborne Reynolds. It consists in feeding a thin thread of highly coloured liquid into the centre of a tube through which the same liquid, uncoloured, is flowing (Fig. 4.7). The velocity of flow of the liquid can be altered over a considerable range by raising or lowering the reservoir from which the uncoloured liquid enters the tube *A*.

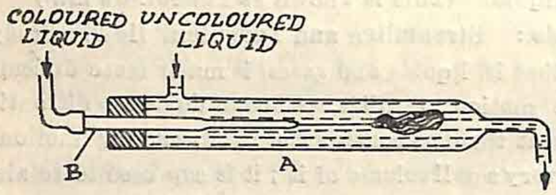

Fig. 4.7

At low speeds the coloured thread of liquid remains unbroken and continuous. This is steady flow. As the *critical speed* (depending on the size of the tube *A*, roughness of its inner wall and the nature of the liquid) is gradually approached the coloured thread is more and more agitated. Finally, its continuity is broken. Random, irregular, local circular currents, called **vortices**, develop throughout the fluid, and the resistance to flow greatly increases. A flow of this kind is called *turbulent flow.*

Turbulent flow is characterised by the formation of vortices. These may be easily seen when a solid object is moved through a fluid, or, what is the same thing, when a fluid is allowed to flow past a stationary object at any but the smallest velocities.

In all experiments on measurement of viscosity the fluid flow must be 'steady' (or so called 'streamline').

Exercises

1. Explain what are meant by *surface tension* and *viscosity*.

Give examples to show that the free surface of a liquid behaves like a stretched membrane and has a tendency to contract.

2. What happens when a glass capillary tube is partly immersed in (i) water, (ii) mercury?

How does a wick supply kerosene to a kerosene lamp?

How does blotting paper soak ink?

3. What is meant by *angle of contact*? What effect do (i) wetting agents (or detergents) and (ii) waterproofing agents have on the angle of contact?

4. Does a liquid at rest show viscosity? When do viscous forces come into play? In what sense can we call viscosity 'internal friction'?

What effect has viscosity on the motion of a body through a liquid or air?

5. Distinguish between *streamline* and *turbulent flow*.

Describe a simple experiment to demonstrate the difference. What is a streamline?

HEAT

CHAPTER 1

HEAT AND TEMPERATURE : RECAPITULATION

1-1. Heat. We derive the sensations of warmth and cold through our sense of touch. When a vessel containing cold water is placed over a fire, it becomes warm, then hot and finally begins to boil. *The external agent which turns a cold body hot is called heat.* When a body is heated we say that *heat* has been added to it.

All bodies contain heat. However cold a body is, it contains some heat. Water in a lake, pond or river is colder than boiling water ; ice is colder than water in a lake. If ice is added to water in a tumbler it melts. The water becomes colder than before, but remains warmer than ice. We explain it by saying that some heat has passed from the water into the ice and melted and warmed it. The water in the tumbler, which was colder than boiling water undoubtedly contained heat.

Even ice contains heat. Liquid air is much colder than ice. When a vessel containing liquid air is placed over ice, liquid air boils like water over a fire. Heat passes from ice into the liquid air.

The same agent 'heat' is responsible for both the sensations of heat and cold. When heat enters our body from the object touched, we feel the sensation of warmth. When heat leaves our body, we feel the sensation of cold.

1-2. Temperature. We have said that all bodies contain heat. Our sensation of warmth or cold does not depend on the total amount of heat that a body contains. When a quantity of cold water is poured into a vessel containing hot water, the total amount of heat in the water of the vessel is larger than before ; but the water appears colder than it was.

If the sensation of warmth is not to depend on the total quantity of heat in a body, it must depend on some other factor. This factor is called *temperature*. In trying to define temperature we observe that when two bodies are kept in contact, heat may pass from one body into the other. One of the bodies gets colder and the other warmer. The former loses heat to the latter. This behaviour of bodies helps us in defining temperature. As an elementary definition we may say.

The temperature of a substance is a number which expresses its degree of hotness on some chosen scale.

Heat flows from a body at a higher temperature to one at a lower temperature. When the temperature is the same for two bodies heat ceases to flow from one to the other.

If we take a cupful of water out of a bath tub, the water in the cup has the same temperature as that in the tub. But as the tub may contain a thousand cupfuls of water, the total heat in the tub is much greater than that in the cup.

1-3. Difference between heat and temperature. The points of difference between heat and temperature may be stated as follows :

(i) Heat is a form of energy. Temperature is a thermal state which determines the direction in which heat will flow. Heat always flows from a body at a higher temperature to a body at a lower temperature.

(ii) Temperature is not determined by the amount of heat in a body*. (Consider a bucketful of water, and a cupful taken out of it. Both have the same temperature ; but the water in the bucket contains many times the heat contained in the cup.)

(iii) If equal amounts of heat are added to different bodies, their temperature rise, in general, will be different.

(iv) Temperature may be compared with the water level in a vessel and heat with the amount of water in it. Addition of heat raises the temperature just as addition of water raises the water level. When two vessels with different water levels are connected together, water flows from the one having the higher level to the one having the lower level. The amount of water in either vessel does not determine the direction of flow.

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*Strictly speaking, it is not proper to speak of the 'amount of heat in a body'. In chapter 10 we shall learn that 'heat is energy in transit due to temperature difference'. The amount of heat we can get from a body depends on the temperature of the other body in contact.

1-4. Effects of heat. Almost all physical properties are affected more or less by heat. It can also bring about chemical changes. Most chemical actions take place faster at higher temperatures. This also applies to life processes ; but life cannot continue at high temperatures. Heat raises the temperature ; the other changes are direct effects of the rise of temperature. In the study of heat, temperature is the most fundamantal quantity.

1-5. Thermometers. *A thermometer is a device for measuring temperature.* To measure temperature we fix upon some property of a substance which changes much with change in temperature. Such a substance is called a *thermometric substance.* The property we use for measuring temperature is called its *thermometric property.*

The most common form of thermometer is the mercury-in-glass thermometer. Mercury is the thermometric substance. Its expansion with rise in temperature is fairly large. Expansion of mercury with temperature is the thermometric property we use for measuring temperature. Strictly speaking, it is the difference of expansion between mercury and its glass container that we utilize. We may, therefore, say that in a mercury thermometer, the change in length of the mercury column in the glass tube is the thermometric property.

1-6. Scales of temperature. To give a temperature a numerical value, it is necessary to devise a scale of temperature. For this purpose we take two temperatures fixed by Nature. One is the melting point of ice and the other is the boiling point of pure water, both at a pressure of one standard atmosphere. The former is called the *ice-point* and the latter, the *steam point.* In preparing a temperature scale, these two temperatures are given arbitrary values. The most used temperature scale is the *centigrade scale.* In it, the ice point is assigned the value $0°C$ and, the steam point is given the value $100°C$. The temperature interval between the two fixed points is called the *fundamental interval.* In the centigrade scale the fundamental interval is divided into 100 equal parts, called degrees. The scale is extended above and below the two fixed points. Temperatures below $0°C$ (the ice-point) are marked negative, such as. $-1°C$, $-20°C$, etc. (In 1948, the International Organization for Standards directed use of the word 'celsius' in place of 'centigrade' in honour of Anders Celsius, a Swede, who invented the scale in 1742. The symbol for a degree on this scale continues to be °C, but we now call it 'degree celsius'.)

On the Fahrenheit scale, the ice point is assigned the value 32°F and the steam point is given the value 212°F. The fundamental interval is thus divided into 180 equal parts or degrees. Since a difference of 180 fahrenheit degrees equals a difference of 100 celsius degrees, we have

$$1 \text{ fahrenheit degree} = \frac{100}{180} = \frac{5}{9} \text{ celsius degree.}$$

1-7. How to express a value of temperature and a difference of two temperatures in symbols. What will you understand when we write 20°C or 20°F? Obviously, it means a temperature of 20°C or 20°F. But how shall we express a difference of temperature of 40°C − 20°C = 20 celsius degrees, in symbols? Many authors prefer to indicate the difference as 20 celsius degrees or 20C°, putting the degree mark after the symbol C. (Similarly, 20 fahrenheit degrees or 20F°.) Unless we use the word 'difference', it is better to write 20C° 20C deg. There is, however, no uniformity of writing in this regard. But the point is worth remembering.

Of late, the General Conference of Weights and Measures and the International Organization for Standardization have recommended the use of the symbol, °C, to represent not only a specific temperature, but also a range of temperature. The exact meaning is to be understood from the context.

1-8. Conversion from celsius to fahrenheit reading and vice versa.

Suppose a temperature reads c on the celsius scale and f on the fahrenheit scale. f is $(f-32)$ F° away from the ice-point (0°C). Since 1 $F° = \frac{5}{9}C°$, $(f-32)F° = \frac{5}{9}(f-32)C°$. Hence c is $\frac{5}{9}(f-32)$ celsius degrees away from the ice-point.

$$\therefore \quad c = \frac{5}{9}(f-32) \text{ and } f = \frac{9}{5}c + 32.$$

Though India uses the celsius scale, our clinical thermometers (Doctor's thermometers) were being graduated in fahrenheit degrees. Very recently, however, we have changed to the celsius degree. The normal body temperature is taken as 37°C.

CHAPTER

2

EXPANSION OF SOLIDS

2-1. Some demonstration experiments. When a solid is heated it expands ; when it is cooled it contracts. The expansion or contraction is so small that it cannot be seen with the unaided eye. It therefore requires carefully designed experiments to demonstrate the effect.

Solids expand on heating. (i) Expansion of a solid on heating can be easily demonstrated with the arrangement shown in Fig. 2.1.

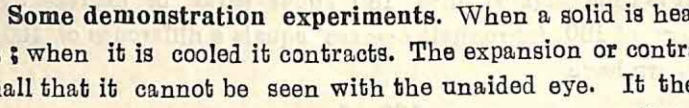

Fig. 2.1

One end of a thin metal rod is clamped to a stand. The free end is placed on a pin which carries a light pointer. A weight is hung from near the free end of the rod so that the rod presses on the pin and keeps the pointer in position. The rod is heated by running a flame along it. It will be seen that the pointer is deflected from its position. When the rod is allowed to cool the pointer attains its original position. The explanation lies in the expansion of the rod due to heating and subsequent contraction on cooling.

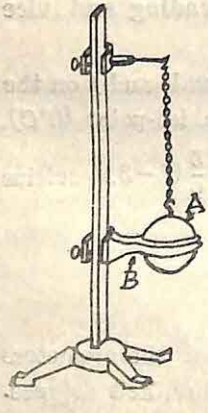

Fig. 2.2

(ii) *Ball and ring experiment* : Fig. 2.2 shows a ball *A* which, when cold, just passes through the ring *B*, but does not do so when heated.

(iii) *Expansion of wire when heated electrically.* A nickel wire about 0.5 mm in diameter is hung from a hook and stretched vertically by a small weight. An electric current of several amperes is passed through the wire so that it becomes red hot. As the wire is heated the weight descends rapidly. It rises when the current is stopped. This provides a vivid demonstration of the expansion of a wire on heating.

Different solids expand differently. Different solids expand by different amounts under identical conditions. This may be shown in the following way.

Two metal strips of exactly equal length are riveted together (Fig. 2.3). One may be of copper or brass and the other of iron. On heating, this compound bar bends with the brass or the copper on the outside. On cooling it straightens again. The bending is due to the fact that one strip becomes longer than the other on heating. Brass or copper expands more than iron under the same conditions.

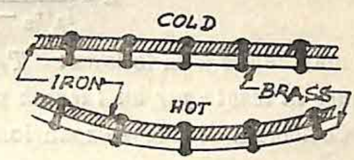

Fig. 2.3

Force of expansion or contraction is enormous. The breaking bar experiment (Fig. 2.4) demonstrates that an enormous force is brought into play when a solid contracts or expands. A stout iron rod fits across two slots in a heavy iron stand. The rod has a hole through one end and a screw at the other. A cast iron pin (about 5 mm in diameter) is passed through the hole. The rod is then heated and clamped tightly while hot by means of the screw. When the rod is allowed to cool it is found that the cast iron pin snaps owing to the enormous force of contraction. A slight modification of the rod (using another hole) enables us similarly to demonstrate the force of expansion.

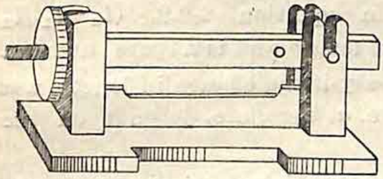

Fig. 2.4

2-2. Co-efficient of linear expansion. When a solid is heated it expands in all directions; but very often we are concerned with its expansion only in one direction, viz., length. We speak of this as the *linear expansion* of the solid.

Let l_1 be the length of a bar at a temperature t_1, and l_2, its length at temperature t_2. Experiment shows that the increase in length, l_2-l_1, is approximately proportional both to l_1 and to the rise in temperature t_2-t_1. We may, therefore, write

$$l_2-l_1=\alpha l_1(t_2-t_1) \tag{2-2.1}$$

where α is a small constant whose value depends on the material. It is called the coefficient of linear expansion of the material. It should properly be called the *mean coefficient of linear expansion* between the temperatures t_1 and t_2. Writing

$$\alpha=\frac{l_2-l_1}{l_1(t_2-t_1)} \tag{2-2.2}$$

we may define α as follows: *The coefficient of linear expansion is the change in length per unit length per degree rise of temperature.*

Coefficient of linear expansion (abbreviated C. L. E.)

$$=\frac{\text{Increase in length}}{\text{Original length}\times\text{rise in temperature}}$$

It follows that if l_1 is the length at a temperature t_1 of a piece of material of coefficient of linear expansion α, its length l_2 at temperature t_2 will be given by

$$l_2=l_1\{1+\alpha(t_2-t_1)\} \tag{2-2.3}$$

It is easy to see that *α is independent of the unit of length, but is dependent on the temperature scale.* In equation 2-2.2, $(l_2-l_1)/l_1$ represents the change in length per unit length and is a pure number. In mentioning the value of α we must say if the change in length was caused by 1°C or 1°F rise of temperature. α, therefore, depends on the scale of temperature adopted.

The meaning of a statement like "the coefficient of linear expansion of brass is 19×10^{-6} per °C" should be clearly understood. It means that for each 1°C rise of temperature a rod of brass expands by 19×10^{-6} part of its original length. In other words

for 1°C rise of temperature a 1 cm long brass rod increases
by 19×10^{-6} cm ;
" " " " 1 metre " 19×10^{-6} metre ;
" " " " 1 yard " 19×10^{-6} yard ;
" " " " l units " $19\times10^{-6}\times l$ units.

For a t°C rise in temperature a rod of length l units will increase by $l\alpha t$ units where α is expressed in "per °C" unit.

2-2.1. True relation between length and temperature

If we take a large enough range of temperature, we find that l does not increase linearly with t. For example, in the case of copper over a range from $-100°C$ to $400°C$

$$l_t = l_o(1 + \alpha t + \alpha_1 t^2 + \alpha_2 t^3 + \cdots) \qquad (2\text{-}2.4)$$

Here l_t is the length of a rod of copper at $t°C$, l_o the length at $0°C$, and α, α_1, α_2 are constants having values $\alpha = 16{\cdot}2 \times 10^{-6}$, $\alpha_1 = 4{\cdot}5 \times 10^{-9}$ and $\alpha_2 = -20 \times 10^{-12}$. Eq. 2-2.4 expresses the general form of the relation between length and temperature, but the constants α, α_1, α_2 have different values for different substances.

$$\text{For iron,}\quad l_t = l_o(1 + 11{\cdot}7 \times 10^{-6} t + 4{\cdot}7 \times 10^{-9} t^2 + \cdots)$$

If we ignore the effects of α_1, α_2 etc., then we can write

$$l_t = l_o(1 + \alpha t) \text{ or } \alpha = (l_t - l_o)/l_o t \qquad (2\text{-}2.5)$$

The coefficients α defined by equations 2-2.2 and 2-2.5 have negligible difference unless the temperatures are very different. It is preferable to use Eq. 2-2.2, calling α the *mean* coefficient of linear expansion. Using Eq. 2-2.5 forces us to take the initial length as that corresponding to $0°C$. In most practical cases we hardly do it.

If for extreme simplicity, we assume that $l_t = l_o(1 + \alpha t)$ for all temperatures, then at $t_1°C$, $l_1 = l_o(1 + \alpha t_1)$, and at $t_2°C$ $l_2 = l_o(1 + \alpha t_2)$.

$$\text{This gives } l_2 - l_1 = l_o\,\alpha(t_2 - t_1) \text{ or } \alpha = (l_2 - l_1)/l_o(t_2 - t_1) \qquad (2\text{-}2.6)$$

2-3. Determination of the coefficient of linear expansion. In all solids the expansion is so small that we must either magnify the change in length before measuring it or use some delicate measuring instrument. There are many forms of apparatus of which we describe one which uses a micrometer screw, and is known as *Pullinger's apparatus* (Fig. 2.5).

The experimental rod R, about a metre long, stands vertically in a tube A mounted on the frame F. A has two thermometers T_1, T_2 inserted in it to measure the temperature of R. It has also side tubes for inlet and outlet of steam and is surrounded by felt or some other non-conductor of heat. The top of R passes through a cork in A and through a hole in a glass plate G which rests horizontally on the frame. A spherometer

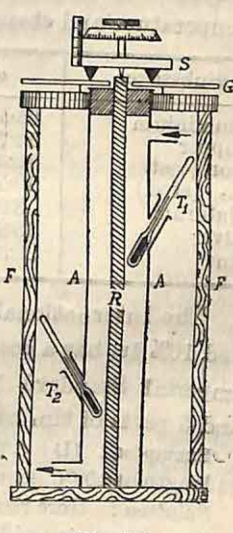

Fig. 2.5

rests on G. After the initial length l of the rod has been measured it is placed in position and the central leg of the spherometer brought into contact with the top of R. (This adjustment may be accurately made by connecting one end of an electric circuit containing an electric bell with a leg of the spherometer and the other end with R. When the contact between the spherometer and the rod R is established the bell starts ringing.) The spherometer reading is taken. The central leg may now be raised without removing the spherometer. The initial temperature t_1 of the rod is read off from T_1 and T_2.

Steam is then allowed to pass through A. Temperature of R as indicated by T_1 and T_2 rises and finally becomes steady. When it has been steady for some time the central leg of the spherometer is again brought into contact with R and its reading taken. The difference in the two readings of the spherometer gives the expansion x of the rod. The final temperature t_2 of the rod is noted from T_1 and T_2. We have all the data to find the coefficient of expansion α.

$$\alpha = \frac{x}{l(t_2 - t_1)}$$

2-4. Values of α for some substances. The following table gives the values of the celsius of linear expansion of some common substances on the celsius scale. They are mean values between room temperature and steam point.

Substance	α per °C	Substance	α per °C
Aluminium	$25{\cdot}5 \times 10^{-6}$	Brass	$18{\cdot}9 \times 10^{-6}$
Copper	16·7 "	Bronze	17·7 "
Iron (cast)	10·2 "	Glass (crown)	8·5 to 9·7 "
" (steel)	10·5 to 11·6 "	" (flint)	7·8 "
Platinum	8·9 "	" (pyrex)	3 "
Silver	18·8 "	Invar (64 Fe, 36 Ni)	0·9 "
Zinc	26·3 "	Quartz (fused)	0·5 "

The International Prototype Metre, made of an alloy of 90% Pt and 10% Ir, has a coefficient of expansion of $8{\cdot}7 \times 10^{-6}$ per °C. The Imperial Standard Yard, made of 32 parts of copper, 2 parts of zinc and 5 parts of tin, has a coefficient of $17{\cdot}7 \times 10^{-6}$ per °C.

Examples : (1) A piece of copper wire has a length of 200 cm at 10°C. Find its length at 100°C. given $\alpha = 17 \times 10^{-6}$ per °C.

Solution : Here rise of temperature = 90C°. (For C°, see Sec. 1-8.)

Now, expansion of 1 cm for 1 C° rise $= 17 \times 10^{-6}$ cm ;

$\therefore$ Expansion of 200 cm for 90C° rise $= 200 \times 90 \times 17 \times 10^{-6}$ cm $= 0{\cdot}306$ cm.

$\therefore$ The length of the wire at 100°C = 200·306 cm.

Alternatively, we could apply equation 2-2.3.

In this equation $l_1 = 200$ cm, $t_1 = 10°C$, $t_2 = 100°C$, $\alpha = 17 \times 10^{-6}$ per °C.

$\therefore\ l_2 = l_1\{(1+\alpha(t_2 - t_1)\} = 200(1 + 17 \times 10^{-6} \times 90) = 200.306$ cm.

(2) A piece of steel has a length of 30 inches at 15°C. At 90°C its length increases by 0·027 in. Find the coefficient of linear expansion of steel.

Solution : Here rise of temp. $= 90 - 15 = 75$ C°.

Expansion of 30 inches for a temperature rise of 75 C° = 0·027 in.

$\therefore$ Expansion of 1 inch for a temperature rise of 1 C° $= 0{\cdot}027/(30 \times 75)$
$= 12 \times 10^{-6}$ in.

$\therefore$ The coefficient of linear expansion $= 12 \times 10^{-6}$ per °C.

Or, apply formula 2-2.2.

(3) A steel scale is correct at 0°C. The length of a brass tube measured by it at 30°C is 4·5 metres. What is the length of the tube at 0°C? (Coefficient of linear expansion of steel $= 11 \times 10^{-6}$ per °C, of brass $= 19 \times 10^{-6}$ per °C.)

Solution : 1 metre on the scale at 30°C is really $1 + 30 \times 11 \times 10^{-6} = 1{\cdot}00033$ metres.

$\therefore$ The true length of the tube at $30°C = 4{\cdot}5 \times 1{\cdot}00033$ metres. If it cooled to 0°C, its length would be $4{\cdot}5 \times 1{\cdot}00033\,(1 - 30 \times 19 \times 10^{-6}) = 4{\cdot}499$ metres nearly.

2-5. Coefficients of surface and volume expansion. When the length of a body changes with temperature, the area of its surface as well as its volume undergoes changes.

The *coefficient of superficial expansion* is the change in area per unit area per degree rise of temperature. If S_1 and S_2 are the areas of any portion of the surface of a body at temperatures t_1 and t_2 respectively, then the coefficient of superficial expansion

$$\beta = \frac{S_2 - S_1}{S_1(t_2 - t_1)} \qquad (2\text{-}5.1)$$

$$\text{or,}\quad S_2 = S_1\{1 + \beta(t_2 - t_1)\} \qquad (2\text{-}5.2)$$

The *coefficient of cubical expansion* is the change in volume per unit volume per degree rise of temperature. If V_1 and V_2 are the volumes of a body at temperatures t_1 and t_2 respectively, then the coefficient of cubical (or volume) expansion

$$\gamma = \frac{V_2 - V_1}{V_1(t_2 - t_1)} \qquad (2\text{-}5.3)$$

$$\text{or,}\quad V_2 = V_1\{1 + \gamma(t_2 - t_1)\} \qquad (2\text{-}5.4)$$

Like the coefficient of linear expansion, these coefficients also depend on the temperature scale only and are expressed in per °C or °F as unit.

The coefficients β and γ defined above are really the *mean coefficients* between t_1 and t_2.

2-5.1. Relation between α, β and γ :

Consider a square of sides l cm. Let it be heated through 1C°. Each side becomes $l(1+\alpha)$ cm and the area $l^2(1+\beta)$ cm^2.

$\therefore \quad l^2(1+\beta)=l^2(1+\alpha)^2 \quad$ or $\quad 1+\beta=1+2\alpha+\alpha^2 \quad$ or $\quad \beta=2\alpha$

(α^2 being neglected)

If a cube of sides l cm be heated through 1 C°, the sides become $l(1+\alpha)$ cm and the volume is changed to $l^3(1+\gamma)$ cm^3.

$\therefore \quad l^3(1+\gamma)=l^3(1+\alpha)^3 \quad$ or, $\quad 1+\gamma=1+3\alpha+3\alpha^2+\alpha^3 \quad$ or, $\quad \gamma=3\alpha$

(α^2, α^3 being neglected).

[Since α is very small, α^2, α^3 are smaller still. For example, if α is 0.00001, then $\alpha^2=0.0000000001$ and $\alpha^3=0.000000000000001$.

We are, therefore, justified in neglecting α^2 and α^3 compared with α].

Thus $$\alpha=\tfrac{1}{2}\beta=\tfrac{1}{3}\gamma \tag{2 5.3}$$

(Also see Secs. 2-10 and 2-11.)

2-6. Practical effects of expansion of solids.

A. Expansion of solids serving a useful purpose :

(*i*) Metal tyres of cart wheels are fitted on when hot. On cooling they contract and attach rigidly to the wheels. Locomotive wheels made of iron are fitted with steel tyres in the same way.

Example : (i) A wheel has a diameter of 50 inches. A steel tyre of inner diameter 49·9 inches is to be fitted to it. Find by how much the tyre must be heated to mount it (α for steel $=11\times10^{-6}$ per °C).

Solution : The inner diameter must increase by heating to 30 inches.

$\therefore$ Increase in length required $=0{\cdot}1$ inch.

From the relation $l_2-l_1=l_1\alpha t$, we have

$$0{\cdot}1=49{\cdot}9\times11\times10^{-6}\,t, \quad \text{whence } t=182{\cdot}2\ C^\circ.$$

(ii) In the above problem find the force with which each square inch area of the tyre holds on to the wheel, given, Young's modulus for steel $=2\times10^{12}$ dynes/cm^2.

Solution : The heated tyre contracts from 50″ to 49·9″.

This contraction may be considered to have been brought about by a compressive force whose magnitude per unit area may be calculated from the value of Young's modulus for steel. The tyre holds on to the wheel with this compressive force.

The strain, in this case = change in length per unit length $=0{\cdot}1/50$.

Young's modulus $=2\times10^{12}$ dyn/cm^2.

$\therefore$ Stress, or the compressive force per unit area

$=$ Young's modulus $\times$ strain $=2\times10^{12}$ dyn/cm$^2\times0{\cdot}1/50=4\times10^9$ dyn/cm^2.

(*i.e.*, every square centimetre area of the tyre holds on to the wheel with a force equal to the weight of over 4000 kg. wt.)

(*ii*) Steel plates of boilers, ships or bridges are held together by rivets. Rivets are fixed while hot. On cooling they contract and make a very firm joint (Fig. 2.6).

(*iii*) Walls which have bulged outwards may be drawn in by passing iron bars through them across the building. To the extremities of the bars are attached iron plates screwed upto the walls with nuts. The bars are heated and the nuts and plates screwed up tightly against the wall. When the bars contract they draw the walls with them.

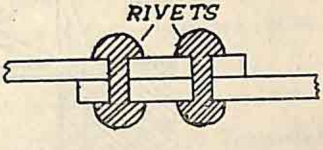

Fig. 2.6

(*iv*) Glass stoppers often stick to bottles too tightly to be opened by ordinary means. Careful heating of the mouth of the bottle enlarges it, when the stopper comes out easily. If a metal cap sticks, the cap itself is heated.

B. *Disadvantageous expansions and their remedy* :

Expansion of solids more often causes inconvenience and calls for suitable remedies.

(*i*) The rails on a railway expand on a hot day and contract at night. The extremes of temperature may be fairly large. To allow for the expansion and contraction of the rails a small gap is always left between the ends of adjacent rails. The arrangement is shown in Fig. 2.7. The rails are joined by fish plates bolted to the rails. Bolt holes are oval in shape and allow the rails to move in the direction of their length.

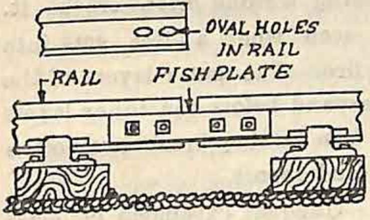

Fig. 2.7

In tramways however it is the common practice to weld rails together in order that the resistance to the flow of electric current through the rails to the earth may be the least. The rails are imbedded in the road and surrounded by granite blocks and concrete. This gives the rails much protection against temperature changes. The rails of a railway are fully exposed.

(*ii*) The ends of a big steel bridge are not rigidly built into the brick work on which they rest. They are supported on rollers which allow them to expand or contract without damaging the masonry (Fig. 2.8).

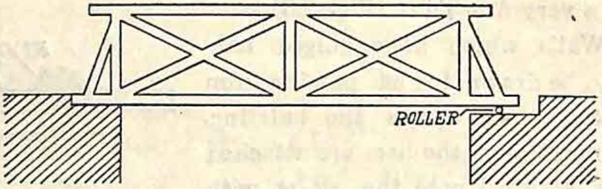

Fig. 2.8

(*iii*) When hot water is poured into a glass vessel with thick walls, the vessel is likely to crack. The inner wall tends to expand; but as glass is a *bad conductor* of heat the outer wall remains cooler and does not expand. This sets up large stresses in the glass, which cracks. The risk is less when the wall is thin or when the glass has a small coefficient of expansion. The linear coefficient for ordinary glass is about 9×10^{-6} per $^\circ C$. Pyrex glass has a coefficient of 3×10^{-6} per $^\circ C$. It can be heated in a flame without cracking. Fused silica (quartz) has such a small coefficient that a crucible made of it can be heated redhot in a furnace and chilled in water without cracking.

Heating a stone often cracks it. This is seen when a piece gets into a coal fire. The outer layers of the stone expand before the inner layers become hot. So they break away often with a loud report.

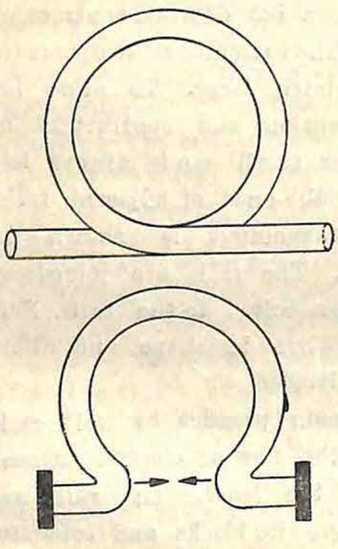

Fig. 2.9

(*iv*) Unequal expansion of glass and many metals makes it impossible to seal a metal wire such as copper or iron into a glass bulb. Platinum and some alloys have very nearly the same expansion as glass, and are used for sealing. In modern electric lamps the leads which pass through the glass stem are made of an alloy of nickel and iron coated with copper This alloy expands equally with the glass.

(*v*) To allow for the expansion and contraction of steam pipes they are provided with expansion bends or loops as shown in Fig. 2.9.

When the pipe expands or contracts the shape of the loop changes slightly. The pipes carrying oil through deserts are also fitted with such bends and loops to allow for their expansion and contraction. They may also be laid in zig-zag fashion loosely on supports.

(*vi*) A telegraph, telephone or a power line is fixed between poles in such a way as to allow for its expansion and contraction. In summer it is longer and looser ; in winter it is shorter and more taut.

(*vii*) The distance between the graduations of a scale increases in summer and diminishes in winter. The graduations are correct only at a certain temperature. For very accurate readings a correction is necessary.

Example : A brass scale is correct at 20°C. The length of a rod as measured by it at 30°C is 50 cm. What is the true length of the rod ? (α for brass = 19×10^{-6} per °C).

Solution : A distance of 1 cm on the scale at 20°C increase to $1+19\times10^{-6}\times10$ $=1{\cdot}00019$ cm at 30°C. Therefore the distance which the scale reads as 1 cm when it is at 30°C, is really 1·00019 cm.

$\therefore$ The true length of the rod $=50\times1{\cdot}00019=50{\cdot}0095$ cm.

(*viii*) The time period of a **pendulum** depends on its length. Since the length increases in summer, the time period also increases and the clock goes 'slow'. The reverse takes place in winter. Arrangement must be made in order that a clock or a watch may keep correct time in all seasons' Such clocks or watches are called 'compensated'. *The effective length of a pendulum is the distance between the point of suspension and the centre of gravity of the suspended system.* If this length remains constant the pendulum keeps correct time. The pendulum bob is mounted on rods of two different materials in such a way that the downward expansion of one rod is equal to the upward expansion of the other. This is diagrammatically shown in Fig. 2.10. Let l and l' be the lengths of the rod and α and α' their coefficients of linear expansion. Their expansions for t° rise of temperature are $l\alpha t$ downwards and $l'\alpha' t$ upwards. The distance of the centre of gravity of the pendulum from the point of suspension will remain unchanged if $l\alpha t=l'\alpha' t$

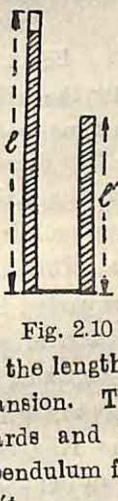

Fig. 2.10

$$\text{or, } \quad l/l'=\alpha'/\alpha \qquad (2\text{-}6.1)$$

The coefficients of iron and zinc are 0·00001 and 0·000028 per °C

respectively. Hence zinc rods used for compensating iron rods will have less than half the length of the latter. The clock 'Big Ben' on the Parliament House in London uses zinc for compensation. Its pendulum is as shown in the Fig. 2.11. Smaller clocks may have the pendulum rod made of 'invar', an alloy of 64% iron and 36% nickel. (The name is derived from 'invariable'.) It has an expansion coefficient of $0{\cdot}9 \times 10^{-6}$ per °C (about 1/13 that of iron). The very small expansion of the invar rod is compensated by a brass tube (Fig. 2.12).

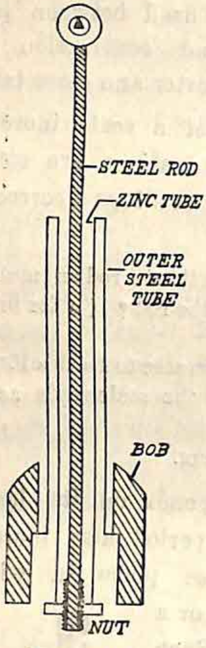

Fig. 2.11

2-7. Compensated balance wheels of watches. In watches the movement of the hands is controlled by balance wheels which execute rotational oscillations under the influence of a steel hair-spring. The time of oscillation depends on the effective distance of the rim of the wheel from the axis of rotation The rim is made of two metals, the outer being more expansible. This double rim is made in two or three parts (Fig. 2. 13) each supported by an arm of the wheel. The rim also carries small screw weights. When the temperature rises the arm expands, but the outer rim of more expansible metal curls in and brings the weights nearer to the axis, thus keeping the effective radius of the ring constant. With change of temperature the stiffness of the hair-spring changes and alters the time-period. The rim is arranged to compensate for this effect also.

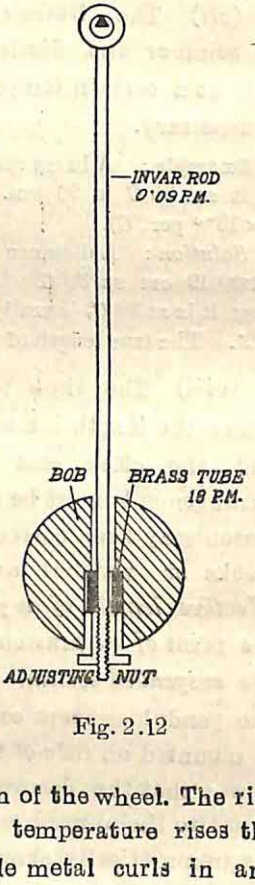

Fig. 2.12

2-8. Bimetallic strips. Strips made of two different metals riveted together curl when temperature changes. Such bimetallic strips may be used as thermometers. The method of use is illustrated in Fig. 2.14.

They also constitute the temperature sensitive element in thermographs and thermostats.

The action of one type of fire alarms depends on the bending of bimetallic strips on heating. When the heat of the fire causes the strip to curl, it closes an electric circuit which rings the alarm. Such fire-alarms may be installed in warehouses, in holds of ships or in other places where a fire may break out without being quickly discovered.

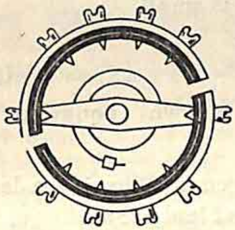

Fig. 2.13

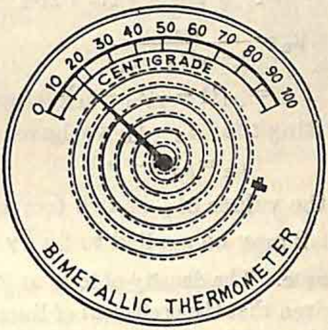

Fig 2.14

2·9. Expansion of a hollow vessel. A hollow vessel will expand as if it were solid. Its expansion will be outwards, not inwards. Inward expansion will reduce its surface area. But heating causes expansion of the surface area also.

If an annular ring is heated, its inner diameter will increase. The circumference of the inner ring will undergo linear expansion with rise in temperature. Hence its diameter will also increase.

A hollow cylinder of diameter d and length l will change its dimensions to $d(1+\alpha t)$ and $l(1+\alpha t)$ where α is the coefficient of linear expansion of its material and t the rise in temperature. The old volume $V=\frac{1}{4}\pi d^2 l$. The new volume $V_t=\frac{1}{4}\pi d^2(1+\alpha t)^2 l(1+\alpha t)=\frac{1}{4}\pi d^2 l$ $(1+\alpha t)^3=V(1+3\alpha t+3\alpha^2t^2+\alpha^3t^3)=V(1+3\alpha t)=V(1+\gamma t)$, where $\gamma=3\alpha$ = coefficient of cubical expansion. (α^2t^2 and α^3t^3 are ignored because of their smallness.)

2·10. Change of density with temperature. The density of a substance is defined as its mass per unit volume. Since the volume of a given mass of a substance changes with temperature while its mass remains constant, the density undergoes a change with the temperature.

Consider a body of mass m. Let V_1 and ρ_1 be its volume and density respectively at temperature t_1 and V_2 and ρ_2 the corresponding values at t_2. Then from the definition of density

$$m=V_1\rho_1=V_2\rho_2=V_1\{1+\gamma(t_2-t_1)\}\,\rho_2 \text{ frcm equation 2-5.4.}$$

$$\text{Or}\quad \rho_2=\frac{\rho_1}{1+\gamma(t_2-t_1)} \qquad (2\text{-}10.1)$$

$$=\rho_1\{1-\gamma(t_2-t_1)\}, \text{ since } \gamma(t_2-t_1) \text{ is small.}$$

Writing t for t_2-t_1 we have

$$\rho_2=\rho_1(1-\gamma t) \qquad (2\text{-}10.2)$$

If the values of ρ at two temperatures are known, equation 2-10.1 or 2-10.2 may be applied to find γ.

Example. The density of lead at $0°C$ is 11.34 g/cm³. What is the density at $100°C$, given that the coefficient of linear expansion of lead $=28\times10^{-6}$ per $°C$?

Solution : The coeff. of volume expansion

$$\gamma=3\times28\times10^{-6}=84\times10^{-6} \text{ per } °C$$

From equation 2 – 10.2, $\rho_2=\rho_1(1-\gamma t)$

$$=11.34\,(1-84\times10^{-6}\times100)=11.25 \text{ g/cm}^3.$$

Exercises

1. Define coefficient of linear expansion. What is the unit in which it has to be expressed? How does its value change if the temperature scale is changed from celsius to fahrenheit?

2. Show how the coefficient of volume expansion is related to that of linear expansion.

3. Give three examples where expansion on heating is made to serve a useful purpose.

4. Give three examples where expansion onheating causes inconvenience, and say how it is remedied.

How can pendulums be constructed to have an invariable length?

5. An aluminium bar is 240 cm long at $35°C$. To what temperature must it be heated to expand by 0·1 cm? α for aluminium $=0.000026$ per $°C$.

[ANS. : $51°C$]

6. Find the temperature to which a zinc rod 50 cm long at $15°C$ must be heated in order that its length may be 50·05 cm. α for zinc $=0.000025$ per $°C$.

[ANS. : $55°C$]

7. α for steel is 0·000012 per $°C$. What is its value in $°F$? If the length of a steel rail laid at $20°C$ is 5 m, what interval must be left between the rails to allow for a rise in temperature to 45 $C°$?

[ANS. ; 0·27 cm.]

8. The old Howrah bridge was 656 m long. Between a winter night and a summer noon is temperature changes from 8°C to 50°C. Find how much it expanded if $\alpha = 12 \times 10^{-6}$ per °C. [ANS.: 33 cm]

9. A 5 m long girder increases by 0·15 cm when temperature changes from 40°C to 55°C. Find the coefficient of linear expansion of the material of the girder. [ANS.: 20×10^{-6}/°C]

10. The diameter of a sphere at 35°C is 2·5 cm. Find the temperature to which it should be raised so that it will just fail to pass through a circular hole of diameter 2·501 cm. Coefficient of cubical expansion of the material of the sphere is 0·000075 per °C. [ANS.: 51°C]

11. A brass metre scale is correct at 30°C. A length measured by it at 55°C is 40 cm. What is the true length ? α for brass is 0·00002 per °C. [ANS.: 40·02 cm.]

12. A wheel has a diameter of 75 cm. A steel tyre of inner diameter 74·8 cm is to be fitted on it. Calculate the range of temperature in degrees celsius through which the tyre must be heated so that it may be mounted on the wheel. α for steel is 0.000012 per °C. [ANS.: 222·8°C]

13. Two bars of brass and steel, standing side by side, have one end of each rigidly attached to the other. The other ends are free to expand. The steel bar is 1 metre long. What should be the length of the brass bar so that the distance between the free ends of the bars remains the same at all temperatures ? α for brass is 0·00002 per °C, and α for steel is 0·000012 per °C. [ANS.: 60 cm.]

14. What is meant by the statement that the coefficient of linear expansion of brass is 19×10^{-6} per °C ? Describe an experiment to measure this coefficient for brass. Explain how to calculate the result.

15. Two metal strips, one of iron and the other of brass, have the same length at 60°C. They are riveted together so as to form a single strip of two layers. Describe and explain what happens when the compound piece is successively exposed to temperatures of 30°C and 100°C.

16. Calculate the total length of the gap to be left in a line 1500 km long when the temperature change is 50 C°. $\alpha = 10 \times 10^{-6}$ per °C. [ANS.: 750 m]

17. Describe an experiment to show that the force of expansion on heating is very great. Give an example of its application.

18. A pyrex flask has a capacity of 1 litre at 15°C. Find its capacity at 35°C. $\alpha = 3 \times 10^{-6}$ per °C. [ANS.: 1·00018 litres]

19. A steel sphere has to be passed through a brass ring. At 20°C, the diameter of the sphere is 25·0 cm and the internal diameter of the ring is 24·9 cm. If the sphere and the ring are heated together what is the temperature rise necessary ? α for steel $= 12 \times 10^{-6}$ per °C, for brass $= 20 \times 10^{-6}$ per °C. [ANS.: About 500 C°]

20. The density of aluminium at 20°C is 2·55 g per cm^3. Compare its densities at 0°C and 100°C, given $\alpha = 25 \times 10^{-6}$ per °C. [ANS.: $\rho_0 : \rho_{100} = 1.005 : 0.994$]

CHAPTER

3

EXPANSION OF LIQUIDS

3-1. Real and apparent expansions of a liquid. A liquid has no shape of its own ; it readily takes the shape of the container. We cannot therefore speak of the linear or superficial expansion of a liquid. Liquids have volume expansion only. The expansion of liquids is roughly ten times that of solids.

The volume of a liquid varies in a complicated way with temperature. In general, the relation is of the form

$$V_t = V_0(1+at+bt^2+ct^3+\cdots)$$

where a, b, c are constants of diminishing magnitude.

For example, in the case of mercury

$$V_t = V_0(1+1.8144\times10^{-4}t+7.016\times10^{-9}t^2+2.86\times10^{-11}t^3+\cdots)$$

Mercury is a liquid whose expansion is rather much more uniform than that of most other liquids, for here b is much smaller than a.

When a liquid is heated the container is also heated along with it and expands. The expansion of the vessel masks the expansion of the liquid and makes the latter appear smaller than its real value. In other words, the apparent expansion of a liquid (i.e., the expansion of a liquid that we see) is less than its real expansion.

To show the expansion of a liquid and that of the containing vessel. The expansion of the container may be demonstrated as follows. Take a litre flask filled with coloured water. Fit to it a cork through which passes a glass tube with a narrow bore (Fig. 3.1). By pushing in the tube or pulling it out as necessary, the level of water in the tube may be made to stand at any desired height (say at A). Put a mark there. If now the flask is suddenly immersed in hot water, the level of water in the tube sinks at first. The reason for it is that the flask has expanded due to heat, but no heat has yet entered the liquid. As the liquid heats up with time the water in the tube rises and moves past the former mark A.

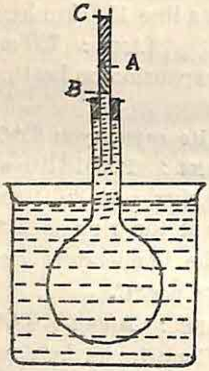

Fig. 3.1

3-1.1. Relation between real and apparent expansion. In Fig. 3.1 let A be the initial level of water in the tube at a temperature t_1, B the position which the level would occupy if the vessel alone expanded to temperature t_2 without any expansion of the liquid and C the final position of the level when the liquid is also at t_2.

The apparent expansion of the liquid is AC while its true expansion is BC ; AB is the expansion in volume of the vessel, all for the same rise of temperature from t_1 to t_2.

From the figure $BC = AC + AB$

or *the true expansion = the apparent expansion + the expansion of the vessel.*

Coefficients of real and apparent expansion. From the above it is clear that in the case of liquids we shall have two coefficients to consider, namely, the coefficient of *real* (or *absolute*) expansion, and the coefficient of *apparent* expansion.

(i) The *coefficient of real* (or *absolute*) *expansion* (γ_r) is the *true* increase in volume per unit volume per unit rise of temperature.

(ii) *The coefficient of apparent expansion* (γ_a) is the *observed* increase in volume per unit volume per unit rise of temperature. γ_a is the value of the coefficient of expansion as we find it when we ignore the expansion of the containing vessel. In finding γ_r we have to take the expansion of the container into consideration.

3-1.2. Relation between coefficients of real and apparent expansion.

Let V_o = the volume of a given mass of liquid at the lower temperature,

V = its *true volume* at a temperature t°C *higher* than the former,

V' = its *apparent volume* at the same *higher* temperature,

γ_r and γ_a = its coefficients of real and apparent expansion,

γ_v = the coefficient of volume expansion of the material of the containing vessel.

Then $V - V_o$ = the real expansion of the liquid,

$V' - V_o$ = the apparent expansion,

and $V_o \gamma_v t$ = the expansion of the vessel.

From the relation

Real expansion = Apparent expansion + Expansion of the vessel we have $V-V_o=(V'-V_o)+V_o\gamma_v t$

Dividing throughout by $V_o t$ we get

$$\frac{V-V_o}{V_o t}=\frac{V'-V_o}{V_o t}+\gamma_v$$

By definition, the term on the left hand side of the equation is the coefficient of real expansion, and the first term on the right hand side is the coefficient of apparent expansion.

$$\therefore\ \gamma_r=\gamma_a+\gamma_v \qquad (3\text{-}1.1)$$

The initial temperature may be $0°C$ or have any other value, This shows that whether we consider the zero coefficient or the mean coefficient we get the result that

The coefficient of real expansion of a liquid = its coefficient of apparent expansion + the coefficient of volume expansion of the material of the container.

[As in the case of solids, if the initial temperature is $0°C$, we call the coefficient the *zero coefficient*. If the initial and final temperatures are $t_1°C$ and $t_2°C$, then the coefficient is the *mean coefficient* between t_1 and t_2. (Note that the zero coefficient is also a mean coefficient — that between $0°C$ and $t°C$. There is no special virtue in taking V_o always as the value at $0°C$, while the temperatures of observation are $t_1°$ and $t_2°$, both different from $0°C$.]

If V_1 = volume of a mass of liquid at $t_1°C$,

V_2 = volume of the same mass of liquid at $t_2°C$,

then the mean coefficient γ between $t_1°$ and $t_2°$ is

$$\gamma=\frac{V_2-V_1}{V_1(t_2-t_1)} \qquad (3\text{-}1.2)$$

$$\text{or}\quad V_2=V_1\{1+\gamma\,(t_2-t_1)\}. \qquad (3\text{-}1.3)$$

γ, so defined, will be the *apparent* or *true* coefficient according as the volumes are the apparent or true volumes of the same mass,

The following tble gives tahe values of mean γ_r of some liquids over a small range around $18°C$, unless otherwise stated. The values are per °C. The expansion coefficient generally increases with rising temperature.

Substance	γ_r	Substance	γ_r
Water (5° to 10°C)	$5{\cdot}3\times10^{-5}$	Mercury	$18{\cdot}1\times10^{-5}$
,, (10° to 20°C)	$15{\cdot}0\times10^{-5}$	Glycerine	47×10^{-5}
,, (20° to 40°C)	$30{\cdot}2\times10^{-5}$	Benzene	122×10^{-5}
,, (40° to 60°C)	$45{\cdot}8\times10^{-5}$	Sulphuric acid (pure)	56×10^{-5}
,, (60° to 80°C)	$58{\cdot}7\times10^{-5}$	Olive oil	70×10^{-5}
Chloroform	127×10^{-5}	Alcohol (Ethyl)	108×10^{-5}
Pentane	155×10^{-5}	Turpentine	96×10^{-5}
Ether	163×10^{-5}		

3.2. Change of density of a liquid on heating. Though expansion of the vessel masks expansion of the liquid, it cannot affect the change in density that a liquid undergoes with rise in temperature. The coefficient of real expansion bears a simple relation to the change in density and may be determined from observations on the latter.

Let V_o = volume of a given mass m of a liquid at the lower temperature.

V = its *true* volume at a temperature t°C *higher* than the colder temperature,

ρ_o = density of the liquid at the lower temperature,

ρ = its density at the higher temperature,

and γ_r = the *mean coefficient of real expansion* between the two temperatures.

Then $\rho_o=\dfrac{m}{V_o}$, $\rho=\dfrac{m}{V}$ and $V=V_o\,(1+\gamma_r t)$;

$$\text{so}\qquad \rho=\frac{m}{V_o(1+\gamma_r t)}=\frac{m}{V_o}.\frac{1}{1+\gamma_r t}\,;$$

$$\text{or}\qquad \rho=\frac{\rho_o}{1+\gamma_r t} \tag{3-2 1}$$

When $\gamma_r t$ is small enough we may write

$$\rho=\rho_o(1+\gamma_r t)^{-1}=\rho_o(1-\gamma_r t) \tag{3-2.2}$$

3.3. Determination of coefficient of apparent expansion of a liquid. This can be easily done with the help of a specific gravity bottle. The method is known as the **weight thermometer method.**

Let M_1 be the mass of the liquid required to fill the bottle at temperature t_1°C and M_2 the mass required to fill the same bottle at t_2°C. As the expansion of the vessel is small compared with that of the liquid, we may disregard it and *assume that the vessel has the same* *volume at both temperatures.* If ρ_1 and ρ_2 are the densities of the liquid at $t_1°$ and $t_2°C$ respectively, then the volume V of the vessel is given by

$$V=\frac{M_1}{\rho_1}=\frac{M_2}{\rho_2}$$

But by Eq. 3-2.1, $\rho_2\{1+\gamma(t_2-t_1)\}=\rho_1$. Here γ should be looked upon as the coefficient of apparent expansion since we have *disregarded* the expansion of the vessel.

$$\therefore\ \frac{M_1}{M_2}=\frac{\rho_1}{\rho_2}=\frac{\rho_2\{1+\gamma_a(t_2-t_1)\}}{\rho_2}\ \text{or}\ \frac{M_1}{M_2}=1+\gamma_a(t_2-t_1)$$

$$\text{or}\quad \gamma_a(t_2-t_1)=\frac{M_1}{M_2}-1\quad\text{or}\quad \gamma_a=\frac{M_1-M_2}{M_2(t_2-t_1)} \tag{3-3.1}$$

M_1-M_2 is the mass of the liquid expelled from the vessel when it is heated from $t_1°$ to $t_2°C$, and M_2 is the mass retained by the vessel at the higher temperature. Hence we may write

$$\gamma_a=\frac{\text{mass expelled}}{\text{mass retained}\times\text{temp. rise}} \tag{3-3.2}$$

The experiment consists in

(*i*) Weighing a dry and clean specific gravity bottle when empty (Mass$=m_1$),

(*ii*) Completely filling it with the liquid, inserting the stopper, wiping it dry and weighing again (Mass$=m_2$).

(*iii*) Noting the temperature of the bottle ($t_1°C$).

(*iv*) Heating the bottle in a water bath which maintains a steady temperature ($t_2°C$) for several minutes. The extra liquid is expelled through the bore in the stopper.

(*v*) Noting the temperature of the bath ($t_2°C$).

(*vi*) Wiping the bottle dry, allowing it to cool, and weighing it again (Mass$=m_3$).

Then $m_2-m_1=M_1$ and $m_3-m_1=M_2$. Eq. 3-3.1 is then applied. This gives the coefficient of apparent expansion.

To get the coefficient of real expansion of the liquid, we may then apply Eq. 3-1.1, if the coefficient of volume expansion of the bottle is known.

Example : A weight thermometer weighs 50 g when empty, 163.13 when filled with glycerine at 0°C and 157.65 g when filled with glycerine at 100°C. Find the coefficient of real expansion of glycerine, given that the coefficient of cubical expansion of glass is 27×10^{-6} per °C.

Solution : From equation 3-3.1, the coefficient of apparent expansion of glycerine

$$= \frac{\text{mass expelled}}{\text{mass retained} \times \text{temp. rise}}$$

$$= \frac{163.13 - 157.65}{(157.65 - 50) \times 100} = \frac{5.48}{107.65 \times 10} = 0.000509 \text{ per } ^\circ C.$$

$$\therefore \text{ Coefficient of real expansion} = 0.000509 + 0.000027$$

$$= 0.000536 \text{ per } ^\circ C.$$

3-4. Determination of the coefficient of real expansion of a liquid : Dulong and Petit's method. This is also known as the *method of balancing columns.* The apparatus is represented in Fig. 3.2. The arms AB and CD of a U-tube are surrounded by a jacket each. The experimental liquid, say mercury, is poured into the U-tube. When the temperatures of the two arms are the same, mercury stands at the same level in both. Cold water is then circulated through one of the jackets and steam through the other. As the temperatures of the mercury columns change, the density changes also. Since under all circumstances the hydrostatic pressure due to one column is equal to that due to the other, the heights of the columns in the two limbs of the U-tube will be different when the temperatures are different. To prevent the flow of liquid from one limb into the other the horizontal part AC of the U-tube is kept cold. The temperatures in the jackets are read off from thermometers T_1 and T_2.

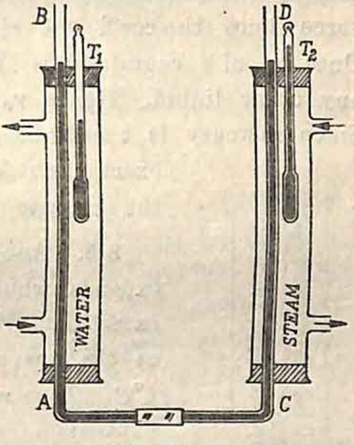

Fig. 3-2

Let $t°C$ = difference of temperature between the hot and the cold limbs,

h_0 and h = heights of the liquid levels in the cold and hot limbs respectively above the axis of the cross tube AC,

ρ_0 and ρ = densities of the cold and the hot liquid columns respectively.

Since the hydrostatic pressures due to the liquid columns balance each other, we have

$$h_o\rho_o g = h\rho g \quad \text{or,} \quad \frac{h}{h_o}=\frac{\rho_o}{\rho}=\frac{\rho(1+\gamma_r t)}{\rho}=1+\gamma_r t.$$

$$\therefore \quad \gamma_r=\frac{h-h_o}{h_o t} \qquad (3.4.1)$$

As in all experiments, there are some sources of error in this experiment too. For example, the temperature in either limb is not constant and equilibrium is never established. Besides, the exposed columns of liquid are at temperatures different from those in the limbs. The arrangement was later modified to minimize such errors.

The determination of γ_r for mercury is a matter of great importance. From a knowledge of this value γ_g for any container may be calculated. This is a better method than taking γ_g to be three times the coefficient of linear expansion, particularly for glass. Once γ_g of a container is known, it may be used to find γ_a for any other liquid. Hence γ_r for the latter may also be found out. Since mercury is the liquid used in pressure gauges and barometers, exact knowledge of its density is necessary for the accurate determination of pressures.

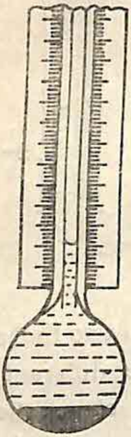

Fig. 3.3

3-5. Anomalous expansion of water. A liquid expands when heated; but water at $0^\circ C$ is an exception. *When heated from $0^\circ C$ water contracts as the temperature rises. This continues to about* $4^\circ C$, above which point a rise in temperature causes expansion. Water, therefore, has maximum density at about $4^\circ C$. An exact knowledge of the temperature is important, since the litre has been defined as the volume of one kilogram of water at its maximum density.

The anomalous expansion of water may be demonstrated by means of a simple experiment. Fig. 3.3 represents a glass vessel with a tube of narrow bore fitted to it. The tube carries a scale. The vessel contains mercury which occupies one-seventh part of the volume of the vessel. The rest is filled with water, which rises to a distance in the tube.

Since the cubical expansion of mercury is seven times that of glass the inner volume of the vessel (not occupied by mercury) will not change when the vessel is heated. It is kept in melting ice for some time, when it acquires the temperature 0°C. The position of the water column in the tube is noted. If the vessel is now slowly heated it will be found that the water column descends at first, showing that water at 0°C contracts on heating. This goes on till 4°C is reached. Beyond that temperature the column ascends.

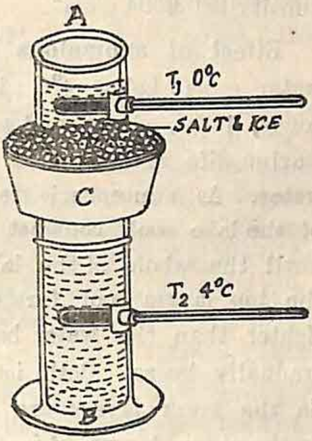

Fig. 3.4

Hope's experiment. The anomalous expansion of water may be demonstrated and temperature at which the density of water is maximum may be determined by a simple experiment due to Hope. A tall glass vessel AB (Fig. 3.4) full of water has a freezing bath C of ice and salt round about its middle. Two thermometers T_1 and T_2 are fitted into AB near about the top and the bottom. Their readings are noted before the freezing mixture is applied. As the freezing mixture cools the water in its neighbourhood, the water becomes cooler and denser and sinks to the bottom. The temperature recorded by the lower thermometer T_2 gradually falls until it reaches about 4°C. By this time the upper thermometer T_1 has shown practically no change. While the reading of T_2 remains steady at 4°C, that of T_1 begins to fall slowly at first, and then more rapidly until 0°C is reached. This shows that water is densest at about 4°C. Careful measurements show that the temperature at which the density of water is maximum is 3.98°C. The manner in which the two thermometers change with time after the freezing mixture has been applied, is represented in Fig. 3.5.

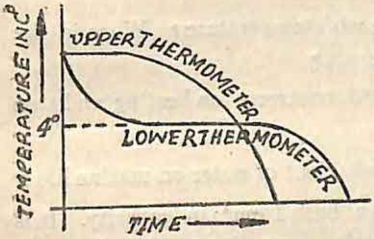

Fig. 3.5

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* The reason for the behaviour of T_1 is as follows : After the water column from the bottom B to the freezing bath C has reached the temperature 4°C, further cooling at C makes the water lighter. It practically stays where it is until with further cooling ice begins to form. Ice being lighter than water rises to the top and melts, thus gradually lowering the temperature of T_1.

The density of water rises from 0.99987 g/cm^3 at $0°C$ to exactly 1 at $4°C$ (by definition). It then diminishes, and at $8°C$ acquires the same value as at $0°C$. The diminution continues ; at $100°C$ the density is 0.9584 g/cm^3.

Effect of anomalous expansion of water on marine life. As water cools below $4°C$ it expands and becomes lighter. When ponds, lakes, rivers and seas in very cold countries freeze in winter, marine life is saved from extinction by the anomalous expansion of water. As atmospheric temperature comes down the upper layers of the lake cool, contract and sink to the bottom. This goes on until the whole of the lake acquires the temperature of $4°C$. When the top layers cool further, the cooler water does not sink as it is lighter than the water below. With further cooling the top layers gradually freeze. Both ice and water are bad conductors of heat. So the lower layers are protected to a great extent against freezing by the upper layers of ice and cold water. The result is that water continues to exist at the bottom though a thickness of ice may have formed at the top. This enables marine life to continue.

Exercises

1. Distinguish between absolute and apparent expansions of a liquid, and find the relation between the corresponding coefficients.

2. How does the density of a liquid change with temperature ? What is the peculiarity in the behaviour of water in this respect ?

3. How would you show that water at $0°C$ contracts on heating and that water has its maximum density at $4°C$?

What is the consequence of this anomolus behaviour of water on marine life ?

4. A mercury in glass thermometer has a bulb 1 cm^3 in capacity. It is desired to make each celsius degree on the scale 5 mm long. Calculate the cross-sectional area of the capillary tube. The coefficient of apparent expansion of mercury in glass is 0.00016 per $°C$. [Ans. : 0.0032 cm^2]

5. What should be the volume of mercury to be put in a glass flask of capacity 720 cm^3 so that the volume above the mercury remains the same at all temperatures ? The coefficient of real expansion of mercury is 0.00018 per $°C$ and the coefficient of cubical expansion of glass is 0.000025 per $°C$. [Ans. : 100 cm^3]

6. The coefficient of expansion of water between 4°*C* and 20°*C* is 0.00015. Calculate the weight of 1 litre of water at 20°*C*. [Ans.: 997.6 g]

A specific gravity bottle is marked "50 c.c., 20°*C*". Say what it means. How can you test its accuracy ? (c.c. = cubic centimetre)

7. A thin copper sphere of diameter 10 cm has a uniform tube of diameter 0.5 cm attached to it. Water fills the sphere and stands at a height of 10 cm in the tube. What will be the change in the level of water in the tube if the sphere is (*i*) *suddenly*, (*ii*) *slowly*, heated from 30°*C* to 60°*C* ? (C. L. E. of copper = 16.7×10^{-6} per °*C*, coeff. of abs. exp. of water = $45 \times 10 \times 10^{-5}$ per °*C*.)

[Ans.: (*i*) Will drop 4 cm. (*ii*) Will rise 32 cm]

8. A column of mercury at 100°*C* is balanced by a column at 0°C. Their heights are 50·90 cm and 50.00 cm respectively. Calculate the coefficient of real expansion of mercury. [Ans.: 0.00018]

9. How much mercury must be placed inside a glass flask, having an internal volume of 300 cm^3, so that the volume of the remaining space inside the flask may be constant at all temperatures ?

(Coefficient of cubical expansion of mercury = 0.00018 per °*C* ;
Coefficient of linear expansion of glass = 0.000009 per °*C*)

[**Hint :** Expansion of mercury must be equal to the expansion of the flask for the same rise of temperature. If V is the volume in cm^3 of the mercury, then the expansions are $V \times 0.00018 \times t$ for mercury and $300 \times 3 \times 0.000009 \times t$ for the flask, whence $V = 45$.]

CHAPTER

4

EXPANSION OF GASES

4-1. Gases expand much more than solids or liquids for a given rise of temperature. Moreover a gas expands when its pressure is reduced. In determining the effect of temperature on the volume of a given mass of gas, it is therefore necessary to keep the pressure of the gas constant. Such a restriction is not necessary for solids or liquids, as a change of pressure causes but an inappreciable change of volume in them.

In discussing the expansion of gases it should be remembered that **the volume V of a given mass of a gas depends both on its temperature t and pressure P.** When the temperature is constant the relation between V and P is given by Boyle's law (Sec. 4-5).

To show that a gas expands when heated. Fit up a round bottomed flask with a rubber cork through which one end of a narrow glass tube projects a little into the flask. Invert the flask and introduce the other end into a beaker of water as in Fig. 4-1. On gently heating the flask with a flame, air bubbles will be seen to come out. The flame heats up the air inside the flask causing expansion of air within the flask, which escapes in the form of bubbles. On allowing the flask to cool the air contracts and some water is sucked up into the tube.

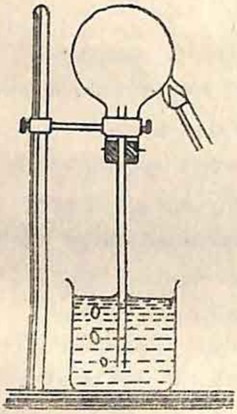

Fig. 4.1

4-2. Charles' law. The relation between volume and temperature when the pressure is kept constant is known as Charles' Law. **The coefficient (α_p) of increase in volume at constant pressure** is defined as the fraction of the volume at $0°C$ by which the volume of a given mass of gas increases for a rise in temperature of $1°C$, the pressure remaining constant. If V_o= volume of a fixed mass of gas at 0°C, and V= its volume at t°C, both at the same pressure,

then
$$\alpha_p=\frac{V-V_o}{V_o t}, \tag{4-2.1}$$

which gives
$$V=V_o(1+\alpha_p t). \tag{4-2.2}$$

Charles showed that *the coefficient of increase in volume of all gases at constant pressure is the same.* This statement is referred to as Charles' or Gay-Lussac's law. (Gay-Lussac discovered the law independently a few years after Charles.) α_p is often called the **volume coefficient.**

According to Regnault, who measured the coefficient accurately,

$$\alpha_p=1/273 \text{ per } ^\circ C \text{ or } 0\cdot00366 \text{ per } ^\circ C. \tag{4-2.3}$$

In the light of Regnault's value for α_p **Charles' law** may be restated as follows :

For a fixed mass of any gas heated at constant pressure, the volume increases by 1/273 of its volume at 0°C for each celsius degree rise in temperature.

4·3. The absolute scale of temperature. Since for a gas α_p= 1/273 per °C, its volume V_t at any temperature t°C is given by

$$V_t=V_o(1+t/273)$$

Thus the volume at 1°C is $V_1=V_o(1+1/273)$,

the volume at −30°C is $V_{-30}=V_o(1-30/273)$ etc.

If α_p remains the same whatever the temperature, then at a temperature of −273°C,

$$V=V_o(1-273/273)=0$$

At a temperature below −273°C the volume would be *negative.* Since we cannot imagine a negative volume, temperatures lower than −273° will have no sense to us. We therefore take the temperature at which the *calculated* volume of a gas becomes zero, to be the lowest temperature possible and call this temperature the **absolute zero.** Any scale of temperature measured from the absolute zero is called an **absolute scale of temperature.** On the celsius (centigrade) scale, the absolute zero will be taken as −273°C.

The above is not, however, a happy way of defining the absolute zero. When you reach a higher stage of study, you will learn that *Nature has put a limit the lowest temperature possible in the universe. This limit is called the absolute zero.* Careful experiments show that the value is about −273·15°C. The agreement of this value with the one at which the calculated volume of a gas becomes zero is *accidental* and is due to the fact that most permanent gases (such as H, He) behave practically as perfect gases (*i.e.*, satisfy Boyle's law and Charles' law with a high degree of accuracy.)

The absolute scale of temperature in which there are 100 degrees between the ice point ($0°C$) and the steam point ($100°C$), *i.e.*, the absolute scale in which each degree *interval* is equal to a degree interval on the celsius scale, is called the **Kelvin scale.** The readings on this scale are indicated as *degrees Kelvin*, or in symbols, $°K$ (now merely as *Kelvin*, with K as the symbol. The degree sign has been dropped.) It should be noted that

a difference of temperature of $1°K$

$=$ a difference of temperature of $1°C$

For ordinary purposes, we take the absolute zero on the celsius scale as $-273°C$, the ice point on the Kelvin scale as $273°K$ and any temperature $t°C$ as $(273+t)°K$, *i.e.* $t°C=(273+t)°K$ (4-3.1)

4-4. Another form of Charles' law. From Charles' law, we have for a given mass of gas at constant pressure,

$$V=V_o(1+t/273)=V_o(273+t)/273 \tag{4-4.1}$$

where V is the volume of the given mass at $t°C$ and V_o, that at $0°C$.

Now $t°C=(t+273)°K=T°K$ (say),
while $0°C=273°K=T_o°K$ (say).
$\therefore$ From Eq. 4-4.1, $V=T(V_o/T_o)$

$$\text{or} \quad \frac{V}{T}=\frac{V_o}{T_o} \tag{4-4.2}$$

Since V_o has a fixed value for the given mass, and T_o is a constant, V_o/T_o is also constant. We can therefore say that for a given mass of gas V/T is a constant, *i.e.*, *at constant pressure the volume of a given mass of gas is directly proportional to its absolute temperature.* This assertion may also be taken as the statement of Charles' law. In symbols, $V \propto T$ when P is constant.

Strictly speaking, the temperature T as defined here is called the temperature on the **perfect gas scale**, or briefly the *gas scale.* The kelvin scale is defined in Thermodynamics. The definition is different from the one given here. But the perfect gas scale and the kelvin scale have been found to be *identical.* So we need not differentiate between the kelvin scale and the perfect gas scale at this stage.

4-5. Boyle's law. Gases are highly compressible. Consider a mass of gas confined in a cylinder by an air-tight piston. If the pressure on the gas is increased by pushing the piston inwards, the volume of the gas decreases. As the pressure is reduced, the volume increases.

The relation between the pressure (P) and volume (V) of a fixed mass of gas was discovered by Robert Boyle (1627-1691), an English philosopher. It is known as **Boyle's law** and may be stated as follows :

For a fixed mass of gas at a given temperature, the volume varies inversely as the pressure, *i.e.*, the product of the pressure and volume is constant.

In symbols, $PV=\textbf{constant}=k.$ (4-5.1)

If P_1 and V_1 are the initial values of the pressure and volume and P_2 and V_2 the final values, we shall have

$$P_1V_1=P_2V_2=k. \quad (4\text{-}5.2)$$

Examples. (1) The volume of a mas of gas at 740 mm pressure is 1250 cm^3. Find its volume at 760 mm, if the temperature is unchanged.

Solution : Let V be the new volume. Then from the relation $P_1V_1=P_2V_2$ we have $V\times 760$ mm $=740$ mm $\times 1250$ cm^3 or $V=1217$ cm^3.

(2) A bubble of gas, 100 mm^3 in volume, is formed at a depth of 100 metres of water. Find its volume when it reaches the surface, the atmosperic pressure being 76 cm. Assume unchanged temperature.

Solution : Atmospheric pressure $=76$ cm of mercury $=76\times 13.6$ cm of water $=$ 10.34 metres of water. This is the final pressure P_2 of the bubble. The initial pressure $P_1=(100+10.34)=110.34$ metres of water. If V is the final volume, then $V\times 10.34$ metres of water $=100$ mm$^3\times 110.34$ metres of water, whence $V=1067$ mm^3.

The value of the constant k (Eq.s 4-5.1 and 4-5.2) depends **upon**

(i) the **mass** of the gas taken, (ii) its **temperature** and (iii) the **units** in which P and V are expressed.

Examples. (i) At a pressure of one atmosphere 32 g of oxygen occupy a volume of 22·4 litres when the temperature is 0°C. Therefore, for 32 g of oxygen at 0°C the product

$PV=k=22{\cdot}4$ litres $\times$ 1 atmosphere $=22{\cdot}4$ litre-atmospheres.

Since 1 litre $=1000$ cm^3 and one atmosphere $=1{\cdot}013\times 10^6$ dyn/cm^2, the same product is also equal to

$22{\cdot}4\times 1000$ cm$^3\times 1{\cdot}013\times 10^6$ dyn/cm$^2=22{\cdot}69\times 10^9$ erg.

(ii) If we consider 16 g of oxygen at 0°C, the volume is 11·2 litres at a pressure of 1 atmosphere,

$\therefore$ for 16 g of oxygen at 0°C the product

$PV=k=11{\cdot}2$ litre-atmospheres $=11{\cdot}35\times 10^9$ erg.

(*iii*) At 27°C the volume of 16 g of oxygen is 12·3 litres if the pressure is 1 atmosphere. Under these conditions the product

$PV=k=12{\cdot}3$ litre-atmospheres $=12{\cdot}46\times10^9$ erg.

At a given temperature, the product

$PV=k$ is directly proportional to the mass of the gas taken.

4-5.1. Density and pressure.

Let ρ_1 and ρ_2 be the densities of a gas at pressures P_1 and P_2, and V_1 and V_2, the volumes of the same mass m of the gas at these pressures, the temperature remaining constant. Then

$$m=V_1\rho_1=V_2\rho_2.$$

By Boyle's law $P_1V_1=P_2V_2$.

Dividing the first relation by the second, we have

$$\frac{\rho_1}{P_1}=\frac{\rho_2}{P_2}$$

which means that *the density of a gas is proportional to its pressure, so long as the temperature is constant.*

Example. Find the mass of 1 cubic metre of air at 0°*C* and 19 mmHg pressure given that the density of air at S.T.P. is 0.001293 g/cm³.

Solution : If ρ is the density at 19 mmHg pressure, then

$$\frac{\rho}{19\text{ mmHg}}=\frac{0{\cdot}001293\text{ g/cm}^3}{76\text{ cmHg}}$$

$$\text{or}\quad \rho=\frac{0{\cdot}001293}{76}\times1.9\text{ g/cm}^3$$

$$\therefore\ \text{mass of 1 cu. metre}=\frac{0.001293}{76}=1{\cdot}9\ \frac{\text{g}}{\text{cm}^3}\times(100)^3\text{ cm}^3=32.3\text{ g}.$$

4-6. The ideal or perfect gas equation.

A gas which obeys both Boyle's Law and Charles' law strictly is called a *perfect or ideal gas*. Permanent gases (such as hydrogen, helium, oxygen and nitrogen) obey the laws with sufficient accuracy to be treated as perfect gases. The two laws may be combined to give a single relation.

Let the initial pressure, volume and absolute temperature of a given mass of gas be P_1, V_1 and T_1.

(i) Keeping the pressure P_1 constant, let the absolute temperature be changed to T_2, and the new volume to V'. According to Charles' law $V'=V_1T_2/T_1$.

(ii) Keeping the temperature T_2 constant, let the pressure be changed to P_2. Then, according to Boyle's law, the new volume V_2 is such that

$$P_2V_2 = P_1V' = \frac{P_1V_1}{T_1}T_2$$

$$\text{or} \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \text{a constant} \tag{4·6.1}$$

This is *the equation of state* for an ideal gas*, and is also called the *ideal or perfect gas equation.* It contains both Charles' law and Boyle's law. For if $T_1 = T_2$, we get $P_1V_1 = P_2V_2$, which is Boyle's law. And if $P_1 = P_2$, we get $V_1/T_1 = V_2/T_2$, which is Charles' law.

We may write Eq. 4-6.1 in the form

$$PV = RT \tag{4-6.2}$$

in which P stands for the pressure, V for the volume and T for the absolute temperature of a *given mass* of gas. The value of R will depend on the mass. If the mass is doubled, R will also be doubled. For a gram-molecule of *any gas* R has the same value. This value of R (for a gram-molecule) is variously called the **universal gas constant, gram-molecular gas constant** or **molar gas constant.** We shall denote it by R_M.

4-6.1. Value of the molar gas constant. Let us consider a mole (gram-molecule) of a gas at S.T.P. (standard temperature and pressure). Then we have

$P_o = 1$ standard atmosphere $= 1{\cdot}013 \times 10^6$ dyn/cm^2,

$T_o =$ temperature of melting ice $= 273^\circ K$

$V_o =$ volume at S.T.P. of one mole of a gas (supposed perfect)
$= 22{\cdot}4$ litres.

$$\therefore \quad R_M = \frac{P_oV_o}{T_o} = \frac{1{\cdot}013 \times 10^6 \text{ dyn/cm}^2 \times 22400 \text{ cm}^3\text{/mole}}{273^\circ K}$$

$$= 8{\cdot}31 \times 10^7 \text{ erg per } ^\circ K \text{ per mole (erg } ^\circ K^{-1} \text{ mole}^{-1})$$

If the mass of the gas taken is n moles, then $PV = nR_MT$.

If m is the mass of the gas in grams and M its molecular weight, then

$$PV = mR_MT/M. \tag{4-6.3}$$

For unit mass, therefore, $R = R_M/M$. (4-6.4)

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* The relation between the volume, temperature and pressure of a substance is called its **Equation of state.**

For different masses of the same gas, we get from Eq. 4-6.3,

$$\frac{P_1V_1}{m_1T_1}=\frac{P_2V_2}{m_2T_2}=\frac{R_M}{M}=R \tag{4-6.5}$$

Eq. 4-6.1 or 4-6.2 enables us to solve problems when any two of the quantities pressure, volume and temperature of a fixed mass of gas change at the same time. Eq. 4-6.5 is useful when masses differ (see Sec. 4-8).

Examples. (1) A litre of air is heated from 27°C to 177°C at constant pressure. Find its volume.

Solution : From Charles' law that $\frac{V_1}{T_1}=\frac{V_2}{T_2}$,

$T_1=273+27=300°K$, $T_2=273+177=450°K$,
$V_1=1$ litre, whence $V_2=1.5$ litres.

(2) The pressure of a given mass of gas at 33°C is 75 cm of mercury. Find the temperature in °C at which the pressure is doubled, the gas being heated at constant volume. Find also the temperature at which the pressure is halved.

Solution : Apply equation 4-6.1. Since volume remains constant, $V_1=V_2$.
(*i*) $T_1=273+33=306°K$. If $P_2=2P_1$, $T_2=P_2T_1/P_1=2\times306°K$
$=612°K=(612-273)°C=339°C$.
(*ii*) When $P_2=P_1/2$, $T_2=(P_2/P_1)\times306°K=153°K$
$=(153-273)°C=-120°C$.

(Note that since the ratio P_2/P_1 is given, the value of P_1 is unnecessary.)

(3) The volume of a mass of gas at 47°C and a pressure of 75 cm of mercury is 640 cm³. Find the volume at *S.T.P.*

Solution : Here $P_1=75$ cm of mercury, $V_1=640$ cm³, $T_1=273+47=320°K$, $P_2=76$ cm of mercury, $T_2=273°K$; to find V_2.

∴ From the relation $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$, $V_2=\frac{P_1}{P_2}\cdot\frac{T_2}{T_1}\cdot V_1$

∴ $V_2=\frac{75}{76}\times\frac{273}{320}\times640=539$ cm³ (approximately)

(4) Calculate the value of the gas constant for 1 g of hydrogen, given that the density of hydrogen at S.T.P. $=0.00009$ g/cm³.

Solution : Here $P=76\times13.6\times980=1.013\times10^6$ dyn/cm².

$$V=\frac{1}{0.00009}\,\frac{\text{cm}^3}{\text{g}};\quad T=273°K.$$

$$\therefore \text{ The required value}=\frac{PV}{T}=\frac{1.013\times10^6\text{ dyn}}{273\times0.00009\text{ cm}^2}\times\frac{\text{cm}^3}{\text{g}}\times\frac{1}{°K}$$
$$=4.12\times10^7 \text{ erg per }°K\text{ per g (erg }°K^{-1}\text{g}^{-1}).$$

4-7. Conversion of densities from one set of conditions to another. Since the gas equation applies to a given mass, say *m* g of a gas, the density $\rho_1 = m/V_1$. Under the new set of conditions $\rho_2 = m/V_2$. Then from the relation

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}, \text{ we get } \frac{P_1 m}{\rho_1 T_1} = \frac{P_2 m}{\rho_2 T_2}$$

$$\text{or, } \frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2} \;\; i.e., \;\; \frac{P}{\rho T} = \text{Constant} \qquad (4\text{-}7.1)$$

Example. A litre of dry air at S.T.P. weighs 1.293 g. What would be the mass of 3 litres at 115°C and a pressure of 4 atmospheres ?

Solution: Here $P_1 = 1$ atmosphere, $T_1 = 273°K$, $\rho_1 = 1.293$ g/litre and $P_2 = 4$ atmospheres, $T_2 = 273 + 115 = 388°K$. First find ρ_2.

From Eq. 4-7.1 $\rho_2 = \frac{P_2}{P_1} \cdot \frac{T_1}{T_2} \cdot \rho_1 = 4 \times \frac{273}{388} \times 1.293$ g/litre

$\therefore$ mass $= V_2 \rho_2 = 3 \times 4 \times \frac{273}{388} \times 1.293$ g $= 10.9$ g.

Or, the volume might first be reduced to S.T.P. Let V be the volume of the given mass of air at S.T.P. Since

$$\frac{PV}{T} = \frac{P_0 V_0}{T_0}, \text{ we shall have } \frac{1 \times V}{273} = \frac{4 \times 3}{388} \text{ or } V = \frac{12 \times 273}{388} \text{ litres,}$$

whence mass $= V \times 1.293 = 10.9$ g.

4-8. Gas law for different masses. Let M be the molecular weight of a gas and R_M the value of the molar gas constant (Sec. 4-6). Then the value of the gas constant for unit mass will be R_M/M. If V_1 is the volume of a mass m_1 of the gas at pressure P_1 and temperature T_1, and V_2 the volume of a mass m_2 of the same gas at P_2 and T_2, then from Eq. 4-6.2 we shall have

$$\frac{P_1 V_1}{T_1} = \frac{m_1 R_M}{M} \quad \text{or,} \quad \frac{P_1 V_1}{m_1 T_1} = \frac{R_M}{M}$$

$$\text{and} \quad \frac{P_2 V_2}{T_2} = \frac{m_2 R_M}{M} \quad \text{or,} \quad \frac{P_2 V_2}{m_2 T_2} = \frac{R_M}{M}$$

$$\therefore \quad \frac{P_1 V_1}{m_1 T_1} = \frac{P_2 V_2}{m_2 T_2}. \qquad (4\text{-}8\ 1)$$

This equation (the same as Eq. 4-6.5) may be applied to cases where the masses of the gas are different. Note that this is also the same as Eq. 4-7.1 for $m_1/V_1 = \rho_1$ and $m_2/V_2 = \rho_2$.

Example. A gas cylinder contains 20 kg of air at a pressure of 12 atmospheres. In driving a pneumatic drill for some time, the pressure falls to 10 atmospheres. How much gas has been used ?

Solution : We may apply Eq. 4-8.1. Here $V_1 = V_2$, $T_1 = T_2$, $P_1 = 12$ atmos, $P_2 = 10$ atmos and $m_1 = 20$ kg. To find m_2.

From Eq. 4-8.1, $\frac{12 \text{ atmos}}{20 \text{ kg}} = \frac{10 \text{ atmos}}{m_2}$ or $m_2 = \frac{200}{12} = 16\frac{2}{3}$ kg.

The quantity of gas used $= m_1 - m_2 = 3\frac{1}{3}$ kg.

4-9. Increase of pressure with rise of temperature at constant volume. If the volume remains constant, it follows from the perfect gas equation that

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} = \text{constant}, \tag{4-9.1}$$

i.e., the pressure of a perfect gas at constant volume is directly proportional to its absolute temperature. This shows that when a given mass of gas at constant volume is heated, its pressure increases. Experiment shows that *at constant volume the pressure of a given mass of gas increases by a constant fraction of its pressure at 0°C for each degree celsius rise of temperature.* This fraction is called the **pressure coefficient** (α_v) of the gas *at constant volume.*

If P_0 = pressure of a given mass of gas at 0° C,

P = pressure at temperature t° C,

then the fractional increase of pressure $= (P - P_0)/P_0$, and the fractional increase of pressure per 0° C rise of temperature (*i.e.* α_v)

$$= (P - P_0)/P_0 t \quad \text{or} \quad P = P_0(1 + \alpha_v t) \tag{4-9.2}$$

It is further found from experiment that α_v *is approximately equal to* 1/273 *per* °C *for practically all gases.*

Example. A gas is at 27°C. To what temperature must it be heated at constant volume so that the pressure is doubled ?

Solution : Let the initial pressure be P_1. The initial temperature is $T_1 = 27 + 273 = 300$°K. The final pressure $P_2 = 2P_1$. The final temperature T_2 (in °K) will be given by Eq. 4-9.1. Therefore,

$P_1/300 = 2P_1/T_2$, whence $T_2 = 600$°K or $600 - 273 = 327$°C.

4-9.1. To show that for a perfect gas the pressure and volume coefficients are equal ($\alpha_v = \alpha_p$). Perfect gases satisfy the relation PV/T = constant. It may be shown that α_p and α_v for such gases are the same.

Suppose a fixed mass of a perfect gas has a volume V_o and a pressure P_o at 0° C. If the pressure be kept constant and the temperature changed to t° C, its volume V will be $V = V_o(1 + \alpha_p t)$. Now keep the temperature constant at t° C, but increase the pressure from P_o to

P so that the volume V diminishes and reaches the former value V_o. Then from Boyle's law we get $P_oV=PV_o$ (the temperature being $t°C$).

Hence, we shall have $P_oV_o(1+\alpha_pt)=PV_o$

$$\text{or,} \qquad P=P_o(1+\alpha_pt) \qquad \text{(A)}$$

The changes described above are such that the new temperature is $t°C$ and the pressure is P, while the volume is unchanged. As the pressure rises from P_o at $0°C$ to P at $t°C$ at constant volume, we have, from the definition of α_v,

$$P=P_o(1+\alpha_vt) \qquad \text{(B)}$$

Hence, from the relations (A) and (B) we find that $\alpha_v=\alpha_p$.

Values of α_p and α_v for some gases are given in the table below. They are per °C.

Gas	α_p	α_v
Air	0.00367	0.00367
H_2	·00366	.00366
N_2	·00367	.00367
O_2	·00367	.00367
He	·00366	.00366
CO_2	·00374	.00372

As the initial pressure is reduced it is found that the values of α_p and α_v for all gases (even of CO_2) approach the same value. This *limiting value* is found to be 0·0036608 = 1/273·15 per °C.

Exercises

1. State Charles' law and show how it gives rise to the idea of an absolute zero of temperature. What is an absolute scale of temperature?

2. (*a*) A certain mass of gas is at 40°C. Calculate the temperature at which the volume will be doubled, pressure remaining the same.

(*b*) In the above example, if the volume is kept constant, what will be the temperature if the pressure increases to three times its initial value?

[Ans.: 353°C; 666°C]

3. Write down the ideal gas equation and clearly state the meaning of each symbol.

4. (*a* A gas occupies 50 c.c. at 67°C and pressure 70 cm. of mercury. If the volume becomes 40 c.c. at 17°C, what is the new pressure?

(*b*) A gas occupies 125 c.c. at 60°C and 75 cm of mercury. If the pressure be increased to 80 cm at temperature 30°C, what is the new volume?

[Ans.: 74.6 cm of mercury; 106.5 cm^3.]

5. A thin glass bulb is sealed at 27°C, the inside pressure being 1 atmosphere. The maximum internal pressure the bulb can withstand is 95 cm of mercury. At what temperature will the bulb burst? [Ans.: 102°C]

6. A bicycle tyre contains air at a pressure of 2 atmospheres at 30°C. If the temperature rises 40°C, what should be the pressure of air in the tyre, the volume remaining constant. [Ans.: 2.07 atmos.]

7. At constant pressure 1 litre of a certain gas expands by 128 cm³ when heated from 0°C to 35°C. Calculate from these data the value of the absolute zero. [Ans.: −273°C.]

8. Compare the density of air at 10°C and 750 mm pressure with that at 15°C and 760 mm pressure. [Ans.: $\rho_{10} : \rho_{15} = 1 : 1.004$]

9. A litre of gas weighs 1.562 g at 0°C under a pressure of 76 cm of mercury. The temperature rises to 250°C, and the pressure to 78 cm of mercury. What is the weight of one litre of gas under the new condition ? [Ans.: 1·47 g/litre.]

10. A 10-litre vessel containing air at one atmosphere is heated from 0°C to 100°C. What will the pressure be if the vessel was a closed one ? What fraction of the mass of air will escape if the vessel was open to the atmosphere ?

[Ans.: 1.37 atmos.; 100/373]

11. A glass globe of capacity 1 litre is heated at sea-level from 0°C to 100°C with its mouth open. Find the mass of air expelled thereby. Density of air at $N.T.P.$ = 0·001293 g/cm³. [Ans.: 0·3466 g.]

12. At constant pressure the volume of a certain gas increases in the ratio 1 : 1.035 between 15°C and 25°C. Calculate from these data the value of the absolute zero. [Ans.: −270°C.]

13. Mention and briefly explain the gas laws connecting pressure, volume and temperature of gases, Deduce from these the general law PV/T = Constant.

14. Name and state the laws connecting (*a*) volume and temperature, (*b*) volume and pressure, in a gass. Deduce from them; showing clearly the steps in your proof, the law connecting the pressure and temperature at constant volume.

A glass vessel contains air at 60°C. To what temperature must it be heated to expel one-third of the air, the pressure remaining constant ?

15. State the two fundamental laws of change of pressure, volume and temperature of gases and show that they may be expressed in the form of a single equation.

The density of oxygen at N.T.P. is 1.429 g/litre. A certain mass of the gas is enclosed in a cylinder, whose volume is 2.5 litres, under a pressure of 780 mm at a temperature 27°C. What is the mass of the gas in the cylinder ?

[Ans.: 3.336 g.]

16. State Boyle's law aad Charles's law. Deduce a relation between the volume, the temperature and the pressure of a given mass of gas, on the basis of these two laws.

A gas at 13°C has its temperature raised at constant pressure so that its volume is doubled. What is the final temperature ? [Ans.: 299°C.]

CHAPTER

5

CALORIMETRY

5-1. Heat is a measurable quantity. In this chapter we shall consider how to measure heat and a few other thermal quantities. Such measurements form the subject matter of *calorimetry*.

Units of heat. Before we can measure a quantity it is necessary to fix upon a unit in terms of which it can be expressed. The unit of heat is defined as the quantity of heat required to raise the temperature of unit mass of water by unity. This gave rise to three different units, *viz.*,

(i) the **calorie**, which is the unit of heat in the cgs system. *It is the quantity of heat required to raise the temperature of one gram of water one degree centigrade.* A *kilocalorie* (also called a *large calorie*) is 1000 calories.

(ii) the **British thermal unit** (abbreviated Btu), which is the unit of heat in the fps system. It is defined as the *heat required to raise the temperature of one pound of water one degree fahrenheit.*

A commercial unit of heat is the **therm.**

1 therm = 100,000 Btu.

(iii) the **pound-degree-centigrade unit** or *centigrade heat unit* (abbreviated c.h.u. or lb °C), which *raises the temperature of one pound of water by* 1°C.

It is clear from the definitions that the heat required to raise m grams of water through $t^\circ C$ is $m \times t$ calories, while H calories will raise the temperature of m grams of water by $H/m\ ^\circ C$. We are thus led to the following relations between the units :

$$1 \text{ Btu} = 1 \text{ lb} \times 1^\circ F = 453.6 \text{ g} \times \tfrac{5}{9}{}^\circ C = 252 \text{ calories};$$

$$1 \text{ lb}\,^\circ C = 1 \text{ lb} \times 1^\circ C = 453.6 \text{ g} \times 1^\circ C = 453.6 \text{ calories}.$$

With improvement in the accuracy of measurement, we gradually came to know that water does not require the same quantity of heat for a one-degree rise at different temperatures. So, in defining the heat units, it became necessary to state at what temperature this rise should take place. The present *calorie* (symbol, cal) was defined as :

The calorie is the quantity of heat required to raise the temperature of one gram of pure water from 14·5° C to 15·5° C (nder a pressure of one atmosphere*). It is called the *15°-calorie*.

Heat is energy, and mechanical or electrical energy can be measured with much greater accuracy than heat. It was therefore decided at the Ninth General Conference of Weights and Measures (1948) to use the *joule* (J) *as the unit of heat*. The 15°-calorie was found to be equal to 4·1855 J. This conference not only discouraged the use of calorie as a heat unit, but also changed the name of the centigrade scale to the celsius scale. Though all international bodies have accepted the recommendations, many authors and scientists have not given up the use of the words 'centigrade' and 'calorie'. So you should be prepared to come across both the older and newer terms.

West Germany has passed a law to the effect that the joule (J) should be used instead of the calorie even for domestic purposes—such as in stating the calorific value of food or fuel.

5-2. Specific heat. The specific heat is the quantity of heat required to raise the temperature of unit mass of a substance by one degree. In the cgs system, *the specific heat of a substance is the heat in calories required to raise the temperature of* 1 *gram of the substance through* 1° *C*. The unit in which to express specific heat is *calories per gram per* °*C*. In symbols, we write it as cal/g °C or cal $g^{-1\circ}C^{-1}$. For 'cal', you may find J in some books.

[The term **specific heat capacity** is now preferred to the old term 'specific heat'.]

Specific heat was once defined as the ratio of the heat required to raise the temperature of a given mass of the substance through any range to the heat required to raise the temperature of an equal mass of water through the same range. Or, to be more precise,

$$\text{Specific heat} = \frac{\text{heat to raise the temp. of unit mass of substance by unity}}{\text{heat to raise the temp. of unit mass of water by unity}}$$

As the ratio of two similar quantities is a pure number, specific heat, according to this definition, is a pure number. In the cgs system the denominator is evidently 1 calorie. Thus if s be the specific heat of a substance,

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* Increase of pressure slightly lowers the specific heat. Mention of pressure is necessary for high precision work.

the heat required to raise 1 g of the substance by 1°C
$= s \times 1$ calorie $= s$ calories.

(This second definition, an old one, is no longer encouraged. The first definition is simpler and more meaningful. The second depends on it. Note that the two definitions are similar to those of density and specific gravity, the latter being the ratio of two densities. For simplicity of writing, sp. heat is written in many cases as a pure number. This means that the second definition is being used.)

We shall generally use cgs units. From both definitions of specific heat it follows that if s be the specific heat of a substance, heat required to change the temp. of 1 g of a substance by $1^\circ C = s$ calories ;
heat required to change the temp. of m g of a substance by $1^\circ C = ms$ calories ;
heat (Q) required to change the temp. of m g of a substance by $t^\circ C = mst$ calories.

In symbols, $$\mathbf{Q = mst} \qquad (5\text{-}2.1)$$

Heat gained = mass × specific heat × temperature rise.
Heat lost = mass × specific heat × temperature fall.

Eq. 5-2.1 is the *fundamental equation* in calorimetry.

These relationships also apply to the other units. Thus if s is the specific heat given as a number, Q will be in Btu if m is in lb, and t in °F. When m is in lb and t is in °C, Q will be in centigrade heat units (lb °C).

Table. Specific heats (in calories per gram per °C)

Substance	Sp. heat	Substance	Sp. heat	Substance	Sp. heat
Aluminium	0·210	Nickel	0·109	Glass	0·12 – .19
Copper	0·091	Platinum	0·032	Ice	0·502
Gold	0·030	Silver	0·056	Marble	0·22
Iron	0·105	Tin	0·054	Castor oil	0·508
Lead	0·030	Zinc	0·092	Olive oil	0·47
Mercury	0·033	Brass	0·088	Turpentine	0·42

Examples. (1) If the specific heat of iron is 0.1 cal/g °C how much heat will 100 g of iron require to be heated from 30°C to 100°C ?

Solution : From equation 5-2.1, we have

heat gained = mass × sp. heat × temp. rise

$$= 100\ \text{g} \times 0.1\ \frac{\text{cal}}{\text{g}^\circ\text{C}} \times (100 - 30)^\circ\text{C} = 700\ \text{cal.}$$

(2) If the sp. heat of ice is 0.5 cal/g °C how much heat will 4 kg of ice give up in cooling from 0°C to -10°C ?

Solution : Since heat lost = mass × sp. heat × fall in temp., we have

$$\text{heat lost} = 4000\ \text{g} \times 0.5 \frac{\text{cal}}{\text{g}^\circ\text{C}} = 10^\circ\text{C} = 20{,}000\ \text{cal.}$$

(3) In cooling from 100°C to 20°C a mass of 50 g of brass gives up 360 calories of heat. Find the specific heat of brass.

Solution : If s is the required specific heat we have from relation 5-2.1

$$360\ \text{cal} = 50\ \text{g} \times 80^\circ\text{C} \times s$$

$$\therefore\ s = \frac{360}{50 \times 80} \cdot \frac{\text{cal}}{\text{g}^\circ\text{C}} = 0{\cdot}09\,\frac{\text{cal}}{\text{g}^\circ\text{C}}$$

(4) Express a specific heat of 0.1 cal/g °C in fps units.

Solution : Let $0.1\ \text{cal g}^{-1}\ {}^\circ\text{C}^{-1} = x\ \text{Btu lb}^{-1}\ {}^\circ F^{-1}$.

$$\text{Then } x = 0.1 = \frac{\text{cal}}{\text{Btu}} \times \frac{\text{lb}}{\text{g}} \times \frac{{}^\circ F}{{}^\circ\text{C}}$$

$$= 0.1 \times \frac{1}{252} \times 453.6 \times \frac{5}{9} = 0.1$$

N.B. In the same way you can show that this specific heat is also equal to 0.1 c.h.u/lb°C.

5-2.1. Experiment to show difference in specific heats :

The fact that different substances differ in the values of their specific heats may be demonstrated by means of a simple experiment. Take a number of spherical balls of different materials but of the *same mass.* Heat them together in boiling water and place them on a thick sheet of wax. It will be seen that different balls penetrate to different depths within the wax. The reason for it lies in the difference in the heats given out by the balls in cooling from the temperature of boiling water to that of melting wax. The ball that gives up more heat melts more wax and penetrates deeper. Heat given out by any of them is equal to its mass × specific heat × fall in temperature. Since the mass and the fall in temperature are the same for all, their specific heats must be different.

5-3. Thermal capacity (or heat capacity) and water-equivalent.

The **thermal capacity** *of a body is defined as the heat required to raise the temperature of the body by* 1°C. If m be the mass of the body in grams and s the specific heat of its material (in cal/g°C) it required $Q = mst$ calories for a rise of t°C. Then its heat capacity C = heat required for 1°C rise of temperature $= Q/t = ms$. In symbols,

$$C = ms\ \text{cal}/^\circ\text{C} \tag{5-3.1}$$

It is clear from the above that **specific heat (s) (specific heat capacity)** is the **thermal** (or heat) **capacity per unit mass.**

The **water equivalent** *of a body is defined as the mass of water in grams which will be heated through* 1°C *by the heat that raises the temperature of the body itself by* 1°C. If m is the mass of the body in grams and s its specific heat, the body requires ms calories in order that it may be heated by 1°C. Since 1 calorie heats 1 g of water by 1°C, ms calories will heat ms grams of water by 1°C. Therefore, the *water equivalent W of the body is ms grams.*

$$W = ms \text{ grams} \qquad (5\text{-}3.2)$$

From equations 5-3 1 and 5-3.2 it is clear that the numerical values of the thermal capacity and the water equivalent of a body are the same, but they are expressed in *different units.*

When the water equivalent W of a body is known, it follows from the definition of this quantity that the heat Q which the body requires for a rise of temperature of t°C is

$$Q = Wt \qquad (5\text{-}3.3)$$

Since the specific heat of water is unity.

[The use of the term 'water equivalent' is no longer encouraged. The term 'heat capacity' is used instead.]

5-4. The fundamental principle of calorimetry. In most experiments on calorimetry bodies at different temperatures are brought into close contact inside a calorimeter. It is a cylindrical copper vessel (Fig. 5.1) containing a liquid and a stirrer for stirring the liquid. Heat flows from the hotter to the colder bodies until all of them acquire a common temperature. The method is known as the **method of mixture.**

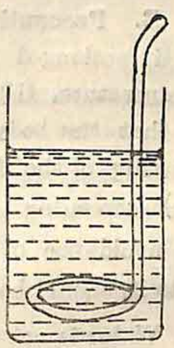

Fig. 5.1

Let us assume that no heat enters or leaves the calorimeter after the bodies have been brought into contact and that no chemical action takes place between the bodies. Then, from the principle of conservation of energy, we may say that.

Heat lost by the warmer bodies = heat gained by the colder bodies.

This is the *fundamental principle of calorimetry.*

5-4.1. Precautions. The assumption that no heat enters or leaves the calorimeter must be fulfilled in calorimetric experiments. The calorimeter must be designed in such a way as to eliminate loss or gain of heat by conduction, convection and radiation.

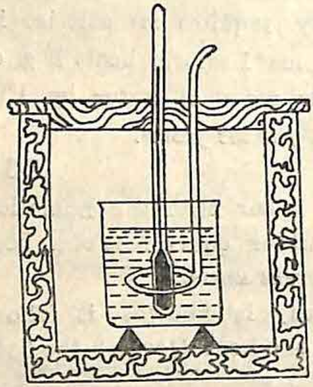

Fig. 5.2

A. Precautions in design. To minimize conduction the calorimeter is placed on a thermal insulator, such as cork or felt. It is better however to support it on spikes of cork (Fig. 5.2) or strips of cardboard set on edge or to suspend it by fine threads. Convection is reduced by surrounding the calorimeter by an outer vessel and packing dry cotton wool or felt round it. The lid of this vessel is provided with holes to allow the thermometer and stirrer to pass. To reduce radiation loss the calorimeter is highly polished. The heat lost by radiation—rather by convection and radiation—can however be determined experimentally and taken into consideration in calculating the results.

B. Precautions in method. Besides the above, precautions such as (i) prolonged heating of the solid so that it may attain a steady temperature, (ii) quick transference of the body to the calorimeter so that the body may not lose heat on its way, (iii) continuous stirring of the liquid so that it may attain a uniform temperature, (iv) screening the calorimeter from other sources of heat, and (v) avoidance of splashing of the liquid while dropping the solid into it, should be taken.

Water is not suitable as a calorimetric liquid. Its specific heat is much higher than that of any other liquid. So the rise in temperature of water for a given supply of heat will be much less than that of other liquids. The accuracy with which a temperature difference can be measured diminishes as this difference itself becomes smaller. Therefore, with water as the calorimetric liquid the result is likely to be less accurate than if some other liquid, such as an oil, were used.

5-5. Basic equation of the method of mixture. All calorimetric problems based on the method of mixture can be solved with the help of one equation only. The equation is established below. Let

mass of calorimeter + stirrer $= m_1$(g),
specific heat of its material $= s_1$(cal/g °C),
the mass of the solid (to be heated) $= m$(g),
its specific heat $= s$(cal/g°C),
mass of liquid in the calorimeter $= m'$(g), (this may be water),
specific heat of liquid in calorimeter $= s'$ ($s' = 1$ for water)
water-equivalent of the calorimeter = mass of cal. × sp. heat of its material $= W$(g),
initial temperature of liquid in calorimeter $= t_1$°C,
initial temperature of the hot body $= t_2$°C,
final common temperature $= t$°C.

Here the body loses heat while the calorimeter and its contents gain heat.

Heat lost by the body $= ms(t_2 - t)$ cal ;
Heat gained by liquid in the calorimeter $= m's'(t - t_1)$ cal ;
Heat gained by the calorimeter $= W(t - t_1)$ cal ;

$$\therefore \quad ms(t_2 - t) = (m's' + W)(t - t_1) \qquad (5\text{-}5.1)$$

From this equation any of its unknown quantities, such as s, s', W, t, etc., may be determined, when all other quantities are known.

Examples. (i) Sp. heat of a solid. 10 g of common salt are heated to 97°C and dropped into a calorimeter containing oil of turpentine. If the mass of the oil is 125 g, its specific heat 0.43, and temperature 32°C, find the specific heat of common salt when the water equivalent is 15 g and the final temperature 35°C.

Solution : Let s be the required specific heat.

Heat lost by common salt $= 10 \times s \times (97 - 35)$ cal.
Heat gained by the calorimeter and oil $= (15 + 125 \times 0.43)(35 - 32)$ cal.

$$\therefore \quad 10 \times s \times 62 = (15 + 125 \times 0.43) \times 3$$

whence $s = 0.333$ (in cal g^{-1} $°C^{-1}$).

(ii) Determination of a high temperature with a calorimeter. An iron ball weighing 50 g is, heated in a furnace and dropped into 240 g of water at 30°C contained in a vessel of water equivalent 10 g. If the temperature rises to 50°C, find the temperature of the furnace, given that the specific heat of iron = 0.1.

Solution: Let t be the required temperature

Heat lost by the ball $= 50 \times 0.1 \times (t-50)$ cal.

Heat gained by water and the vessel $= (10+240)(50-30)$ cal.

$\therefore\ 5(t-50) = 250 \times 20$ or $t = 1050°C$.

(iii) **Determination of the specific heat of a liquid.** A piece of glass weighing 100 g is heated to 95°C and dropped into olive oil contained in a calorimeter. The mass of the oil is 120 g, the mass of the calorimeter 150 g and its specific heat 0.1. If the temperature rises from 30°C to 45°C, find the specific heat of the oil, given that the specific heat of glass is 0.22.

Solution: Heat lost by glass $= 100 \times 0.22 \times (95-45) = 1100$ cal.

Heat gained by the calorimeter and the oil

$= (150 \times 0.1 + 120s)(45-30) = 225 + 1800s$ cal,

$\therefore\ 1800s + 225 = 1100$ or $s = 0.486$.

(iv) **Determination of water equivalent of a calorimeter.** A calorimeter contains 70.2 g of water at 15.3°C. If 143.7 g of water at 36.5°C are mixed with it the common temperature is 28.7°C. Find the water equivalent of the calorimeter.

Solution: Let the water equivalent be W g.

Heat gained by the calorimeter $= W(28.7-15.3) = 13.4\ W$ cal.

Heat gained by 70.2 g of water in being heated from 15.3°C to 28.7°C

$= 70.2 \times 13.4$ cal $= 940.68$ cal.

Heat lost by the warm water $= 143.7\ (36.5-28.7) = 1120.86$ cal,

$\therefore\ 13.4W + 940.68 = 1120.86$ whence $W = 13.4$ g.

(v) **Calculation of common temperature.** 50 g copper are heated to 98°C and dropped into a calorimeter containing 100 g of water at 30°C. If the water equivalent is 10 g what is the final temperature? Specific heat of copper = 0.09.

Solution: Let the final temperature be t°C.

Heat lost by copper $= 50 \times 0.09 \times (98-t)$ cal.

Heat gained by the calorimeter and water $= (10+100)(t-30)$ cal.

$\therefore\ 110(t-30) = 50 \times 0.09 \times (98-t)$ whence $t = 32.7°C$ (approximately).

5-6. How to handle thermal units properly. Handling units is never a problem when, in an equation, you write all the quantities with the unit symbols after their numerical values. If the unit for any quantity is not given, its value will automatically come out from the equation, provided you *treat the unit symbols as algebraical quantities*. This applies not only to thermal problems, but to all cases.

To illustrate it, let us treat the last solved example above (Example v) in the way stated. Take specific heat not as a ratio but in cal $g^{-1}\,°C^{-1}$.

Then, heat lost by copper $= 50\text{g} \times 0{\cdot}09 \text{ cal g}^{-1}\,°C^{-1} \times (98-t)°C$

$= 50 \times 0{\cdot}09 \times (98-t)$ cal (as before).

Heat gained by calorimeter and water $=(10\text{ g}+100\text{g})(t-30)^\circ\text{C}\times 1\text{ cal g}^{-1}\,{}^\circ\text{C}^{-1}$ because heat gained is mass of water × sp. heat of water × temp. rise, and the specific heat of water is $1\text{ cal g}^{-1}\,{}^\circ\text{C}^{-1}$.

Equating these you get t (in °C).

Determination of latent heats of fusion and condensation has been treated in the chapters on change of state (Chap.s 6 and 7) (see Sec.s 6-4 and 7-7).

Exercises

1. Define the calorie. In what other unit (used in mechanics) can heat be expressed? Express the calorie in this unit. Why is it necessary to mention the temperature in the definition of calorie?

2. Define specific heat and show that the heat lost or gained by a substance is equal to the product of its mass, specific heat and change in temperature.

3. (*a*) What is the fundamental principle of calorimetry? What precautions are necessary in a calorimetric experiment by the method of mixture?

(*b*) Define thermal capacity and water equivalent. What are the units in which they are expressed?

4. Fill up the gaps in the following table:

	Mass	*Specific heat*	*Change of temperature*	*Heat*
(*a*)	50 g	0.09	30°C to 90°C	...
(*b*)	10 lb	...	100°F to 212°F	1120 Btu
(*c*)	100 g	0.21	40°C to ...	1680 calories
(*d*)	...	0.25	20°F to 200°F	225 Btu
(*e*)	20 kg	0.4	20°C to 30°C	...

[Ans.: (*a*) 270 calories, (*b*) 1, (*c*) 120°C, (*d*) 5 lb, (*e*) 80,000 cal.]

5. An iron ball weighing 50 g was placed in a furnace for some time and then quickly transferred to a calorimeter containing 1 kg of water at 35°C. It raised the temperature of the calorimeter and its contents to 40°C. The water equivalent of the calorimeter is 125 g and the sp. heat of iron 0·12. Calculate the temperature of the furnace. [Ans.: 977.5°C.]

6. A metal ball at 130°C and weighing 20 g is dropped into a calorimeter (water equivalent 10 g) containing 50 g of a liquid (specific heat 0.5) at a temperature of 40°C. The final temperature is 50°C. Calculate the specific heat of the metal. [Ans.: 0.22]

7. A calorimeter of water equivalent 25 g contains 100 g of an oil at 40°C. A solid of specific heat 0.1, weighing 50 g is heated to 120°C and quickly dropped into the calorimeter. The resulting temperature is 45°C. Calculate the specific heat of the oil. [Ans.: 0.5]

8. A calorimeter contains 500 g of water at 30°C. 200 g of water at 90°C are poured into it. If the water equivalent of the calorimeter be 10 g what is the resulting temperatue? [Ans.: 46.9°C]

9. 200 g of an oil of sp. heat 0.42 are dropped at a temperature of 60°*C* into a calorimeter containing 36 g of water at 23°*C*. The resulting common temperature is 30°*C*. What is the water equivalent of the calorimeter ? If the mass of the calorimeter be 100 g what is the specific heat of the material ? [ANS. : 10 g ; 0.1]

10. Which of the two following cases requires the greater quantity of heat ?

(i) 500 g of water heated from 35°*C* to 95°*C*.

(ii) 4 lb of water heated from 100°*F* to 212°*F*. [ANS. : The second case]

11. A vessel contains 60 litres of air (sp. heat 0.24) at 30°*C* weighing 1.3 g/litre. How much heat is required to raise the temperature of the air to 40°*C* ?

[ANS. : 187.2 calories]

12. On pouring 1 kg of water at 90°*C* into a hot water bottle at 35°*C*, the temperature of the water falls to 85°*C*. Calculate the weight of the bottle. The specific heat of the material of the bottle is 0·2. [ANS. : 500 g]

13. A can placed on a flame absorbs heat at the rate of 200 calories per sec. How long will it take to raise the temperature of 3 kg of water from 30°*C* to the boiling point assuming that the can alone absorbs 3000 calories of heat.

[ANS. : 17 min. 45 sec.]

14. Suppose you are given a thermometer reading only from 50°*C* to 100°*C* and some water of which the temperature is below 20°*C*. Describe an experiment to determine roughly the temperature of the water, without using any other thermometer.

15. How would you find the temperature of a red-hot iron ball if only mercury thermometers were available ? Show, in detail, how the result is calculated from your observations.

16. Explain what is meant by the 'specific heat' of a material.

How would you proceed to determine the specific heat of a metal ? Point out the sources of error and mention the precautions necessary to avoid them.

A cylinder of metal weighing 450 g at 150°*C* is dropped into 200 g of water at 15°*C* contained in a brass calorimeter weighing 200 g and the temperature rises to 38°*C*. What is the *thermal capacity* of the metal ? (sp. heat of brass = 0.092)

[ANS. : 44.85 cal/°C.]

17. Define specific heat, thermal capacity, and calorie. Compare the thermal capacities of equal volumes of water and mercury. (sp. heat of mercury = $\frac{1}{30}$ and sp. gr. of mercury = 13. 6). [ANS. : 1 : 0·453]

18. Define : (i) Calorie, (ii) Specific heat, (iii) Water equivalent, (iv) Thermal capacity, (v) Specific heat capacity.

Describe a calorimeter and explain its use in finding the specific heat of a solid insoluble in water.

CHAPTER

6

CHANGE OF STATE (SOLID TO LIQUID)

6-1. Melting and freezing. When a substance changes from the solid to the liquid state the process is called *melting*. The reverse process. *i.e.*, a transformation from the liquid to the solid state is called *solidification*, or *freezing*.

The temperature at which melting takes place under a pressure of one standard atmosphere is called the **normal melting point.** Similarly, the temperature at which solidification takes place under a pressure of one standard atmosphere is called the *normal temperature of solidification*, or **normal freezing point**. (For effect of pressure on melting point see. Sec. 6-6.)

For a pure crystalline substance these two temperatures are the same and a sharply defined one. Each pure substance has its own particular temperature at which it melts (unless chemical change occurs before melting). The change from solid to liquid begins only when this temperature is reached. *The substance remains at this temperature until the change is complete.*

Non-crystalline substances such as fat, wax, glass etc., pass through an intermediate soft state before melting. They do not possess a fixed or sharply defined melting point. In some of these substances melting and freezing do not take place at the same temperature. Thus butter melts between $28^\circ C$ and $33^\circ C$ but solidifies between $23^\circ C$ and $20^\circ C$. Impure substances and mixtures do not as a rule have a sharp melting point. They melt over a range of temperatures. Sharpness of melting point is, therefore, a test of purity, and is used by chemists as a test of purity.

The following table gives the melting points of a few substances :

Substance	Melting point	Substance	Melting point
Brass	$800^\circ C$ to $1000^\circ C$	Paraffin-Hard	$52^\circ C$ to $58^\circ C$
Carbon	about $3500^\circ C$	Soft	$38^\circ C$ to $52^\circ C$
Copper	$1083^\circ C$	Platinum	$1769^\circ C$
Gold	$1063^\circ C$	Silver	$960{\cdot}8^\circ C$
Iron (cast)	$1100^\circ C$ to $1300^\circ C$	Sulphur	$119^\circ C$
Lead	$327{\cdot}3^\circ C$	Tin	$231{\cdot}9^\circ C$
Mercury	$-38{\cdot}87^\circ C$	Tungsten	$3380^\circ C$
Naphthalene	$80{\cdot}2^\circ C$	Zinc	$419{\cdot}5^\circ C$

Melting point is generally lowered by the presence of other substances. Plumber's solder, which is an alloy of two parts by weight of lead and one part of tin, melts at 180°*C*, though lead melts at 327°*C* and tin at 232°*C*.

By adding bismuth (m.p. 271°*C*) to a mixture of 37% lead and 63% tin, an alloy melting at 94°*C* may be obtained. *Wood's metal* which contains 50% bismuth, 25% lead, 12·5% antimony (m.p. 630·5°*C*) and 12·5% cadmium (m.p. 320·9°*C*), melts at 65·5°*C*.

A fire-fighting and alarm device used in factories makes use of such low melting alloys. Water pipes are fixed to the ceiling, and at intervals nozzles filled with the alloy are fitted. If fire breaks out the alloy melts and water is sprayed into the room. Another part of the device rings a warning bell at the same time.

6-2. Latent heat and its action. When heat is supplied to a substance its temperature rises. It is supposed that its molecules vibrate with greater and greater amplitude as the temperature rises. The energy required for the larger vibrations comes from the heat supplied. This heat is sometimes called *sensible heat*. It increases the kinetic energy of motion of the molecules.

But the heat supplied to a solid during melting does not cause any rise in temperature. Where does then the energy go ?

In crystalline solids, as you may know, the molecules are arranged in regular geometrical patterns in space. They are held in their equilibrium positions by forces of mutual attraction and repulsion. Instead of being at rest, the molecules vibrate about their equilibrium positions with frequencies of the order of 10^{13} per second, and kinetic energy proportional to the absolute temperature.

The energy supplied to a solid at its melting point is spent in destroying its geometrical arrangement, thus converting the solid into a liquid without change of temperature. The heat which has to be supplied to a solid at its melting point to bring about a change in state *without change in temperature* is called **latent heat.** The latent heat increases the potential energy of the molecules, and *is given out by the liquid when it freezes.*

The **specific latent heat of fusion** (or **melting**) is the amount of heat required to convert one gram of solid into liquid without any change of temperature. The same amount of heat will be given out by 1 gram of the liquid when it solidifies at its freezing point without change in temperature.

Use of the word *specific* to mean *per unit mass* is of late origin. Older texts often write 'latent heat' to mean 'specific latent heat', that is, latent heat per unit mass. The statement that "Latent heat of ice is 80" is a loose statement and means 80 calories of heat will be required to convert 1 gram of ice at $0°C$ to 1 gram of water at $0°C$. Latent heat is expressed in heat units per unit of mass, such as *calories per gram* (or *Btu per lb* etc.).

The following table shows the melting points and latent heats of a few substances :—

Substance	Melting point (in °C)	Latent heat (in cal/g)
Acetic acid	16·7	44·7
Ammonia	−77·7	108
Benzene	5·5	30·1
Glycerine	20	48
Ice	0	79·7
Lead	327	6
Mercury	−39	2·8
Sulphuric acid	10·5	24
Tin	232	14

The latent heat of water may be made to serve a useful purpose in preventing damage to meat, fruits and vegetables in storage. Extreme cold may freeze their sap or otherwise reduce their food value. To prevent such freezing a few buckets of water are kept in the store room. When the temperature of the room goes below $0°C$ the water in the buckets gradually freezes and gives out its latent heat. This raises the temperature of the room and it may not go down far enough to freeze the sap. Freezing releases the latent heat ; freezing may thus be considered a heating process.

No freezing unless latent heat is extracted. A liquid will not freeze by merely keeping it at its freezing point. The latent heat of fusion must be extracted from it. This can be easily proved by keeping water in a small phial fully surrounded by ice. The water will soon reach the freezing point ($0°C$), but will not freeze. It must be cooled below $0°C$, even though slightly, so that the latent heat may be extracted. (Loss of heat lowers the temperature of a body. On this view, latent heat is not heat, but internal potential energy of the molecules of the liquid.)

The same applies to melting. A solid kept at its melting point will not melt unless the latent heat is supplied.

Supercooling. Sometimes, a pure liquid, cooled undisturbed, does not freeze even though it has been cooled below the normal freezing point. This phenomenon is known as **supercooling.** Sodium thiosulphate (popularly known as 'hypo'), thymol, naphthalene, etc., can be melted and then cooled carefully much below their normal freezing points without freezing. Supercooled liquids are very unstable ; a slight disturbance or the addition of a crystal of it may be enough to start solidificiation of the whole mass. The temperature rises to the normal freezing point at the same time.

6-3. Sublimation. There are some solids which, when heated, directly pass on to the gaseous state without passing through the liquid state. This phenomenon is called *sublimation.* Camphor, iodine, dry ice (solid CO_2) sublime even at the room temperature. Sublimation also requires latent heat.

6-4. Determination of the latent heat of ice. Take a clean dry calorimeter with a wire gauze stirrer and weigh it empty. Fill it partly with water and weigh again. The difference gives the weight of water taken (m g).

Put a thermometer in it and record the initial temperature. Let it be t_1°C.

Take one or two *small* pieces of ice and soak away all water from them by means of a piece of blotting paper. Holding the ice in the blotting paper, drop the former into the calorimeter. Stir the water keeping the ice below the stirrer. Note the lowest temperature reached. Let it be t°C.

Wait till the calorimeter and its contents are again at the room temperature. Weigh them again. The excess in weight gives the amount of ice added. Let the mass of ice added be m' g.

Let W be the water equivalent of the calorimeter, and L, the latent heat of ice per gram.

Then, heat lost by the calorimeter $=W(t_1-t)$;
heat lost by the water $=m(t_1-t)$;
heat gained by ice on melting $=m'L$;
heat gained by water formed of ice $=m't$.

Since heat lost = heat gained

we have $(W+m)(t_1-t)=m'L+m't$

$$\therefore\quad L=\frac{(W+m)\,(t_1-t)}{m'}-t \qquad (6\text{-}4.1)$$

Note : To guard against such a fall in temperature of the calorimeter as will deposit water from the atmosphere on its sides, only one or two small pieces of ice should be added.

Examples. (1) 100 g of ice at $-10°C$ are heated till fully converted into water at $30°C$. Find the amount of heat necessary. (Given, latent heat of ice $=80$ cal/g and specific heat of ice $=0.5$ cal $g^{-1}\,°C^{-1}$.

Solution : Heat required to raise 100 g of ice from $-10°C$ to $0°C = 100 \times 0.5 \times 10 = 500$ cal.

Heat required to melt 100 g of ice at $0°C$ into water at $0°C = 8000$ cal.

Heat required to raise 100 g of water from $0°C$ to $30°C = 100 \times 30$ cal.

$\therefore$ Total heat required $= (500 + 8000 + 3000)$ cal $= 11500$ cal.

(2) 75 grams of water $100°C$ are added to 20 grams of ice at $-15°C$. Find the resulting temperature. (Given, specific heat of ice $=0.5$ and latent heat of ice $=80$ cal/g.)

Solution : Let t be the final temperature.

Heat lost by the warm water $= 75 \times 1 \times (100 - t)$ cal.

Heat gained by ice for being raised from $-15°C$ to $0°C = 20 \times 0.5 \times 15$ cal.

Heat required for melting of ice $= 20 \times 80$ cal.

Heat gained by water (formed of the ice) for being raised from $0°C$ to $t°C$ $= 20 \times 1 \times t$ cal.

$\therefore$ $75 \times (100 - t) = 150 + 1600 + 20t$ whence $t = 60.52°C$

(3) 20 grams of water at $100°C$ are added to 75 grams of ice at $-15°C$. Find the result.

N.B. Note that this problem is of the same nature as the previous one. If treated in the same way it will give a common final temperature lower than that of the ice. Obviously this is absurd as by adding hot water the temperature of ice cannot be lowered.

The *fallacy* lies in assuming that all ice is melted. This is true in the first case, but does not hold for the second.

The following method of treatment is recommended.

Heat required for raising 75 g of ice from $-15°C$ to $0°C = 75 \times 0.5 \times 15 = 562.5$ cal.

Heat required for melting 75 g of ice at $0°C$ to water at $0°C = 75 \times 80 = 6000$ cal.

Now if the hot water cools up to $0°C$ the heat given out will be $20 \times 1 \times 100 = 2000$ cal. Of this heat 562.5 cal will be required to bring the ice up to $0°C$. The remaining 1437.5 cal will be available for melting ice.

$\therefore$ The quantity of ice melted would be $1437.5/80 = 17.97$ g. The result will be 57.03 g of ice and $20 + 17.97 = 37.97$ g of water at $0°C$.

(4) 900 g of iron at $500°C$ are transferred into a hole in a block of ice and 680 cm³ of water collected from it. If the specific heat of iron is 0.12, find the latent heat of ice.

Solution: The mass of 680 cm^3 of water = 680 g. Iron cooled from 500°C to 0°C. Hence heat lost by iron $= 900 \times 0.12 \times 500$ cal. The heat gained by ice $= 680 \times L$.

$$\therefore\ 680L = 900 \times 0.12 \times 500 \quad \text{or} \quad L = 79.4 \text{ cal/g}.$$

(5) A calorimeter at 30°C contains 220 g of paraffin oil. The water equivalent of the calorimeter is 40 g. When 15 g of ice at 0°C are added to it the temperature falls to 20°C. Find the specific heat of paraffin.

Solution: Heat lost by paraffin $= 220 \times s \times (30 - 20) = 2200s$.

Heat lost by the calorimeter $= 40 \times 10 = 400$ cal.

Heat gained by ice in melting $= 15 \times 80 = 1200$ cal.

Heat gained by water so formed in rising from 0°C to 20°C $= 15 \times 20 = 300$ cal.

$2200s + 400 = 1500$, whence $s = 0.5$.

6-5. Change of volume on melting. The volume of most solids increases on melting; but there are a few substances, *viz.*, ice, cast iron, bismuth, antimony and type metal, which show a decrease in volume on melting. This property of cast iron and type metal is taken advantage of in making castings of various designs from iron and in the manufacture of types for printing. The fact that ice floats in water is of great consequence in keeping aquatic animals safe in the seas of the arctic regions or in waters which freeze in winter.

In expanding on solidification water exerts a considerable force:

In cold countries water pipes are sometimes found to burst in winter, hot water pipes bursting more often than cold water pipes. Hot water contains much less dissolved air than cold water. In cold water pipes the dissolved air remains as air pockets when water cools and finally freezes. The increase of volume of water on solidification compresses these air pockets. The cold water pipes may thus be saved from bursting.

Boulders which have retained some water in their cracks may fall into pieces when, in cold winter nights, the water in the cracks freezes and expands.

6-6. Effect of pressure on melting point. Melting points of substances are slightly affected by pressure. Most liquids contract on cooling and this continues when they turn solid. In these cases, increase of pressure assists the change from liquid to solid. Such a liquid will therefore, freeze more easily, that is, at a higher temperature when under pressure. Liquids like water which expand on freezing, will freeze at a lower temperature when subjected to pressure.

Here pressure opposes the change ; hence it becomes necessary to reach a lower temperature before freezing can take place. Ice melts at $-1^{\circ}C$ under a pressure of about 134 atmospheres.

Regelation. The familiar phenomenon of fusion of two pieces of ice to form a single piece by pressure is an example of lowering of melting point under pressure. When two pieces of ice are pressed together the points of contact between them are subjected to high pressure. Hence the melting point at these places is lowered. The actual temperature at these points, *i.e.*, $0^{\circ}C$, is higher than the melting point under the condition of high pressure. Hence ice melts at these points taking the latent heat from their immediate surroundings. When the pressure is released the water around the points of contact freezes as it is lower than $0^{\circ}C$ due to the abstraction of latent heat from it during melting. The two pieces are thus cemented together. The melting of ice by pressure followed by solidification of the water so formed on removal of pressure is known as **regelation.**

Skating on ice is another example of regelation. The steel edge of the skate exerts a high pressure on the ice. The ice below the steel edge of the skate immediately melts.

This reduces friction to the motion of the skate. As soon as the skate moves away, the water solidifies as the pressure on it has been reduced.

Bottomley's experiment. A single turn of *bare, thin copper wire* is wound round a large block of ice supported on two wooden blocks as shown in Fig. 6.1 and is loaded by a weight of several kilograms. The copper wire will be seen to penetrate through the block of ice and pass out of it, while the block remains intact. Due to the load the thin wire exerts a rather high pressure on the ice just below it and lowers the melting point at that place. The ice at that place, therefore, melts taking the latent heat from the surroundings through the copper wire. Copper is a good conductor of heat. The water so formed flows round the wire which sinks. This water freezes again due to the removal of the pressure on it. The latent heat it gives out passes through the wire and

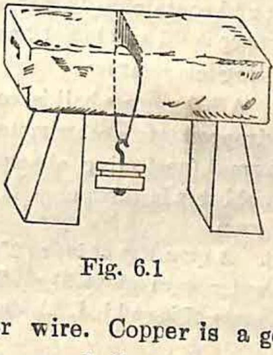

Fig. 6.1

supplies the latent heat of fusion for the ice below the wire. This process is repeated and each layer is cemented no sooner the wire cuts through it.

For the success of the experiment it is necessary that heat be allowed to flow from the solidifying water to the ice to be melted below the wire. The wire must, therefore, be made of a material which conducts heat well. The experiment will not succeed if the wire is a bad conductor of heat.

Exercises

1. What is latent heat? What does it do? What do you understand by the statement that latent heat of ice is 80?

Is it a proper statement to say only 80 without any mention of units? What should be a correct statement?

Describe an experiment for measuring the specific latent heat of ice.

2. How is melting point affected by pressure? What is regelation? Explain how a loaded copper wire can pass through a block of ice. Will the experiment succeed if a silk thread replaced the copper wire? Explain your answer.

3. 3 kg of copper heated to 72° *C* are placed on a block of ice. How much of ice will melt? Sp. heat copper is 0.1 and latent heat of fusion of ice is 80 cal/g? [Ans.: 270 g]

4. A piece of ice at 0° *C* and weighing 15 g is dropped into 85 g of water at 20° *C* in a copper calorimeter. When all the ice melts the temperature falls to 10° *C*. If the latent heat of fusion of ice is 80 calories per gram, calculate the water equivalent of the calorimeter. [Ans.: 50 g]

5. 40 g of ice at $-15°C$ (sp. heat 0.5) are mixed with 150 g of water at 90° *C*. What is the resulting temperature? [Ans.: 45.25° *C*]

6. 250 g of copper at 90° *C* are dropped into a copper calorimeter weighing 100 g and containing 10 g of ice and 25 g of water at 0° *C*. The specific heat of copper is 0·1 and latent heat of fusion of ice, 80 calories per gram. Find the resulting temperature. [Ans.: 20.7° *C*]

7. A 100 g brass ball is cooled to the temperature of liquid air, *i.e.*, $-190°C$ and dropped into a calorimeter containing water at 0° *C*. Calculate the mass of ice formed (neglecting absorption of heat from the calorimeter). The specific heat of brass in this range is 0.08 and latent heat of fusion of ice is 80 calories per gram. [Ans.: 19 g]

8. A quantity of ice at 0° *C* is added to 50 g of water at 30° *C* in a calorimeter of water equivalent 10 g. The final common temperature is 10° *C*. Calculate the mass of ice added. Latent heat of fusion of ice is 80 calories per gram. [Ans.: $13\frac{1}{3}$ g]

9. A brass ball weighing 10 g and heated to 250° *C* is put inside a cavity in a block of ice, of which 2.7 g melt. If the latent heat of fusion of ice is 80 calories per gram, calculate the specific heat of brass. [Ans.: 0.09 cal $g^{-1}\,°C^{-1}$]

10. A metal vessel containing 250 g of water at 30° *C* is placed in a refrigerator which abstracts heat at the rate of 275 calories per minute. Calculate the time taken by the water to be converted into ice at 0° *C*. The latent heat of fusion of ice is 80 calories per gram. The weight of the vessel may be neglected.

[ANS. : 1 hr. 40 min.]

11. 100 g of ice at 0° *C* are added to 200 g of water at 30° *C*. What will be the final temperature ? [ANS. : 0° *C*]

(Note that only a part of the ice will melt)

12. Suppose that the latent heat of ice were suddenly reduced to half its value. What may be the effect of such a change on the snows of the Himalayas ? (Answer in ten lines.)

13. Answer the following :

(*i*) Do all substances have sharp melting points ? Give examples.

(*ii*) In what sense is freezing a heating process ? How does the heat thus produced protect plants and temper the climate ?

(*iii*) Brass is not suitable for type making. Why ?

(*iv*) In cold countries hot water pipes burst more often than cold wate pipes rnnning side by side. Why ?

(*v*) Explain if a small tube filled with water will freeze if kept surrounded by melting ice.

14. The latent heat of fusion of sulphur which melts at 113° *C* is 9 cal/g, and the specific heat of solid sulphur is 0·17 cal/g° *C*. What do the statements mean ? Find the *rise* in temperature when 35 g of liquid sulphur at its melting point is poured into a copper calorimeter weighing 40 g and containing 100 g of water at 14° *C*. (sp. heat of copper = 0.1 cal/g ° *C*)

[ANS. : 22.2 − 14 = 8.2° *C*]

15. Explain what you mean by 'latent heat'. Why is it so called ?

Describe how the latent heat of fusion of ice may be determined.

A pitcher holds about 1 litre of water. We wish to cool it from 25° *C* to 10° *C*. How many 20 g ice-cubes must be added ? [ANS. : 8]

16. Explain why two pieces of ice can be joined by pressing them together for a few seconds and then releasing the pressure.

CHAPTER

7

CHANGE OF STATE (LIQUID TO VAPOUR) AND HYGROMETRY

7.1. Vaporization and vapour pressure. When a liquid is heated it changes into vapour. Such change is called *vaporization*. It can take place in two ways, *viz.*, (i) by *evaporation*, and (ii) by *boiling*. The reverse process of change of state from vapour to liquid is called *condensation* or *liquefaction*. Before proceeding to consider the processes of evaporation and boiling in some detail it may be profitable for us to know that *a vapour exerts pressure like a gas*. This may be seen as follows.

Take a barometer tube filled with mercury and invert it over a mercury trough. Introduce a few drops of a liquid, such as water, alcohol or ether, into the tube with the help of a bent pipette (Fig. 7.1). As the first drops rise through the mercury column and reach the vacuous space above, they turn completely into vapour. The vapour so formed depresses the mercury column, showing that it exerts pressure like a gas.

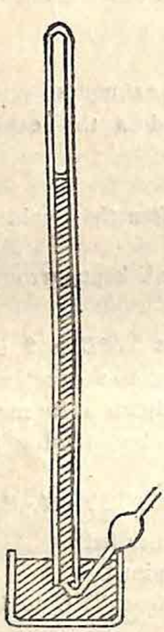

Fig. 7.1

As you introduce more and more of the liquid into the tube, a state will be reached when it no longer evaporates, but accumulates on the mercury surface. When this condition is reached, the pressure exerted by the vapour does not increase any more.

These show that (i) *a vapour exerts pressure*, and (ii) *there is a limit to the amount of vapour that a space* (here the Torricellian vacuum) *can hold*. If we increase the temperature of the Torricellian space by surrounding it with a suitable bath, it will be found that the maximum vapour pressure exerted by the liquid increases with rise of temperature and the space can hold more vapour at the higher temperature.

By using different liquids, we can see that the maximum vapour pressure a liquid can exert at a given temperature depends on the nature of the liquid.

7-1.1. Vapour pressure. When a liquid is confined in a closed space, the escaping molecules move about at random in the available space. They constantly collide with the walls bounding the space. In this way they exert a pressure at every point of the bounding surfaces including the liquid surface. Some of the molecules impinging on the liquid are drawn into it while some others leave the liquid, Soon a state of equilibrium is reached in which as many molecules leave the surface as enter it in a given time. When this condition has been reached the pressure exerted by the vapour is a maximum. This pressure is variously known as the *equilibrium vapour pressure*, *maximum vapour pressure*, *saturated vapour pressure* (abbreviated S.V.P.) or simply, *vapour pressure* of the liquid. For a given liquid S.V.P. is determined only by the temperature. Its value increases with rise of temperature, and depends on the nature of the liquid. The nearer a liquid is to its normal boiling point the greater will be its vapour pressure.

7-2. Evaporation. Evaporation is the slow process of conversion of a liquid into the vaporous state. It is easy to confirm the following facts about evaporation. Many of them are part of our daily experience.

(i) Evaporation takes place at all temperatures and under all pressures.

(ii) The rate of evaporation increases with rise of temperature.

(iii) The rate of evaporation also depends on the pressure. The lower the pressure on a liquid the faster will it evaporate. Evaporation is quickest in a vacuum.

(iv) The greater the area of the exposed surface the greater the evaporation.

(v) Evaporation is speeded up by a current of air over the liquid.

(vi) The rate of evaporation depends on the amount of the vapour of the liquid present over the liquid surface. The higher it is the slower is the evaporation. A moist cloth dries much slower on a wet day than on a dry day because there is more moisture in the air on the wet day.

(vii) Of two liquids the one which has a lower boiling point will evaporate more quickly. Ether evaporates much more quickly than alcohol, and alcohol quicker than water. The boiling points of ether, alcohol and water are 35°C, 78°C and 100°C respectively.

(viii) *Heat is required for evaporation. If it is not supplied the liquid cools itself or its surroundings to get the heat* (Also see Sec. 7-8).

All liquids have a tendency to turn into vapour at all temperatures. This tendency is measured by the maximum vapour pressure a liquid can exert at the temperature at which it is. Vapour pressure rises with increasing temperature. A liquid begins to boil when its vapour pressure equals the pressure on it. (see Sec. 7-3).

The average kinetic energy of the molecules of a liquid depends on its temperature, and diminishes with lowering of temperature. In evaporation, some of the faster molecules near the liquid surface overcome the attraction of the rest of the liquid, and leave the liquid. When a liquid loses its faster molecules, the average molecular kinetic energy is lowered. This means a lower temperature, that is, cooling. Unless heat enters the liquid from the surroundings, its temperature will be lowered due to evaporation.

In rapid evaporation, there can be much cooling because the faster molecules are carried away very quickly (see Sec.s 7-8).

7-3. Boiling. Take some water in a flask fitted with a rubber stopper. Pass through it a thermometer, an open-tube manometer, and a delivery tube provided with an india rubber tubing and a pinch cock (Fig. 7-2).

Heat the flask slowly over a Bunsen flame. As the temperature gradually increases you will find more and more steam rising *from the surface*. (This is not boiling). Also, the dissolved air in the water forms into small bubbles, rises to the surface and escapes. At about 70°—80°C, small bubbles of water vapour will be seen to form at the bottom of the flask. They rise and collapse as they reach the upper layers of colder water. At this stage a simmering sound is heard. In the final stage the bubbles rise to the top and burst at the surface. When this happens we say that the

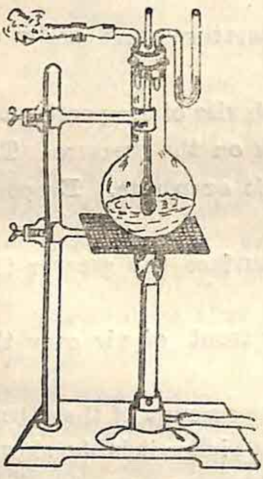

Fig. 7.2

liquid is boiling. *During boiling, vapour rises to the surface from all points throughout the liquid. The temperature of the liquid remains constant so long as the boiling continues.*

The vapour within the flask is invisible like air. As it comes out of the delivery tube it condenses into small water drops which look like a white cloud. This cloud soon vanishes as the droplets evaporate in the open air.

During boiling, the manometer registers a constant pressure. When the delivery tube is wide open, mercury in both limbs of the manometer stands at the same height, showing that the pressure inside the flask is equal to that of the atmosphere. If boiling has continued long enough we may safely conclude that air inside the flask has been completely replaced by water vapour. So the pressure inside the flask is that exerted by the water vapour only. This shows that when **there is a free communication with the atmosphere a liquid boils when its vapour pressure is equal to the atmospheric pressure.**

The constant temperature at which a liquid boils under normal atmospheric pressure is called its **normal boiling point.** The boiling point of water under a pressure of 76 cm of mercury is 100°*C*.

7-3.1. Effect of pressure on boiling point.

(1) **Increase of pressure on a liquid raises its boiling point.** During brisk boiling, close the pinch cock partially (Fig. 7.2). If it is closed enough so that rate of outflow of steam through the tube becomes less than the rate of generation the manometer will show a higher pressure in the flask. The thermometer also shows a higher temperature when the liquid boils under this condition of increased pressure on its surface.

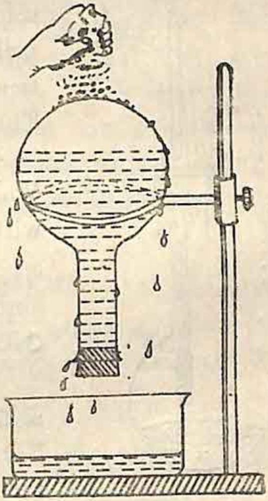

Fig. 7.3

We, therefore, conclude that an increase in pressure on the liquid surface results in an increase in the boiling point of a liquid.

(2) Lowering of pressure on a liquid lowers its boiling point.

(a) Franklin's experiment. Take a flask half full of water and boil it briskly. After boiling has proceeded for some time so that the air inside has been replaced by water vapour, remove the flask from the flame and quickly close it with a rubber stopper. Note that boiling stops immediately. Now invert the flask on a retort holder and pour cold water on it (Fig. 7.3). The water within the flask will be seen to boil again. This boiling lasts for a short time. The experiment can be repeated several times after the first boiling.

The explanation is simple. Part of the water vapour above the surface of the liquid condenses into water due to the cooling of the surface of the flask. This results in a diminution of pressure on the liquid surface. If the pressure is lowered below the vapour pressure of the water in the flask, the liquid boils.

This shows that a liquid may be made to boil at a lower temperature than its normal boiling point. This happens when the pressure on it is reduced below the normal atmospheric pressure.

Boiling under reduced pressure has its application in industry. There are chemicals, such as H_2O_2, which decompose before reaching the normal boiling point. In manufacture, they are separated by distillation at a reduced pressure.

In the sugar industry, sugar is separated by crystallization from solution. The syrupy solution has a high boiling point. It is made to boil at a lower temperature by reducing the pressure on it. This economizes fuel consumption.

(b) Boiling of water at room temperature. Water may be made to boil at room temperature without heating. For this purpose a high speed exhaust pump and towers containing absorbents of water vapour, such as anhydrous $CaCl_2$, are necessary.

Take a rather small quantity of water in a flask and close it with a rubber stopper. Through the stopper pass a thermometer and two glass tubes, one of which is connected to the pump and the other to a closed-tube manometer (Fig. 7.4). The pump is connected to the flask through two

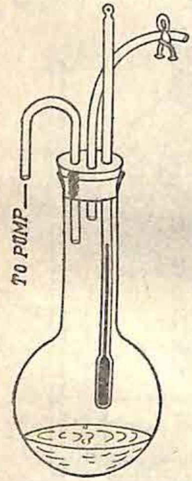

Fig. 7.4

absorbing towers containing fused calcium chloride. See that the system is leak tight. Now work the pump. When the pressure falls considerably the water in the flask begins to boil at the room temperature. As the boiling is continued the thermometer may register a temperature a little lower than the room temperature.

(c) **For a liquid to boil, its vapour pressure must be equal to the pressure on it.** A liquid is said to boil when bubbles of vapour formed within it rise to the top and burst on the surface. The pressure inside a bubble is due to the vapour it contains. To form this bubble the vapour must push the liquid away at the place where it is formed. For this purpose the pressure of the vapour must be at least equal to the pressure at that place of the liquid. Hence the vapour pressure of the liquid must at least be equal to the pressure on the liquid.

At lower depths in a liquid the temperature must be a little higher than at the top so that bubbles may form. Hence in measuring a boiling point, the thermometer should be placed in the vapour, and *not* in the liquid.

7-3.2. Boiling point of a solution. Make an arrangement as in Fig. 7.2. Add some salt to the water in the flask and put the thermometer bulb *within the liquid.* Note that boiling sets in at a higher temperature when the salt is added.

Slowly raise the thermometer above the liquid in the flask. Note that the temperature recorded gradually diminishes until it is the same as we would get if we had pure water boiling inside. This shows that the boiling point of a solution is higher than that of the pure solvent under the same pressure. But the temperature of the vapour is ultimately the same as for the pure solvent boiling under the same pressure.

Boiling point of a solution rises with its concentration.

7-3.3. Characteristics of boiling and factors affecting it.

Summarising, we may say as follows ;

(i) Boiling takes place at a definite temperature under a definite pressure. The temperature at which a liquid boils under normal atmospheric pressure is called the **normal boiling point.**

(ii) For a pure liquid the temperature at which boiling takes place at a given pressure depends on the nature of the liquid.

(iii) *A liquid boils when its vapour pressure is equal to the pressure on its surface. Hence the boiling point of a liquid rises or falls with the pressure on it.*

(iv) At a given pressure, the boiling point of a solution is higher than that of the pure solvent. But the vapour formed has finally the same temperature in both cases.

(v) Boiling is accompanied by absorption of heat. The rate of boiling depends on the rate of heat supply.

(vi) Boiling is accompanied by a large change in volume. $1\ \text{cm}^3$ of water, on boiling at 100°*C*, forms about $1650\ \text{cm}^3$ of steam.

Table. Normal boiling points of some substances in °C

Substance	Boiling point	Substance	Boiling point	Substance	Boiling point
Sulphur	444°	Water	100°	Ammonia	− 34°
Mercury	357°	Alcohol	78°	Oxygen	−183°
Paraffin	350° − 530°	Chloroform	61°	Hydrogen	−253°
Glycerine	290°	Ether	35°	Helium	−269°

Difference between evaporation and boiling. From the characteristics of the two processes already discussed, we may say that the fundamental differences between them are the following :—

(i) Boiling takes place at a definite temperature under a definite pressure, but evaporation takes place at all temperatures under any pressure.

(ii) Boiling takes place throughout the bulk of the liquid while evaporation takes place only from the exposed surface.

7-4. Boiling at high altitudes. Atmospheric pressure is greatest at the sea level and diminishes as we go up. Since the boiling point of a liquid diminishes with lowering of pressure, we should expect that in hill stations water will boil at a lower temperature than on the plains. This is actually so. On Mt. Blanc (15, 782 ft) water boils at 83°*C* ; on Mt. Everest (29,002 ft) it would boil at 70°*C*. It has been estimated that at a height of 65,000 ft water would boil at 37°*C*. At this height the water in our body, *if exposed*, would start boiling.

The following table gives the approximate temperatures at which water boils at the heights mentioned.

Height above sea-level (in ft.)	Boiling point (in °C)
2000 (~ 600 m)	98
4000	96
7000	93
10,000	90
15,000	85
20,000	80
25,000	74

It would be seen that the boiling point of water diminishes roughly by 1°C per thousand feet (960 ft to be more precise) or 300 metres of ascent above the sea-level upto about 20,000 ft. A knowledge of the boiling point of water can, therefore, give us an idea of the height of the place above sea-level.

7-5. Boiling under increased pressure. Cooking at temperatures less than 100°C is of great inconvenience. For cooking at high altitudes or in a shorter time, a special kind of cooker, called **pressure cooker** is used. In some industries, such as the paper industry, it is necessary to soften materials with water boiling at a high temperature. Such devices go by the general name of 'digesters'. The principle of the cooker or the digester may be explained with the aid of Fig. 7.5*a*. The vessel is made of thick sheet metal. The lid is attached to it air-tight. A valve closes an opening in the lid. It is kept pressed in position by a suitable weight *W* (which, in industry, may act through a lever). The weight and the position of *W* determine the pressure of the steam in the vessel which will lift the valve. If the steam generated within the vessel attains a greater pressure, the valve will be forced open and the excess of pressure released. By suitably choosing *W* the water inside may be made to boil at a desired temperature. If *W* exerts an extra pressure of about one atmosphere the boiling point of water on the plains will be about 120°*C*. Meat can be cooked in 15 minutes with such a cooker. The external force on the valve can also

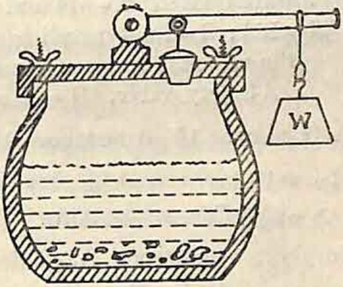

Fig. 7.5 (a)

be applied by an adjustable spring. Modern pressure cookers (Fig. 7.5 *b*) have a valve with a fixed weight on it. The temperature inside is about 110° − 115° *C*.

Papin's digester is used in manufacturing paper pulp by boiling saw dust and caustic soda under pressure, in the manufacture of artificial silk and for extracting gelatine from bones. In boilers of steam engines steam is generated under high pressure by making water boil at a temperature higher than 100° *C*. Safety valves open when a pre-determined pressure is reached. Boilers of locomotive engines may have a steam pressure of about 250 lb per sq. inch (*i.e.*, over 15 atmospheres), the water boiling at about 200° *C*.

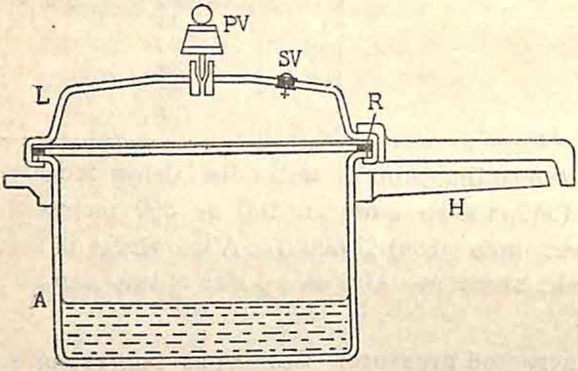

Fig. 7.5(*b*)
[Pressure Cooker. A = Aluminium container L = Lid ; R = Rubber sealing ring ; PV = Pin valve for escape of Steam ; SV = Safety valve, H = Handle]

In autoclaves and hospital sterilizers water may be made to boil at about 135° *C* corresponding to a pressure at 30 lb per sq. inch above atmosphere pressure. At this temperature bacteria and bacterial spores are killed. Dressings, towels etc. are placed in the chambers of the sterilizer. High pressure steam circulates through an outer jacket. Canning food is cooking under pressure. Sealed cans are heated by high pressure steam to about 150° *C* to kill bacteria and also to cook. But it destroys the vitamins.

7-6. Latent heat of vaporization. We have seen that there is no change of temperature when a liquid boils under a constant pressure. Nevertheless, heat must be supplied even at the boiling point in order that the liquid may boil. Heat is also necessary to evaporate a liquid. This heat, which does not raise the temperature of the liquid, but brings about a change of state from liquid to vapour, is called *latent heat of vaporization*.

There is some attraction between the molecules of a liquid, but in a vapour the mutual attraction between molecules is negligible. To convert a molecule of the liquid into a molecule of the vapour, it must be removed beyond the range of attraction of other liquid molecules. In other words, some energy is necessary to effect the change of state and this energy will not, as in the general case, go to raise the temperature of the liquid. Latent heat of vaporization supplies this energy. It may be looked upon as being stored up as potential energy of the vapour molecules, and is released as heat when the vapour condenses.

Quantitatively, **the latent heat of vaporization is the amount of heat required to convert unit mass of a liquid into vapour at the same temperature.** It is measured in *heat units per unit mass*, generally in *calories per gram.* Latent heat is independent of the process by which the change of state is brought about. Whether the liquid changes into the vapour by evaporation or by boiling, it requires the same latent heat at a given temperature.

The statement that *the latent heat of steam is 539 calories per gram* means that when water at 100°*C* is converted into steam at 100°*C*, each gram will require 539 calories for the change of state only. When 1 g of steam at 100°*C* condenses into water at 100°*C*, 539 calories of heat are released.

More heat is released by condensation of steam than by cooling of boiling water. 1 g of water at 100°C will give out 100 cal of heat if cooled to 0°*C*. But 1 g of steam at 100°*C* will give out 539 cal of heat on condensing. Thus the heat available by condenstion of steamis always much greater thanthe heat available by cooling water. This is why in cold countries steam is led through pipes into condensersin a room which it is required to heat. As the steam condenses it gives out its latent heat and the condensed water returns through another pipe to the boiler.

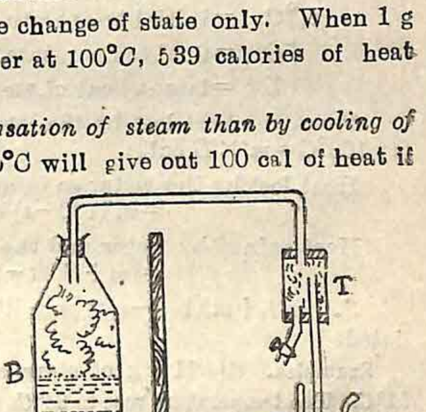

Fig. 7.6

The reason why a scald due to steam is more severe than one due to hot water lies in the larger amount of heat released by the condensation of steam.

7-7. Determination of the latent heat of steam. Take a clean dry calorimeter and weigh it. Fill it more than half with water and weigh again. Note its initial temperature.

Boil water in a vessel *B* (Fig. 7-6) provided with a delivery tube leading to a steam trap *T*. From the trap only steam can enter into the calorimeter *C* through another tube. The condensed water will be arrested in the trap and can be drained off from time to time. A wooden screen separates the boiler *B* from the calorimeter *C*. Allow the steam to condense in the calorimeter for some time. Stir the water well and read off the final temperature after the supply of steam has been cut off.

Wait till the calorimeter and its contents reach the room temperature. Weigh them again. The difference from the second weight gives the mass of the steam condensed.

Let m g = initial mass of water taken,
m' g = mass of steam condensed,
W g = water equivalent of the calorimeter,
$t_1{}^\circ C$ = initial temperature of water,
$t^\circ C$ = final temperature,
L = latent heat of steam in calories per gram.

Then, the heat lost by the steam at $100^\circ C$ to condense into water at $100^\circ C = m' \times L$ cal.

Heat lost by the water so formed to cool through $(100-t)^\circ C$

$$= m'(100-t) \text{ cal.}$$

Heat gained by water and the calorimeter

$$= (m+W)(t-t_1) \text{ cal.}$$

$\therefore$ $m'L + m'(100-t) = (m+W)(t-t_1)$, from which L can be calculated.

Examples. (1) 11·5 g of steam at $100^\circ C$ were passed into 480 g of water at $11^\circ C$. The temperature rose to $25^\circ C$. If the mass of the container is 190 g and the specific heat of its material, 0.1, find the latent heat.

Solution : Let L be the latent heat. Heat lost by 11.5 g of steam at $100^\circ C$ to condense into water at $100^\circ C = 11.5 \times L$ cal.

This water at $100^\circ C$ falls through $(100-25)^\circ C$ and the heat lost $= 11.5 \times 75$ cal $= 862.5$ cal.

Heat gained by 480 g of water
$= 480 \times 1 \times (25 - 11)$ cal $= 6720$ cal.

Heat gained by the vessel $= 190 \times 0{\cdot}1 \times (25-11)$ cal.

Equating the heat lost to the heat gained we have $11.5L + 862.5 = 6720 + 266$ whence $L = 532.5$ cal/g.

(2) 20 g of ice at $-15°C$ are heated till fully turned into steam at $100°C$. The steam is further heated to $110°C$. Find the amount of heat necessary. Given, latent heat of ice = 80 cal/g ; latent heat of steam = 538 cal/g ; sp. heat of ice = 0.5 ; sp. heat of steam = 0.48.

Solution: Heat gained by ice in being heated from $-15°C$ to $0°C$
$= 20 \times 0.5 \times 15 = 150$ cal.

Heat gained by ice in melting at $0°C = 20 \times 80 = 1600$ cal.

Heat gained by water to rise in temperature from $0°C$ to $100°C$
$= 20 \times 1 \times 100 = 2000$ cal.

Heat gained by water at $100°C$ for conversion into steam at $100°C$
$= 20 \times 538 = 10760$ cal.

Heat gained by steam in being heated from $100°C$ to $110°C$
$= 20 \times 0.48 \times 10 = 96$ cal.

$\therefore$ Total heat required = 14606 cal.

(3) Steam at $100°C$ is injected into an ice calorimeter and 100 grams of water at $0°C$ were collected. Find the amount of ice melted, given, latent heat of steam = 537 cal/g, and latent heat of ice = 80 cal/g.

Solution: Let x g of ice melt. Then steam condensed $= (100 - x)$ g.

Heat given out by the steam at $100°C$ in condensing into water at $100°C = 537\,(100 - x)$ cal.

Heat given out by water at $100°C$ in cooling down to $0°C = 100(100 - x)$ cal.

Heat gained by ice at $0°C$ in melting into water at $0°C = x \times 80$ cal.

$\therefore\ 537\,(100 - x) + 100\,(100 - x) = x \times 80$, whence $x = 88{\cdot}84$ g.

7.8. Cold caused by evaporation. A liquid cannot change to vapour unless the requisite amount of latent heat is supplied. Hence when a liquid evaporates, the portion that evaporates draws its latent heat from its neighbourhood including the rest of the liquid if there is no external source of heat. As a result the liquid and its surroundings are cooled.

This explains a number of phenomena which we come across in everyday life.

(i) When spirit is sprinkled on our skin, the spirit evaporates and takes the latent heat from the skin, which feels cold.

(ii) If after a bath we expose ourselves to a wind, or after perspiration sit below a fan the water on our body evaporates taking the latent heat from our body. We therefore feel cold.

In tropical countries an adult may perspire to the extent of a litre or more per day. A considerable part of it evaporates taking from the body the necessary latent heat, which is about 580,000 calories per litre. This is one of the ways our body gets rid of the superfluous heat.

(iii) Water in an earthenware pot is cooled in supplying the latent heat of evaporation to the water that comes out through the pores in the pot and evaporates.

(iv) The dentist sprays the gum with a liquid known as ethyl chloride before he makes an injection. The rapid evaporation of ethyl chloride cools and 'freezes' the gum so as to make it less sensitive.

7-8.1. Freezing by evaporation. (i) Pour a little water on a wooden block and place on it a thin tin can containing ether. Bubble air rapidly through the ether by means of a glass tube and hand bellows. The rapid evaporation of ether cools the water and causes it to freeze. The wooden block can then be lifted by the tin can.

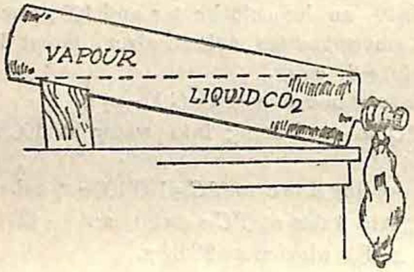

Fig. 7.7

(ii) Preparation of **dry** ice, which is solid carbon dioxide, is an example of freezing by evaporation. Liquid carbon dioxide contained under pressure in a cylinder (Fig. 7.7) issues in a jet through a valve. The rapid evaporation of the liquid cools it and a part freezes. This is collected in a muslin bag attached to the nozzle.

Dry ice has a temperature of −78°C. It is used for cooling. The ice cream vendor uses dry ice to keep his goods from melting.

7-8.2. Refrigeration. Use of refrigerators is on the increase. You often see them in a druggist store, a sweetmeat shop or a household. They provide cold chambers, around 0°C or less, in which you keep food, sera, vaccines etc. for a long time without deterioration. Large rooms can be kept cool in the same way as a household refrigerator and used for large scale storage of meat, fruit and vegetables for months. Ships which transport such items of food from one country to another are provided with cooling plants in their holds as in refrigerators.

Cooling in these devices is produced by the evaporation of a liquid under reduced pressure in a set of coils called the *evaporator coils*. The coolant may be ammonia, sulphur dioxide, ethyl chloride or freon (CF_2Cl_2). The coolant is called the *refrigerant*.

An *ice machine* is in principle a refrigerator which cools brine (*i.e.*, a solution of salt in water). Tin cans containing pure water are imbedded in the brine. Water in these cans is frozen by the cold brine.

Exercises

(*On evaporation and boiling*)

1. How would you show that a vapour exerts pressure and that the maximum pressure a vapour can exert depends on the temperature ?

2. State the principal facts about evaporation and boiling. What is the fundamental difference between them ?

3. How would you show that a liquid boils when its vapour pressure is equal to the pressure on it ?

4. What is the effect of change of pressure on boiling point ? What is meant by normal boiling point ?

Describe one experiment each to show the effect of (*a*) an increased pressure, (b) a reduced pressure on boiling point.

5. Mention some applications of boiling under (*a*) increased, (*b*) reduced pressure.

Water boils (*a*) on a hill station, (*b*) at sea-level, (*c*) the bottom of a deep mine. How do the boiling points differ at the three places ?

6. What do you understand by the statement that 'the latent heat of steam is 540 calories per gram' ? Describe an experiment to measure this latent heat.

7. Give some examples of cold produced by evaporation. Mention some practical uses of it.

8. A can of water-equivalent 100 grams contains 500 g of water at 40°*C*. Steam at 100°*C* is allowed to condense in the vessel. What quantity of steam is required to raise the water to its boiling point ? Latent heat of steam is 540 calories per gram. [ANS. : 66·7 g.]

9. What quantity of steam at 100°*C* should be condensed in a mixture of 300 g of water and 30 g of ice at 0°*C* so as to make the final temperature 30°*C* ? Latent heat of fusion of ice is 80 calories per gram and latent heat of steam, 540 calories per gram. [ANS. : 11·7 g.]

10. A calorimeter of which the heat capacity may be neglected, contains 1000 g of water at 30°*C*. It is placed on an electric stove, when the temperature of water rises to 100°*C* in 10 minutes. Calculate the amount of heat absorbed by the water per minute. How long will the water at 100°*C* take to boil away completely ? [ANS. : 7000 cal/min ; 77·1 min.]

11. By the evaporation of 25 g of liquid ammonia 85 g of ice at 0 °*C* are formed from water at 20° *C*. If the latent heat of fusion of ice is 80 calories per gram, calculate the latent heat of vaporization of ammonia.

[Ans. : 340 calories per gram.]

12. Answer the following :—

(*i*) A flask containing water is heated. After the water has been boiling briskly for some time, the flask is quickly stoppered and removed from the flame. Explain why, when the flask is dipped into cold water, the water inside begins to boil.

(*ii*) To cook food at the top of a high mountain, you employ a method different from that used on the plains. Explain why.

(*iii*) How do earthenware pots keep water cool in summer ?

(*iv*) How is dry ice formed ?

(*v*) Can water in a beaker kept on a table be made to boil by passing steam at atmospheric pressure through it ? Why ?

(*vi*) A piece of blotting paper is placed in a beaker of ether with part of the paper projecting over the edge. Soon frost is noticed on the projecting portion of the paper. Why ?

(*vii*) The temperature recorded by a thermometer decreases when its bulb is covered with a piece of cloth soaked in alcohol. Explain.

(*viii*) When can heat be supplied to a body without causing a rise of temperature ? Can this heat be recovered ?

13. If 70,000 cal of heat are extracted from 100 g of steam at 100°C, what will be the result ? (*L* of steam = 540 cal/g ; *L* of ice = 80 cal/g.)

[Ans. : 75 g of ice and 25 g of water at 0° *C*.]

14. A copper vessel of water equivalent 60 g contains 600 g of water at 30°C. A bunsen burner, adjusted to supply 100 calories per sec. is used to heat the vessel. Neglecting all losses, calculate (*a*) the time required to raise the water to boiling point and (*b*) the time required to boil away 50 g of water. (Latent heat of steam = 540 cal). [Ans. : (*a*) 462 sec. ; (*b*) 270 seconds after (*a*).]

15. A gram of ice at −20°C is heated gradually until the temperature is 120°C. Describe the changes in volume that it would undergo. Draw a rough graph of volume against temperature. State any accurate values of volume changes that you know.

[Note : The volumes of 1 g will be as follows :
as ice at 0°C ... 1 09 cm^3,
as water at 0°C ... 1 cm^3,
as water at 100°C ... 1·04 cm^3,
as steam at 100°C ... nearly 1650 cm^3.

Thereafter the increase in volume will be given approximately by Charles' law.]

16. A mass of ice, 5 g in weight, initially at $-20°C$, is heated till it is all converted into steam at 100°C. Enumerate the various changes that will take place as the temperature increases.

Calculate the total amount of heat required in the above case, assuming the specific heat of ice to be 0.5, the latent heat of fusion of ice 80 cal/g and that of steam 540 cal/g. [ANS. : 3650 cal.]

17. What is meant by the statement that the latent heat of steam is 540 cal/g ?

How would you determine this quantity in the laboratory ? What precautions are necessary for the experiment ?

When steam at 100°C is passed into a mixture of ice and water contained in a vessel it is found that 1.5 g is condensed before all the ice is melted and the mixture rises to 4°C. How much ice was there to begin with ? [Water equivalent of vessel and water = 50 g ; Lt. heat of water = 80 cal/g.]

[ANS. : 9 g nearly.]

18. Describe and explain the experiments you would perform to prove that increase of pressure (*a*) lowers the melting point of ice, (*b*) raises the boiling point of water.

Give one example of application of each of these phenomena.

19. Distinguish between evaporation and boiling and discuss the factors governing them.

How would you show experimentally that (*a*) a liquid boils when its vapour pressure equals the external pressure, (*b*) water can be frozen by its own evaporation ?

20. Describe an experiment to show the cooling of a liquid by evaporation and explain the observed effect. Do you know of any contrivance in which the above principle has been utilized ?

21. 10 g of water at 0° *C* is rapidly evaporated until the remaining water freezes to ice. Assuming that no heat is absorbed from the surroundings, calculate how much water evaporates away when the remaining water is all frozen. (Latent heat of evaporation of water is 600 cal/g ; latent heat of ice = 80 cal/g). [ANS. : 1·2 g nearly]

22. Distinguish between evaporation and boiling.

Describe an experiment to show that considerable cooling can be produced by the rapid evaporation of a liquid. Give reason for this cooling.

What is the relation between the vapour pressure of a liquid at its boiling point and the superincumbent pressure ?

HYGROMETRY

7-9. Water vapour in the atmosphere. Evaporation is constantly taking place from the surface of water which covers more than two-thirds of the face of the earth. Many millions of tons of water are drawn up into the air every hour. Air must, therefore, always contain some water vapour. Though the quantity of water vapour is small compared with the other constituents of air, such natural phenomena as rain, dew, fog etc. are due to the condensation of moisture in air.

We feel comfortable when the moisture in air lies within certain limits. Good living conditions, therefore, require a controlled amount of moisture in the air. A similar control is necessary for carrying out some industrial processes connected with cotton and wool industries, artificial seasoning of timber, manufacture of artificial silk etc. Besides, forecasting of weather requires a knowledge of the amount of moisture in the air. It, therefore, becomes necessary for us to determine the moisture content in air at different places and times. *Hygrometry* is concerned with the measurement of the amount of moisture in the air.

7-10. Saturated and unsaturated vapour. Before proceeding further it will be worth while to go back to Sec. 7-1 where an experiment has been described to show that

(i) water vapour exerts pressure ;

(ii) there is a limit to the amount of water vapour that a given space can hold ;

(iii) the vapour pressure as well as the amount of vapour in the given space increases with rise of temperature.

When a space contains the maximum amount of water vapour it can hold at a given temperature, it is said to be *saturated with the vapour*. Otherwise, it is *unsaturated*. The terms saturated and unsaturated are also applied to the vapour itself. The maximum pressure which a vapour can exert at a given temperature is variously called **saturated vapour pressure** (abbreviated S.V.P. in this book), *maximum vapour pressure*, or simply, *vapour pressure* of the liquid at that temperature. It easily follows that at a given temperature, a saturated vapour exerts the maximum vapour pressure. If the pressure exerted by a vapour is less than the maximum it can exert at the temperature, the vapour is unsaturated.

If a closed space contains both a liquid and its vapour, the vapour is saturated.

7-11. Dew point. Ordinarily air does not contain enough moisture to saturate it. If the air at any place is gradually cooled, it will tend towards saturation. At a certain temperature a given volume of air will be saturated by the moisture it contains. This temperature is called **dew point**. Further cooling causes dew to appear. When you add ice to water in a tumbler, dew forms on the outside of the tumbler due to the cooling of the surrounding air below the dew point.

As the total pressure continues to be the local atmospheric pressure, the pressure due to the vapour actually present in the air is unaffected by this cooling. Since at the dew point the vapour is saturated, *the saturation vapour pressure corresponding to the dew point is also the actual pressure due to the vapour present.*

The S.V.P. of water vapour at different temperatures has been accurately determined and compiled into a table. If the dew point at any place at a given time be determined, the actual pressure of vapour in the air could be found by reference to the table.

Table *S.V.P. of Water*

Temperature (°C)	Pressure (cm of Hg)	Temperature (°C)	Pressure (cm of Hg)	Temperature (°C)	Pressure (cm of Hg)
0	0.46	40	5.53	80	35.51
10	0.92	50	9·25	90	52.59
20	1.75	60	14·94	100	76.00
30	3.18	70	23.37	120	148.9

7-12. Humidity of air. We often use the terms 'dry' or 'moist' with respect to air. The use of these terms is generally dictated by our *feeling* as to whether the air contains little or much moisture. In reality this feeling depends on two factors, *viz.*, (*a*) the actual amount of moisture present in the air and (*b*) the moisture necessary to saturate the air under the existing conditions. Strictly speaking, *our sensation of dryness or dampness of air depends more on the ratio of* (*a*) *to* (*b*) *than on the value of* (*a*). The humidity (or dampness) of air may be expressed in either of two ways, *viz.*, (*i*) by the absolute humidity and (*ii*) by the relative humidity.

The **absolute humidity** is defined as the mass of water vapour present in unit volume of air. It is expressed in grams per cubic metre of air.

The **relative humidity** is the ratio of the actual mass of water vapour present in a certain volume of the air to the mass of water vapour required to saturate the same volume at the same temperature. It is usually expressed as a percentage. If 1 cu metre of air at a particular temperature contains 18 g of water vapour and if 30 g of vapour are required to saturate it at the same temperature, the relative humidity is 60%.

Assuming that water vapour obeys Boyle's law up to the point of saturation, the density of the vapour, and hence the mass present in a given volume, is proportional to the pressure it exerts. This assumption is not quite correct, but the inaccuracy is not important for the purpose. This leabs to another definition of relative humidity, *viz.*,

Relative humidity

$$=\frac{\text{actual pressure of water vapour present in the air}}{\text{S.V.P. of water at the temperature of the air}}\times 100.$$

Since the actual pressure of the vapour present is equal to the saturation pressure at the dew point, we may also define relative humidity as

Relative humidity

$$=\frac{\textit{Saturation pressure of water at the dew point}}{\textit{Saturation pressure at the temperature of the air}}\times 100$$

If the temperature of the air $=35^\circ C$, dew point $=26^\circ C$, S.V.P. at $35^\circ C=42.14$ mm of mercury and S.V.P. at $26^\circ C=25{\cdot}18$ mm of mercury,

$$\text{R.H.}=\frac{25\ 18}{42.14}=0.598=59.8\%.$$

The rate of evaporation of water is determined by the relative humidity of air, and not by its absolute humidity. For comfort the relative humidity of the air in a room should be kept between 50 and 60.

7·13. Measurement of humidity : Hygrometers. Hygrometers are instruments for measuring the humidity of air. There are different types, namely, (i) dew point hygrometers, (ii) absorption or chemical hygrometers and (iii) empirical hygrometers. Of fundamental importance in dew point hygrometry are tables of S.V.P. of water and ice. We shall consider only one dew point hygrometer.

Dew point hygrometer : Regnault's hygrometer.

The action of dew point hygrometers is based on the definition of relative humidity as the ratio of two pressures. They are used with vapour pressure tables which give the S.V.P. of water at different temperatures. The hygrometer determines the dew point.

In Regnault's dew point hygrometer, which is the best of the kind, a glass tube A (Fig. 7.8) closed at its upper end by a stopper carrying a thermometer T, is fitted at its lower end with a silver cap B. Ether is poured into the cap and fills it completely. Air is sucked through the ether via the tubes CD. As the ether evaporates the silver cap is cooled until it reaches the dew point. Dew then deposits on it.

As soon as dew is detected T is read. Another reading is taken when the dew disappears as the apparatus is allowed to warm up. The mean of these two values gives the dew point.

For convenience of comparison, another similar tube with the silver cap only, is mounted on the same stand. Appearance of due dulls the silver surface. Hence its appearance and disappearance can be well judged by comparison with the other silver cap. Observations are made from a distance with the help of a telescope.

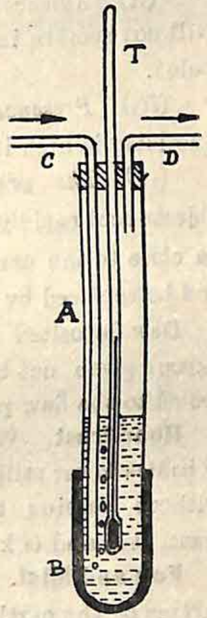

Fig. 7.8

Properly handled, it gives good values of the dew point.

If the dew point thus determined is $T'^{\circ}C$ and the room temperature is $T^{\circ}C$, then

$$\text{Relative humidity} = \frac{\text{Saturation pressure of water at } T'^{\circ}C}{\text{Saturation pressure of water at } T^{\circ}C} \times 100.$$

7-14. Dew, fog and cloud.

Dew. During the day objects on the surface of the earth are heated by direct radiation from the sun, and the air in contact with them is also heated. During the night the bodies lose heat by radiation and those which radiate well quickly cool below the temperature of the surrounding air. Air in contact with them cools too. If in this way the air cools below the dew point, a portion of the vapour is deposited as dew on the surfaces of adjacent bodies.

It is evident that the following conditions are necessary for the deposition of dew :

(*i*) *A clear sky*. If there is cloud in the sky at night, cooling of bodies by radiation is impeded.

(*ii*) *Absence of wind*. If there is wind, air near a cold object will not remain in contact with it long enough to cool below the dew point.

(*iii*) *Presence of copious moisture in the air*. If the initial humidity is high it does not require much cooling to reach dew point.

(*iv*) *The presence of good radiators close to the earth*. Such objects cool rapidly and bring the air below its dew point. They should be close to the earth as otherwise the cooled air will sink downwards and be replaced by warmer air from above.

Dew deposited on grass and leaves of plants is formed by water vapour given out by the leaves when the air around them has been cooled to the dew point.

Hoar-frost. When the dew point is below $0^\circ C$ and the temperature of bodies lower still, water vapour in the air condenses directly as solid without passing through the liquid state. The solid is deposited on grass, etc., and is known as *hoar-frost*.

Fog and Mist. In a windless night the air at regions near the surface of the earth may be cooled below the dew point, when condensation of moisture occurs throughout the mass of air. The result is a *fog* or *mist*. There is no fundamental difference between the two. A mist in which one can no longer see objects at a distance of one kilometre, is usually spoken as a fog.

The cooling is ordinarily due to radiation from the earth's surface. Hygroscopic particles play an important role in the formation of fog and mist.

Mountain mists or fogs are due to the contact of the cold air from the mountain sides with the warm saturated air of the valleys.

Clouds. When warm, moist air rises from a large surface of water, it gradually cools as it rises higher into the atmosphere. When the temperature of the air as a whole is reduced below the dew point, a cloud is formed by the condensation of vapour in the form of small droplets. The nuclei of condensation are minute salt particles originating from the sea and present in the air in sufficient numbers.

A cloud is thus a fog or a mist formed high up in the atmosphere. In the case of a fog or a mist the masses of air involved are often at rest or in very slow motion, but not so in a cloud.

The cooling which leads to the formation of a cloud may occur in a variety of ways. The chief causes are (i) the mixing of warm saturated air with a current of cold air and (ii) cooling by expansion as the air in its ascent moves to regions of lower pressure and increases in volume.

The formation of cloud due to the cooling effect of expansion of air can be shown by the following experiment. A bell jar as in Fig. 7.9 is connected to an exhaust pump through a tube passing through its air-tight stopper. A thermometer is also inserted. Partial vacuum is created within the jar by working the pump. When the inside and the outside temperatures become the same, some air is admitted from outside. The air expands suddenly and a fall in temperature will be noticed. If the admitted air is nearly saturated and contains dust particles, a cloud will be formed as soon as the air enters the jar. After a while the cloud disappears.

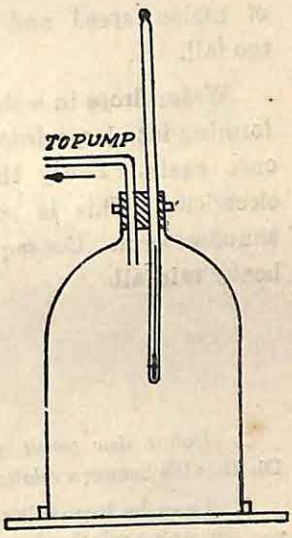

Fig. 7.9

When air saturated with moisture cools on sudden expansion water molecules preferentially condense on any ions that may be present. Wilson, an atomic physicist, constructed his 'cloud chamber' on this principle. This chamber enables us to see or photograph the track of even a single ion passing through it. The **Wilson cloud chamber** is a very important instrument in atomic physics. Wilson was awarded the Nobel prize for this invention.

Rain. The drops of water in a cloud coalesce and increase in size. Whether they will fall to the ground as rain depends on the condition of the air below the cloud, the upward speed of the cloud and various other factors.

The speed with which a rain drop moves towards the earth depends on its size. It is not a case of free fall under gravity, as the resistance due to air increases with velocity. A drop actually moves with a limiting speed, which depends on its size. Rain drops cannot be larger than 5.5 mm in diameter. Larger drops break up into smaller ones as they fall. Drops of this size fall with a speed of 8 metres per second or about 20 miles per hour. Other drops fall more slowly.

In order that a rain drop may fall to the ground, (i) the upward velocity of the cloud must be lower than the limiting velocity of fall of the drop, (ii) it must not encounter an upward moving mass of air of higher speed and (iii) must survive complete evaporation during the fall.

Water drops in a cloud are constantly going through the process of forming into large drops, breaking up into smaller ones and coalescing once again. Every time a drop breaks there is a separation of electricity. This is perhaps the principal source of electricity in a thunderstorm. Consequently thunderstorms are associated with heavy rainfall.

Exercises

1. Define *dew point* and *relative humidity*. How is the latter expressed? Distinguish between *relative* and *absolute humidity*.

2. Describe Regnault's hygrometer for measuring dew point. How would you determine relative humidity with its help?

3. What is *vapour pressure*? How would you show that (i) a liquid can exert a maximum vapour pressure at a given temperature, (ii) this maximum value increases with rise of temperature, and (iii) different liquids have different vapour pressures at the same temperature? (Sec. 7-1 and 7-1. 1)

4. Explain how dew and fog are formed.

5. Answer the following:

(*a*) The bulb of a thermometer is wrapped round with cotton, which is wetted in turn with (*i*) water, (*ii*) ether, (*iii*) oil. How will the readings differ and why?

(*b*) Explain why on a hot summer day immediately after a rain, a block of ice on a cart appears to *steam* copiously.

(*c*) What becomes of the steam which a boiling kettle discharges into a room?

(*d*) A piece of glass is dimmed when you blow on it with your mouth on a winter morning, but not on a summer noon. Explain.

(*e*) Why is summer heat often oppressive before a shower ?

(*f*) What kind of weather would you expect to find when the dew point and the air temperature are the same ?

(*g*) Why does warm moist air cause more discomfort than warmer dry air ?

6. Find the relative humidity when the room temperature is 40°C and the dew point, 30°C. (Use table in 7-11 for vapour pressure.) [Ans : 54%]

7. What would be the effect on the readings of a barometer if (*i*) a little air, (*ii*) a little water, were left in the Torricellian vacuum ? How would you detect whether it was air or water ?

8. What do you mean by the following statements ?

(*a*) The aqueous tension at 20°C is 17·5 mm.

(*b*) The relative humidity of an atmosphere is 60%.

(*c*) The dew-point on a certain day is 19.5°C.

9. Explain what effect, if any, there will be on the dew point and humidity if :—

(*a*) a quantity of water is gradually sprinkled in the room ;

(*b*) the temperature of the atmosphere in the room is raised.

CHAPTER

8

TRANSFER OF HEAT

8·1. The three ways of transfer of heat. There are three different ways is which heat can be transferred from one place to another, *viz.*,

(1) *Conduction.* When one end of an iron rod is put in the fire and the other end held in the hand, it will be noticed that this end gradually becomes warmer. Heat (vibrational energy) has passed from molecule to molecule along the rod from the warmer to the cooler end without any sensible movement of the molecules. This process of transfer of heat is called **conduction.**

(2) *Convection.* When the heated rod is removed from the fire and the hand is placed a few inches *above* the hot end, it feels warm. Air, heated by contact with the iron, carries the heat to the hand. Here the heat is actually carried by the particles of air, the movement of which is due to a change in density, warmer air being lighter than colder air. Transfer of heat due to the bodily motion of the warmer portions of a medium, brought about by a difference of density, is called **convection.** It should be clear that convection is impossible in solids, but occurs in liquids and gases.

(3) *Radiation.* If instead of holding the hand above the hot end of the rod, it is held a few inches *below* the rod, the hand feels warm, but not to the same extent as before. Heat could not have been transferred by conduction or convection. The process by which heat is transmitted under this condition is called radiation. The heat that reaches the earth from the sun is transferred by radiation. We may say that **radiation** *is the process by which heat passes from one body to another across a space without heating the space between the two.*

8·2. Conduction of heat. Transfer of heat by conduction requires a material medium. The particles of the medium take up heat energy from their neighbours on one side and hand it over to their neighbours on the other side. None of them leave their position.

Substances differ in their ability to conduct heat. One end of a glass rod may be put in the fire and brought to red heat while the other end is held in the hand. But if instead of glass you take an iron rod if will not be possible to hold it for long. This shows that heat is more easily transferred by conduction through iron than glass. We say that conductivity of iron is higher than that of glass.

8-2.1. Comparing conductvities of different solids—Ingenhausz's experiment. Conductivities of solids may be compared by Ingenhausz's method. The experimental materials (copper, iron, brass, glass etc.) are taken in the form of long thin rods of equal length, identical in area of cross-section and in surface finish. They are coated with wax and arranged as in Fig. 8.1 with one end projecting into a tank in which water is maintained at the boiling point. Heat is conducted from the water through the rods and melts the wax. After a steady state has been reached, the wax is observed to have melted to differrnt distances along the rods. Conductivity is proportional to the square of the distance over which the wax melts.

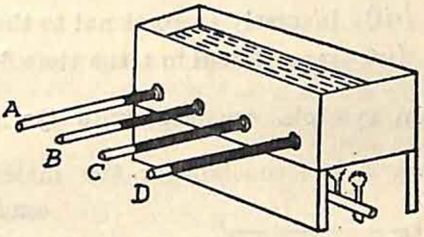

Fig. 8.1

Metals are the best conductors, silver heading the list with copper as a close second. Solids are in general better conductors than liquids and liquids better than gases,

The low conductivity of water may be demonstrated in the following way. Load a piece of ice so that it sinks in water. Put it in a test-tube about two-thirds full of water (Fig.8.2). Heat the tube near the top. The water at the top will be seen to boil before much of the ice has melted.

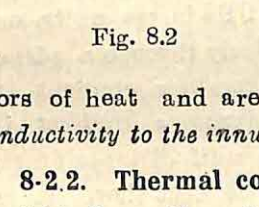

Fig. 8.2

Conductivity of gases is very low. Copper conducts heat 700 times better than water, and water, 25 times better than air. Wool, cotton wool, felt etc. are bad conductors of heat and are used as thermal insulators. *They owe their conductivity to the innumerable air pockets that they enclose.*

8-2.2. Thermal conductivity. Suppose heat is flowing in parallel straight lines through a medium. Consider a rectangular slab of material of cross-sectional area A perpendicular to the lines of flow and of thickness x in the direction of flow (Fig. 8-3). The quantity of heat Q that passes through the slab in time t is

(*i*) proportional to the area A,

(*ii*) proportional to the temperature difference $T_1 - T_2$ between the opposite faces,

(*iii*) inversely proportional to the thickness x, and

(*iv*) proportional to t, the time flow.

In symbols, we may write $Q \propto A \frac{T_1 - T_2}{x} t$ or $Q = KA \frac{T_1 - T_2}{x} t$, where K is a constant for the material and is called its *thermal conductivity* (also *coefficient of thermal conductivity*). $(T_1 - T_2)/x$ is called the *temperature gradient*. If we put A, $(T_1 - T_2)/x$ and t equal to unity in the equation for Q above, we get $K = Q$. In this way, we can define K by saying that **thermal conductivity** of a substance is the heat transferred in unit time by conduction through a plane of unit area under a temperature gradient of unity, the lines of heat flow being perpendicular to the plane.

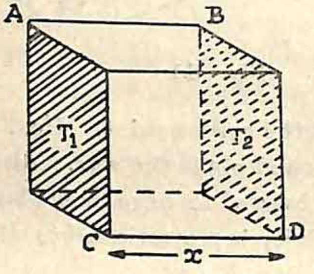

Fig. 8.3

Units. The definition helps us in finding the units in which thermal conductivity can be expressed in a given system. In the cgs system Q will be in calories, A in cm^2, $T_1 - T_2$ in °C, x in cm and t in seconds. Thus

$$K = \frac{Q\,(\text{cal})\,x\,(\text{cm})}{A\,(\text{cm}^2)\,t(\text{s})\,(T_1 - T_2)(^\circ\text{C})} = \frac{Qx}{At(T_1 - T_2)}\,\frac{\text{cal}}{\text{cm s}\,^\circ\text{C}}$$

Thus the cgs unit of K is 1 cal/cm s °C or cal $\text{cm}^{-1}\,\text{s}^{-1}\,{}^\circ\text{C}^{-1}$.

If we say thermal conductivity of copper is 0·92 in cgs units, we mean that 0·92 cal of heat will pass perpendicularly through a plane of unit area in copper in one second if the temperature gradient at the position of the plane is 1°C per cm. *Temperature gradient* is the temperature difference between two planes unit distance apart, the planes being perpendicular to the direction of heat flow. It is also the rate at which temperature changes with distance.

Table of thermal conductivities in cgs units

(cal $\text{cm}^{-1}\ \text{s}^{-1}\ {}^{\circ}\text{c}^{-1}$)

Aluminium	0.50	Lead	0.083	Cork	0.00011
Brass	0.26	Mercury	0.020	Asbestos	0.0003
Copper	0.92	Water	0.0014	Brick	0.0003
Silver	0.97	Glass	0.0025	Felt	0.00009
Gold	0.72	Oak	0.0006	Air	0.00006
Iron	0.115	Sand	0.00013		

Examples : (1) Find the quantity of heat that will be transmitted in 10 minutes across a plate of copper 1 sq. metre in area and 5 cm thick, the difference between the temperatures of its faces being 10°C. (K of copper = 0.9 cgs unit).

Solution : $Q = KA\,(T_1 - T_2)t/x$.

Here $K = 0.9$, $A = 100^2 = 10^4\ \text{cm}^2$, $T_1 - T_2 = 10°\text{C}$, $t = 600\ \text{s}$, $x = 5\ \text{cm}$.

$\therefore\quad Q = 0.9 \times 10^4 \times 10 \times 600/5 = 1{\cdot}08 \times 10^7$ cal.

Note that the temperature gradient in this case is $(T_1 - T_2)/x = 2°\text{C/cm}$.

(2) A kettle of thickness 1 mm and of total outside area 1000 cm² is filled with water whose temperature is maintained at 100°C. It is found that 120 kg of ice melt in 1 min. when the kettle is fully immersed in a tub containing melting ice. Calculate the thermal conductivity of the material of the kettle, given that the latent heat of ice = 80 cal/g.

Solution : Here the temperature difference between the two faces of the walls of the kettle is 100°C. Since 120 kg of ice melt in 1 min. the heat transmitted through the kettle in this time is 120,000 × 80 cal. If K is the required conductivity we have

$$120{,}000 \times 80\ \text{cal} = K \times 1000\ \text{cm}^2 \times 100°\text{C} \times 60\ \text{s}/0{\cdot}1\ \text{cm}.$$

$$\therefore\quad K = \frac{12{,}000 \times 80\ \text{cal} \times 0.1\ \text{cm}}{1000\ \text{cm}^2 \times 100°\text{C} \times 60\ \text{s}} = 0.16\ \text{cal cm}^{-1}\ \text{s}^{-1}\ {}^{\circ}\text{C}^{-1}.$$

8-2.3. Some consequences of thermal conduction. (*i*) When we alternately touch a piece of iron and a piece of stone, both lying in the sun, the iron feels warmer than the stone though their temperatures are the same. This is due to the better conductivity of iron. Heat flows more quickly from it and produces an impression of greater warmth. When their temperatures are lower than that of our body, the iron gives the impression of being cooler due to the quickness with which heat is transferred from the body to it. If a piece of metal in arctic cold be touched with a finger, the finger freezes to the metal due to the large and quick transference of heat.

(*ii*) **Woollen and cotton garments.** The 'warmth' of woollen garments is due to the innumerable air pockets they enclose. Because of the very low thermal conductivity of air, the garment as a whole has a low conductivity and retards the flow of heat through it. Two garments worn together will be more effective in preventing heat transference than only one of the same total thickness. The layer of air between the two further resists the flow of heat.

Woollen fibres are rough and crooked. So they can enclose innumerable air pockets. Cotton fibres are straight and smoother than wool. So they cannot form such air pockets. Hence it is easier for heat to pass through cotton garments than woollen garments.

(*iii*) Water can boil in a paper bag. The temperature at which paper catches fire is much higher than that of boiling water. This makes it possible to boil a small quantity of water in a thin paper bag. Much of the heat supplied to the paper passes through it and while this heat raises the water to its boiling point, the temperature on the lower side of the paper remains below that at which the paper ignites.

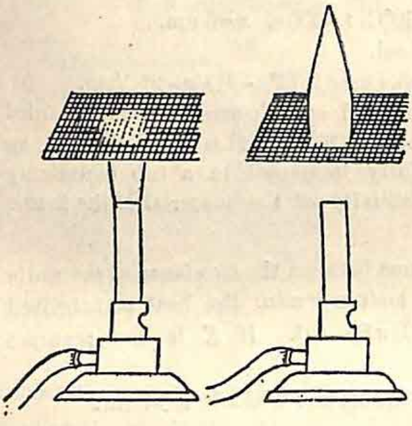

Fig. 8.4

(*iv*) **Copper gauze and flame.** When a clean copper gauze or spiral is lowered on the flame of a burner, it will be found that the flame does not get through the gauze (Fig. 8.4). The gauze conducts the heat away from the flame so rapidly that the temperature on the other side of the gauze is lowered below that at which the gas burns. Combustion is therefore confined to the part of the flame below the gauze. *When the gauze itself is heated to the ignition temperature, it ceases to be effective in preventing the combustion of the gas above it.*

When the gauze is held about an inch above Bunsen burner and the flame lighted from top, it burns above the gauze (Fig. 8.4). The conductivity of the gauze prevents the flame from extending below gauze.

(*v*) Asbestos, a fibrous mineral, is a bad conductor and is non-inflammable. It is extensively used for lining the walls of cookers and refrigerators. Boilers and steam pipes are often covered with an asbestos cement (asbestos fibre and plaster) to reduce leakage of heat.

(*vi*) Hard chalk deposits are found on the inside of boilers and kettles in areas where hard water is used. The deposit is a bad conductor. Unless it is removed from time to time much wastage of fuel is inevitable.

(*vii*) **Davy's safety lamp.** The principle illustrated in the flame-and-gauze experiment (Fig. 8.4) was utilized by Sir Humphrey Davy for the construction of a safety lamp for coal miners.

It can be used in mines without risk of explosion even when combustible gases are present. It is an oil lamp with a wire gauze *B* (Fig. 8.5) arranged above the flame. The flame itself is surrounded by a glass cylinder *A* to increase the illuminating power of the lamp. Air enters the lamp at *C* and burnt gases leave the lamp at *D*.

In an explosive atmosphere the gases which penetrate to the flame are ignited and burn inside the gauze with a peculiar flickering. But the flame produced is unable to extend outwards through the gauze. The gauze conducts the heat away from the burnt gases so quickly that the explosive gas outside the lamp never becomes hot enough to explode.

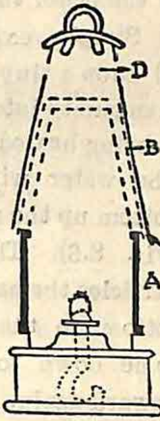

Fig. 8.5

Now-a-days miners use electric lamps. But the leader carries a Davy lamp to detect the presence of 'fire damp' (an explosive gas). If present in strong concentration, the gas burns with a blue flame.

(*viii*) The high thermal conductivity of copper makes it a material of choice in the construction of cooking utensils or small boilers, but its cost is relatively high. Aluminium being lighter and a better thermal conductor than iron is preferred for the construction of piston heads and engine cylinders.

(*ix*) An ice box or a refrigerator is constructed with a double wall. The space inside may be filled with saw dust or cork powder. Ice kept in saw dust melts slowly because of the bad conductivity of the innumerable air pockets enclosed between particles of saw dust.

(*x*) Cotton-wool, asbestos-wool, cork, felt, cellular glass etc. are some of the best non-conductors of heat. They owe this property to the innumerable air pockets that they contain. Their conductivities are a little higher than those of air. Such materials, used for heat insulation, are known as *thermal lagging*.

8-3. Convection. Convection is possible only in liquids and gases, but not in solids. When a fluid is heated it becomes lighter and ascends, while its place is occupied by cooler portions of the fluid. The current so set up within the fluid is called **convection current.** The ascending hot current carries the heat along with it. This process of transfer of heat is called **convection.** It differs from conduction, since in the latter the medium through which heat flows is not displaced.

Simple experiments on convection.

(*i*) Drop a tiny crystal of potassium permanganate into a beaker of water which is being heated by a burner from below. The water will be seen to move from the bottom up the middle and down the sides (Fig. 8.6). This shows that the water particles themselves carry heat from the bottom to the top while cooler particles come down to be heated and then move upward again.

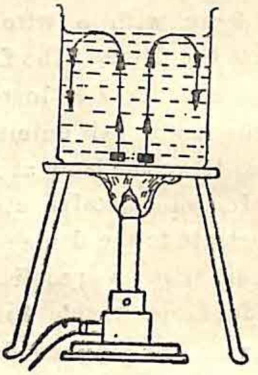

Fig. 8.6

(*ii*) When a candle burns a convection current is created in the air around the flame (Fig. 8.7). The air in contact with the flame is heated and ascends, while the surrounding cold air moves up to the flame. Thus the flame gets a continuous supply of oxygen.

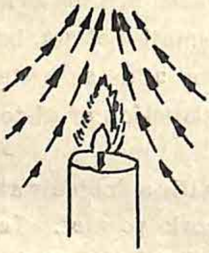

Fig. 8.7

(*iii*) In hurricane-lamps it is also the convection current which feeds the flame continuously with oxygen. Fig. 8.8 shows the path of the current. The burnt gases leave at the top.

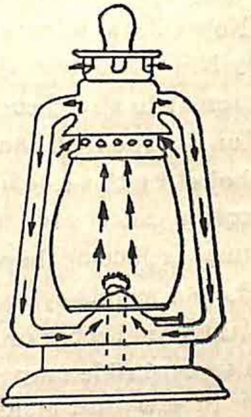

Fig. 8.8

(*iv*) **Cooling system of an automobile engine.** The water cooling the engine of a motor car is kept in circulation by convection.

The water flows in pipes surrounding the engine (Fig. 8.9). As this water is heated by taking heat from the engine it rises to a tank at the front of the bonnet. Before returning to the engine it flows through small pipes cooled by air from a fan driven by the engine.

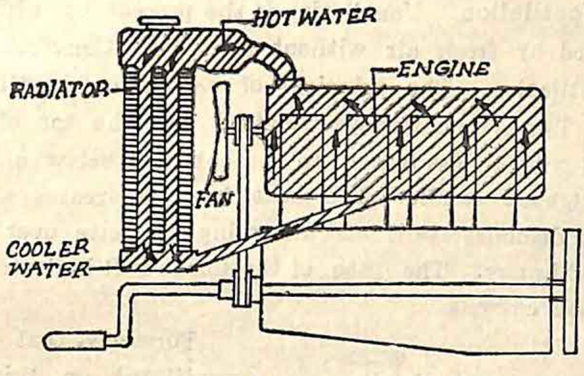

Fig. 8.9

(*v*) When a burning candle is placed inside a chimney in a way that does not allow air to enter through the bottom (Fig. 8. 10) the flame soon goes out owing to lack of oxygen. If a T-shaped cardboard or metal sheet is placed as shown in the figure, the candle continues to burn. The insertion of the sheet makes possible the flow of convection currents as shown.

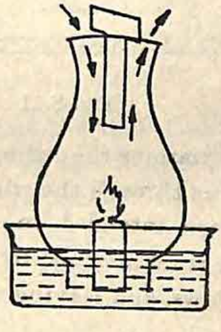

Fig. 8.10

(*vi*) *Ocean currents* are convection currents due to unequal heating of the water masses in the oceans by heat from the sun. Winds are also huge convection currents.

(*vii*) **Land and sea breezes.** Specific heats of rocks and soils are smaller than that of water. So, when the sun rises and shines on neighbouring masses of land and sea, the land warms up more quickly than the sea. The air above the land becomes warmer than that above the sea. This warm air rises and makes room for the flow of cooler air from the sea. The latter is known as *sea-breeze*.

As the sun sets, both land and sea cool down, but the land cools quicker than the sea. The air above the sea becomes warmer than that above the land. The warm air rises and makes room for the flow of cooler air from the land to the sea. This is known as *land-breeze.*

(*viii*) **Ventilation.** Ventilation is the process by which polluted air is replaced by fresh air without draughts. Convection currents produce ventilation. The principle of ventilation is illustrated in Fig. 8.11. Two glass chimneys are fitted into the top of a wooden box provided with a glass front. In the box, just below one chimney, there is a lighted candle. The heat from it creates a convection current. To demonstrate it hold a burning cigarette over the top of the other chimney. The path of the smoke will indicate the path of the convection current.

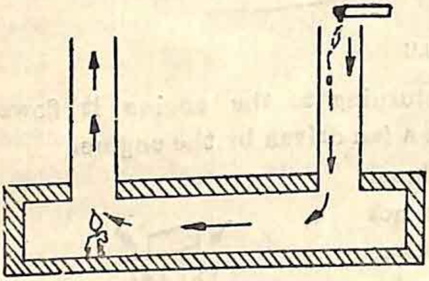

Fig. 8.11

Formerly, coal mines were ventilated on this principle. The mine had two shafts. At the bottom of one a fire was lighted. This caused the air to rise up the shaft. Air travelled down the other shaft and through the mine to maintain the updraught.

Breathed out air is generally warmer than the air in a living room. It therefore rises and escapes through the window or the ventilator. Windows open at the top are intended to allow the warm air to escape. Circulation in a room is maintained by air flowing in at the bottom of open doors and windows, and leaving at the top.

As you learn more, you will come to understand that conduction and radiation are the two basic modes of heat transfer. They obey different laws. Convection is a combination of conduction, radiation and fluid flow.

8.4. Radiation. In radiation heat passes from one body to another without the help of any material medium. The vast distance of 93 million miles from the sun to the earth is a void. (The density of matter in this region does not exceed one or two molecules per cm^3.) Heat cannot therefore come from the sun to the earth by conduction or convection. It reaches the earth by radiation. In reaching the earth, heat from the sun passes through the earth's atmosphere; but this does not heat the air. Maxwell defined radiation as the *transfer of heat from a hot body to a cooler body without appreciable heating of the intervening space.*

Experiment to show transference of heat by radiation : A heating coil is suspended inside a bell jar through its lid (Fig. 8.12). Air from inside the jar can be pumped out by an exhaust pump. A thermometer is placed outside the jar. As the pump works a vacuum is created within the jar. The thermometer still shows a rise in temperature. There is no medium in the bell jar and as such the transference of heat by conduction or convection cannot take place. So there must be a third process by which heat is transferred. This process is radiation.

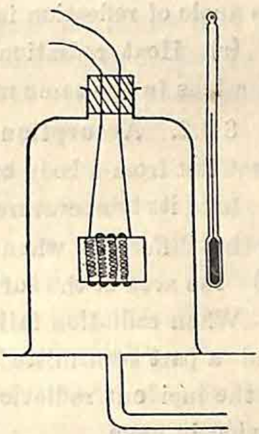

Fig. 8.12

The form of energy when heat is transferred from one body to another by radiation is called **radiant heat, heat radiation** or **thermal radiation.** We now know that it is of the same nature as light and has all properties of the latter except that it does not produce the sensation of sight.

8-4.1. Radiant heat obeys the same laws as light. That radiant heat has the same nature as light will be clear from the similarity of their behaviour as stated below :

(*i*) Both radiant heat and light are transmitted through vacuum with the same speed. During a solar eclipse light and heat from the sun are cut off at the same time. When the eclipse is over they reappear at the same time. Clearly they move with the same speed across the vast void separating the earth from the sun.

(*ii*) Radiant heat travels in straight lines as light does. When an opaque body is held in the sun, it casts a shadow, where no heat is felt. As shadows are due to the rectilinear propagation of light and no heat is felt in the shadow, heat must have travelled in straight lines.

(*iii*) The intensity of light varies inversely as the square of the distance. Radiant heat obeys the same law. When the distance between a Leslie's cube and a thermopile is doubled, the deflection of the galvanometer reduces to a quarter of its value. (Leslie's cube is a simple device for producing thermal radiation at about 100°C. A thermopile is a sensitive device for detecting thermal radiation. It works in conjunction with a galvanometer.)

(*iv*) When radiant heat is reflected from a polished metal surface, its angle of reflection is equal to the angle of incidence.

(*v*) Heat radiation may be focussed at a point by a concave mirror or a lens in the same manner as light.

8.4.2. Absorption and emission of radiation. The amount of heat lost from a body by radiation depends on

(*a*) its temperature and that of the surroundings, being proportional to the difference when it is not large ; (*b*) the nature of its surface ; (*c*) the area of the surface ; and (*d*) the time.

When radiation falls on matter, a part is absorbed, a part reflected and a part transmitted. If a, r and t respectively denote the fractions of the incident radiation aborbed, reflected and transmitted, we shall obviously have

$$a+r+t=1.$$

It will be seen from this equation that *a good absorber*, such as lamp black ($a=0.98$) *is a poor reflector*, and that a *good reflector*, such as a polished metal surface, *is a poor absorber*. Absorbed radiation raises the temperature of the absorber.

It has been found that **good absorbers of radiation are also good emitters.** Black is a better emitter or absorber than white. There is thus good reason for wearing white clothes in the tropics ; it absorbs little heat. A rough surface absorbs or emits radiation more than a polished surface. Calorimeters, therefore, have polished surfaces so that loss or gain of heat by radiation may be a minimum.

Cloud and radiation. Water droplets in the air absorb much of the solar heat. On a cloudy day in summer the temperature of the earth is comparatively low. On a clear day the temperature rises high. Clouds have a screening effect on the earth. Presence of clouds at night prevents radiation loss from the earth.

Greenhouse. Glass absorbs longer heat rays, but transmits shorter rays. Long heat rays radiated by plants in green houses are absorbed by glass and re-radiated into the greenhouse. A greenhouse thus acts as a heat trap.

8-4.3. Prevost's theory of exchanges. The energy that a body radiates by virtue of its temperature is called **temperature radiation** or **thermal radiation.** [Part or whole of the energy that a body receives from its surroundings (or any other object) which goes only to raise its temperature, but produces no other change, is given out by the body as thermal radiation.] It will be wrong to assume that a body does not radiate unless it is heated. Suspend a slab of ice in a vacuum so as to reduce to a minimum the gain of heat by conduction and convection. It receives heat by radiation from the surroundings (if at a higher temperature than that of ice) and melts. If dry-ice (solid carbon dioxide) is similarly kept surrounded by ice, it evaporates. It cannot be logical to assume that ice radiates in the latter case, but not in the former. It is more reasonable to think that radiant energy is given out by ice in both cases. Melting takes place in the first case because the ice receives more heat by radiation than it gives out. In the second case, it gives out more heat by radiation to the dry-ice than it receives from the latter.

Prevost applied the idea of dynamic equilibrium to the thermal radiation emitted or received by a body. If the body is at a higher temperature than its surroundings, it will radiate more energy than it receives, and will be cooled until temperature equilibrium is established. If the body is at a lower temperature it will receive more energy than it radiates and will rise in temperature. If it receives as much energy as it radiates, there will be temperature equilibrium. **The difference of the radiant energy absorbed and the radiant energy emitted is 'heat'.**

Consideration of such exchange of radiation led Prevost (1751-1839) to propose that

"A body radiates heat at a rate which depends only upon its surface and temperature, and that it absorbs heat at a rate which depends only upon its surface and the temperature of its surroundings. When the temperatures are the same the rates of absorption and emission will be equal."

Put in other words, Prevost's theory states that

The amount of energy emitted as radiation from a body depends solely upon what takes place inside the body and is not influenced by its surroundings.

Prevost's theory proved very valuable in establishing the laws of thermal radiation. One of the earliest of these laws is Stefan's law. It is discussed below,

8-5. Stefan's law of radiation. By virtue of its temperature, every body radiates some energy in all directions in the form of electromagnetic waves. Such radiation is called 'temperature radiation' or 'thermal radiation'. The rate of radiation per second per unit area depends upon (*i*) the temperature and (*ii*) the nature of the surface.

Tyndall was the first to carry out some experiments on the amount of energy radiated by a hot platinum wire to its surroundings kept at a fixed temperature. From these experiments Stefan (1879) concluded that *the net energy interchange by radiation between a hot body at temperature* T_1°*K and a colder body at temperature* T_2°*K is proportional to* $T_1^4 - T_2^4$. This is known as Stefan's law of radiation.

Later, Boltzmann (1844-1906) deduced the law on theoretical grounds. To eliminate the effect of the nature of the surface, an ideal radiator had to the imagined. It is called a 'black body' and has the property that the thermal radiation it emits per unit time per unit area is greater than that emitted by any other body at the same temperature. Theoretically, it was found that energy radiated by a black body per unit area per second is given by $E = \sigma T^4$ where T is the absolute temperature of a black body, and σ is a constant of value $5{\cdot}67 \times 10^{-5}$ erg/cm^2.s. K^4. It is called the Stefan-Boltzmann constant.

An actual boby radiates less than an ideal black body at the same temperature. But the radiation from a black surface or a nearly closed space (such as an oven or a furnace) is close to that from the ideal black body. This enables us to calculate the temperature of an incandescent body with the help of Stefan's law. The sun's temperature has been so measured. The temperature of the visible surface of the sun was found with the help of Stefan's law to be about 6160°*K*. Instruments called 'total radiation pyrometers' can measure temperatures above 800°C. They work on the basis of Stefan's law of radiation, and are used in industries for measuring temperatures of furnaces and other very hot bodies.

Exercises

1. Illustrate with suitable examples the different methods of transfer of heat. Bring out clearly the difference between them.

2. How would you show that different substances have different conductivities ?

Describe briefly and explain the action of (*a*) a practical appliance where use is made of the good conducting power of a material, (*b*) an appliance where a bad thermal conductor is employed.

3. Draw a sketch of Davy's safety lamp. Explain the use of the wire gauze surrounding the flame.

4. What is a *convection current ?* Give three examples of convection currents where they are made to serve some useful purpose.

Distinguish between conduction and convection.

5. Hot water is placed in two identical jugs, one with a polished white surface and the other with a black surface. Which one will cool more quickly and why ?

6. Clouds prevent loss of heat by radiation from the earth's surface. How does the presence of night clouds affect formation of mist or fog ?

7. Answer why :

(i) The squirrel wraps its bushy tail round its body during its winter sleep.

(ii) Bird puff their feathers out on a cold day.

(iii) A hen, wishing to sit on eggs to hatch out chicks, casts feathers from her breast.

(iv) A house with a straw roof keeps cool in summer and warm in winter.

(v) The bulb of a delicate thermometer is immersed at a slight depth below the surface of water. The upper surface of the water is heated ; but the thermometer hardly shows any temperature rise. On heating the water from below, the thermometer shows an immediate rise.

(vi) A gas-filled electric lamp feels hotter than an evacuated lamp when lit.

8. To minimize transference of heat between a calorimeter and its surroundings, it is placed on cork tips (or suspended by silk or cotton threads) inside an enclosure. Also, its surface is polished and silvered.

Briefly explain how all these minimize the transference of heat.

9. Explain why (*a*) thermometer with its bulb covered by soot, when kept in the sun, records a higher temperature than an ordinary one.

(*b*) The flame of a bunsen burner cannot get through a piece of wire gauze placed upon it.

10. Explain (i) why the space between the double walls of a thermos flask is evacuated, (ii) why the inner surface of these walls is silvered, and (iii) why the mouth of the flask is closed by a cork stopper.

11. An aluminium pan is placed on a fire for water to boil in it. If the bottom is 1 mm thick, what is the temperature of the side facing the fire, given that 1 g of water vaporizes in 2 minutes per sq. cm area of the bottom. (*K* for Al = 0.5 cgs unit ; latent heat of steam = 540 cal/g.) [ANS. : 101°C nearly.]

12. The metal plate of a boiler is 1.5 cm thick. Find the difference of temperature between its faces if 32 kg of water are evaporated from the boiler per sq. metre per hour. (Latent heat of steam = 540 cal/g; K of boiler plate = 0.15 cgs unit). [ANS.: 4.8°C]

13. What are the different modes of propagation of heat energy? Explain each of them with suitable illustrations.

What device would you employ in the case of a vessel containing a hot liquid for reducing heat transference to a minimum?

14. What is meant by the statement that the thermal conductivity of glass is 0.002 in C.G.S. units? Describe an experiment by means of which the comparative conductivities of a number of metal rods may be found.

The inside of a glass window, 2 mm thick and 1 sq. metre in area, is at a temperature of 15°C and the outside is at −5°C. Calculate the rate at which heat is escaping per hour from the room by conduction through the glass.

[**Hint :** Convert quantities into cgs units.

$1 \text{ m}^2 = 100 \times 100 \text{ cm}^2$. Tem. diff. = 20°C.

$$\text{Heat escaping per hour} = 0.002 \times \frac{100 \times 100 \times 20}{0.2} \times 60 \times 60 \text{ cal}$$

$$= 7.2 \times 10^6 \text{ cal.}$$

15. State the modes of propagation of heat energy and explain their difference. What are your arguments in favour of the conclusion that radiant heat is but invisible light?

A rod of a metal of thermal conductivity 0.9 is 31.41 cm long and 4 cm in diameter. One of its ends is kept exposed to steam at 100°C and the other in contact with a block of ice at 0°C. How much ice will melt per minute in a steady state? [ANS.: 27 g]

16. State Stefan's law. Write a short note on it, stating any use that you know of it.

When Stefan's law is used for measuring the temperature of a molten metal in an open crucible, do we get the correct temperature of the metal? Give reason for your answer.

CHAPTER

9

KINETIC THEORY OF GASES

9-1. Evidence of molecular structure of matter and of random molecular motion.

In science, an *evidence* of a theory is rarely direct. (Consider, for example, Newton's first law of motion.) But when we find that things occur without exception as deduced from the theory, we take it as an evidence of the correctness of the theory. Most evidences in science are of the above nature and indirect. But science accepts such evidence.

Evidence of molecular structure. At least three thousand years ago many Indian and Greek philosophers thought that matter was not continuous in structure. They thought matter was made of particles, but they had no proof to support it. In the eighteenth century, John Dalton, an English school teacher, successfully explained the laws of chemical combination assuming that a given substance (such as iron or sulphur) was made of particles, all of the same kind. He called such particles 'atoms'. Combinations of atoms of the same or different kinds were later given the name 'molecules'. Dalton's laws for the formation of molecules from atoms were fully supported by chemical experiments. This is the first evidence of molecular structure. It came from chemistry and was of an indirect nature.

With progress of science, we could explain various phenomena from different fields on the basis that matter was molecular in structure. We came to know that molecules are so small that we never have any chance of *seeing* them. They have diameters of the order of 10^{-7} cm. The most powerful microscope made of lenses can magnify a small body to about 2000 times. This is not enough to enable us to *see* a molecule. Another kind of microscope, called the 'electron microscope' can magnify upto 10^5 times. Even this is not enough. But the pictures of very thin sections of matter that we get from powerful electron microscopes show that matter has a granular (grain like) structure. Another kind of microscope, known as the 'Field emission microscope', produces pictures of very sharp metal points. These pictures show that the point is made up of layers of granular particles. These are the nearest visual proof that matter has a discontinuous structure and is made up of particles. The granular structures as seen in the pictures mentioned above are not molecules; they are groups of molecules.

We however have enough *indirect evidence* of molecular structure. If alpha particles are allowed to shoot through very thin metal foils, some of them are found to be deflected very much from their original paths. This is possible if the metal foils are made up of atoms whose positive charges are confined to very small regions. You will get more cases of indirect evidence as you learn more of modern physics. Radioactivity will give you much such evidence. With the help of X-rays we have come to know the sizes of molecules and atoms. The 'mass spectrograph' enables us to find the relative masses of atoms accurately.

Evidence of random molecular motion. (*i*) The fact that a gas exerts the same pressure all over its containing wall supports the view that gas molecules move about at random.

(*ii*) If we drop a crystal of copper sulphate in water, the crystal soon dissolves and its blue colour gradually spreads throughout the water. This can easily be explained if we assume that copper sulphate crystals are made up of molecules, and that in solution these molecules move about at random.

(*iii*) When water (or any liquid) evaporates its molecules are lost in the atmosphere. They must have random motion. Otherwise evaporation should have occurred in some special direction only. The same applies to boiling.

(*iv*) When a solid melts, the liquid formed takes the shape of the containing vessel. This can be explained by assuming that the solid is made up of molecules and that these molecules move about at random when in the liquid state.

(*v*) Suspend very fine particles, of diameters about $10^{-3}-10^{-5}$ cm, in water or air. Look at them through a powerful microscope. They will be found to move about at random with a zig-zag motion. (See 'Brownian motion' in the next section.) These particles are not molecules, but are made up of about 10^{6} to 10^{12} molecules. Their zig-zag motion can be explained if we assume that they are constantly being hit by molecules of water or air from all sides. When the particles are small, the number of hits from one side may not be equal to those from the opposite side. Besides, the water or air molecules may not all have the same speed and they may be moving in all possible directions. Such property of the molecules can fully explain the behaviour of the suspended particles as we find it. This is the most direct evidence of the *continuous* and *random* motion of molecules. (The motion of the particles is somewhat similar to the movement of a man who carries a heavy load and is moving in a crowd.)

9-2. Brownian movement. The phenomenon of Brownian movements (or Brownian motion) was discovered in 1827 by an English Botanist, Robert Brown. He observed that minute pollen grains suspended in a liquid drop showed a constant, irregular motion when seen through a powerful microscope. They danced about in the field of view of the microscope without any apparent reason. The movements continue indefinitely and never show any sign of stopping or even diminishing. These movements are known as **Brownian movements.** Aqueous suspensions of solid particles or droplets ($10^{-3}-10^{-5}$ cm in diameter) also show these movements.

The following facts gradually came to be established about Brownian motion :

(*i*) The motions are continuous, eternal, and completely irregular and random. No two particles in the same locality move in the same direction at the same time.

(*ii*) The movements are independent of any mechanical vibration of the container.

(*iii*) The lower the viscosity of the liquid the greater the motion.

(*iv*) The smaller the particles the greater the motion.

(*v*) Two particles of the same size move equally fast at the same temperature.

(*vi*) The higher the temperature the greater the motion.

From these facts the following picture of the processes giving rise to the motions may be drawn. The suspended particles are millions of times larger than the molecules of the liquid. The molecules of the liquid are executing their heat motions. In doing so they constantly bombard the particles from all sides. If the particles are large in size, equal numbers of particles hit them on the average on all sides at each instant. But if they are small enough, the molecular impacts may not be balanced. A small particle may therefore be acted upon by an unbalanced force, which gives it a perceptible motion. (This motion is opposed by the viscous drag of the liquid.) Since the molecular impacts are at random, the unbalanced force on the particle is randomly directed. This gives the particle the motion described. Such particles behave like large molecules in the liquid. They take part in the thermal motion of the liquid and provide a visual proof of the basic assumption of continuous and eternal heat motions of molecules.

9·3. Basic assumptions of the kinetic theory of ideal gases. Gases have the simplest structure, and the ideal gas in the easiest to treat mathematically.

We have at several places pictured a gas as consisting of molecules moving about at random with high speeds. The **kinetic theory of gases** attempts to explain the behaviour of gases on the basis of this picture. The **basic assumptions** made for the purpose are the following :

(1) The molecules of a gas are all alike and behave as if they were hard, smooth, *perfectly elastic* spheres. Even in a small volume the number of molecules is very large. (In air at S. T. P., 1 cm^3 contains $2{\cdot}7 \times 10^{19}$ molecules).

(2) The molecules are in *continual, random* motion, colliding with one another and with the walls of the containing vessel. The collisions are perfectly elastic.

(3) Molecules do not exert any appreciable attraction on one another.

(4) The actual volume occupied by the molecules is negligible compared with the total volume of the vessel.

(5) In between two successive collisions the molecules travel in straight lines with a constant speed. In the most elementary calculations this speed may be taken to be the same for all molecules of a gas at a given temperature. (The speeds are really different.)

(6) The time spent in collision is negligible compared with the time of travel between collisions.

9-4. Explanation of pressure of a gas on the kinetic theory. The molecules of a gas in their random motion constantly collide with and rebound from the walls of the container. A molecule of mass m and speed u colliding perpendicularly with a wall will rebound from it with speed $-u$. The change in momentum of the molecule is $mu-(-mu)=2mu$. The wall receives at each collision an impulse equal to the change in momentum. If during one second a large number of such impacts occur, the average force on the walls, by Newton's second law, is equal to the total change in momentum during that second. The pressure is the force acting on unit area of the wall.

When calculations are made on this basis, it is found that the pressure P of the gas in absolute units is given by

$$P=\tfrac{1}{3}mnc^2 \qquad (9\text{-}4.1)$$

where m = mass of a molecule.

n = number of molecules per cubic centimetre of the gas, and

c^2 = square of the speed of the molecules (properly, the mean value of the squares of the speeds.)

Value of c. In the relation $P=\frac{1}{3}mnc^2$, mn is the mass per unit volume of the gas. Hence $mn=\rho$, the gas density.

$$\therefore\quad P=\tfrac{1}{3}\rho c^2 \quad \text{or} \quad c=\sqrt{3P/\rho} \qquad (9\text{·}4.2)$$

For hydrogen at S. T. P., $\rho=0.00009$ g/cm^3, standard atmospheric pressure $P=76\times13.6\times980$ dyn/cm^2.

$$\therefore\quad c=\sqrt{\frac{3\times76\times13\text{·}6\times980}{0\text{·}00009}}=1.85\times10^5 \text{ cm/s.}$$

Values of c at 0°C for some gases are given below :

Gas	Speed at 0°C
Hydrogen	18.6×10^4 cm/s
Helium	13.1×10^4 ,,
Nitrogen	4.93×10^4 ,,
Oxygen	4.61×10^4 ,,
CO_2	3.92×10^4 ,,

9·5. Concept of temperature from kinetic theory.

Boyle's law follows easily from the relation $P=\frac{1}{3}mnc^2$. Multiplying both sides of the equation by V, the volume of the gas, we get $PV=\frac{1}{3}mnVc^2=\frac{1}{3}mNc^2$, where N = total number of molecules in the vessel. *If c is constant at a given temperature*, $PV=$ constant when the temperature is fixed.

If N_A represents the number of molecules in a gram-molecule of a gas, *i.e.*, $N_A=$ Avogadro's number, we have for one gram-molecule

$$PV=\tfrac{1}{3}\,mN_Ac^2.$$

Comparing it with the perfect gas equation $PV=R_MT$

we have $\tfrac{1}{3}\,mN_Ac^2=R_MT$ (9-5.1)

Now, the kinetic energy of a molecule $=\tfrac{1}{2}\,mc^2$.

$$\therefore\quad \frac{1}{2}mc^2=\frac{3}{2}\,\frac{R_M}{N_A}\,T=\frac{3}{2}kT \tag{9-5.2}$$

where $k=R_M/N_A$ is a constant and is called *Boltzmann's constant*.

From the kinetic theory of ideal gases we therefore find that

The kinetic energy of a molecule is proportional to the absolute temperature.

The statement that the 'kinetic energy of the centre of mass of a molecule is proportional to the absolute temperature' is very important. We apply it to liquids and solids as well.

Eq. 9-5.2 serves as a definition of temperature. So defined, it is called the 'kinetic temperature'. Temperature can also be defined in other ways, such as from thermodynamics. But this is not the place to go into them.

9-6. Limitations of the ideal gas laws. Ideal gases have been defined as gases which obey both Charles' law and Boyle's law at all temperatures and pressures. No real gas is perfect because (i) when sufficiently cooled a gas becomes a liquid, and (ii) at high pressures the product PV of all gases increases. Besides, real gases also show other deviations from Charles' law and Boyle's law.

Real gases do not fully satisfy all the basic assumptions of the kinetic theory of ideal gases. It was assumed that (i) molecules do not exert any appreciable attraction on one another and that (ii) the actual volume occupied by the molecules is negligible compared with the total volume of the vessel. When at high pressures the molecules of a gas are brought close together, they not only experience mutual forces of attraction, but the volume they occupy becomes comparable with the total volume in which they are confined. When real gases at high pressures are sufficiently cooled they turn into liquid due to molecular attraction.

Besides, molecules have a structure and all kinds of molecules cannot be treated as hard, perfectly elastic spheres. In this respect, the inert gases (He, Ne, Ar, etc.) are closest to ideal gases as they are all monatomic. Other gases have two or more atoms to the molecule and gradually cease to behave as perfectly elastic spheres as temperature rises.

As the distance between molecules of a gas increases, the gas behaves more and more like an ideal gas. All the above basic assumptions of the kinetic theory come closer and closer to fulfilment as the pressure of the gas diminishes. At extremely low pressures any real gas behaves practically like an ideal gas (unless the temperature is also extremely low).

Though the ideal gas laws do not strictly apply to real gases under ordinary circumstances, we shall, at this preliminary stage, treat all gases as ideal.

Exercises

1. What evidence can you offer to support that matter has a molecular structure ?
2. What evidence can you offer to support that in liquids and gases molecules have a random motion ?
3. What is Brownian motion ? What is its importance in connection with the kinetic theory of gases ?
4. What are the fundamental assumptions of the kinetic theory of gases ?
5. State briefly how the pressure of a gas is explained on the kinetic theory of gases.
6. How do you derive the concept of temperature from the kinetic theory of gases ?
7. If the pressure and density of a gas are known, how can the velocity of a gas molecule be determined from them ? What relation is there between temperature and velocity of a gas molecule ?
8. The density of oxygen at S.T.P. is 1.43 g/l. What is the velocity of an oxygen molecule ? What will be the velocity at 100°C ?
9. How does a real gas differ from a perfect gas ? Under what condition will a real gas behave closest to a perfect gas ?
10. What are *Avogadro's number*, the *universal gas constant* and *Boltzmann's constant*. How are they related ?

CHAPTER

10

HEAT AND WORK

10-1. Heat is a form of energy. To the question 'What is heat ?' we have long ago answered by saying that 'Heat is a form of energy ; it is not matter'. A body weighs the same whether it is hot or cold. If heat were a form of matter, the body would have weighed more when hot.

It is more difficult to show that heat is a form of energy. We have, however, been led to this conclusion by a variety of phenomena. When we hammer a piece of metal or bend a wire back and forth, it gets hot. Pressing a piece of steel on a grinder produces a crop of sparks. Examples of this kind where heat is produced by motion can be multiplied without limit. Steam engines (and other engines too) produce motion from heat. All these show that kinetic energy and heat are intimately related and one may be converted into the other. Heat must, therefore, be a form of energy.

We now associate temperature with the mean kinetic energy of motion of molecules, absolute temperature being proportional to the mean kinetic energy of molecules according to the kinetic theory. As the temperature rises, the kinetic energy of a molecule—whether in a solid, liquid or gas—also increases. Increase in the pressure of a gas with increase in temperature, heating of a gas by sudden compression and cooling by sudden expansion, cooling of a liquid by evaporation, etc. support this view.

But what is the form of heat energy ? We now define heat as being "energy in transit due to temperature difference". The molecules which make up a piece of matter have both kinetic and potential energies. This total energy is called the **internal energy** of the body. If there are two bodies whose average molecular kinetic energies are different, they are at different temperatures. If these bodies are put in contact, some of the kinetic energy from the stock of internal energy of the warmer body passes on to the cooler body and increases the internal energy of the latter. This goes on until the average molecular kinetic energies of both bodies are equalized. The flow of energy then ceases. The energy that has been so *moving* is heat. When the flow is over the energy ceases to be heat ; it becomes part of the internal energy of the cooler body. You may compare 'heat' with rain drops falling into a lake. So long as they fall in the form of drops they are rain drops. As soon as they reach the lake, they become part of the water of the lake.

Remember the case of kinetic energy. It is kinetic energy so long as the body is *moving*. As soon as the body stops, kinetic energy is transformed into some other form, and ceases to be kinetic energy. Similar consideration applies to heat.

We say mechanical energy is converted into heat, as in compressing a gas. When work is done on a gas, that is, it is compressed, its volume diminishes and pressure increases. This increases the mean kinetic energy of the gas molecules. So the temperature of the gas rises. When a body increases in temperature, we say heat has been added to it, the amount of heat being given by the relation $Q = mst$ (Eq. 5-2 1). So, a temperature rise of t is equivalent to the addition of an amount of heat Q. Really there has been an increase of internal energy of amount Q.

10-2. The mechanical equivalent of heat. When we rub our hands together, heat is produced ; friction always produces heat. This shows that mechanical work can be converted into heat. The equivalence between heat and mechanical work was established by Joule whose experiments show that (*i*) **work can be fully converted into heat** and (*ii*) **whenever mechanical work is fully converted into heat, the heat generated is directly proportional to the work done.** The equivalence is independent of the way in which the work is derived or the means by which the transformation from work to heat is brought about.

If W units of mechanical work, on *full* conversion into heat, produce Q units of heat energy, then

$$\frac{W}{Q} = \textbf{constant} = J \qquad (10\text{-}2.1)$$

where the constant J is called the **mechanical equivalent of heat** or **Joule's equivalent.** The numerical value of J however depends on the units in which W and Q are expressed. If W is in ergs and Q is in calories,

$$J = 4.2 \times 10^7 \text{ ergs per calorie.}$$

(More accurately $J = 4.1855$ joules/calorie)

This means that when 4.2×10^7 ergs of work are *fully* converted into heat, the heat developed is one calorie, which can raise the temperature of 1 g of water 1°C.

If W is in joules and Q in calories, we have

$$J = 4.2 \text{ joules per calorie.}$$

If W is in foot-pounds and Q in British thermal units

$$J = 778 \text{ ft-lb per Btu.}$$

If Q is measured in lb. °C (centigrade heat unit)

$$J = 1400 \text{ ft-lb per lb } ^\circ C.$$

Thus if 1400 ft-lb of mechanical work, derived in any way, be *fully* converted into heat, it will raise the temperature of 1 lb of water 1°C.

Note that using the joule as the unit of heat makes use of the term mechanical equivalent of heat unnecessary.

Examples. (1) A mass of 10 g moving with a speed of 300 metres per sec is suddenly brought to rest. If the whole of its kinetic energy is converted into heat, find (i) the heat developed and (ii) the rise in temperature of the body, assuming that the heat developed is fully retained by it. (Sp. heat = 0.03).

Solution : Speed of the body = 30,000 cm/s ;

kinetic energy $= \frac{1}{2} \times 10 \times (30{,}000)^2 = 4.5 \times 10^9$ erg.

$$Q = \frac{W}{J} = \frac{4.5 \times 10^9 \text{ erg}}{4.2 \times 10^7 \text{ erg/cal}} = 107.1 \text{ cal}$$

Since heat gained = mass × sp. heat × temp. rise,

the rise in temp. of the body $= \frac{107.1}{10 \times 0.03} = 357$°C.

(2) From what height must a gram of ice at 0°C fall in order to melt itself by the heat generated in the impact ?

Solution : In the absence of any statement to the contrary it is assumed that the whole of the heat developed is retained by the body.

Let h cm be the required height. The potential energy of the body at that height is $mgh = 980h$ ergs. When it reaches the ground this energy is fully kinetic. On impact it is converted into heat. The heat developed should be 80 calories, the latent heat of the gram of ice.

$$\therefore \quad 80 = \frac{980h}{4.2 \times 10^7} \text{ or } h = 3.43 \times 10^6 \text{ cm } (= 34.3 \text{ km}).$$

(3) A lead bullet moving with a speed of 400 metres per second strikes a target. Find its rise of temperature assuming that the heat produced is shared equally by the target and the bullet. Sp. heat of lead = 0.03.

Solution : Kinetic energy $= \frac{1}{2}m \times (40,000)^2$ ergs, where m is the mass of the bullet in grams.

$$\text{Heat developed} = \frac{\frac{1}{2} \times m \times (40,000)^2}{4.2 \times 10^7} \text{ cal.}$$

Since the heat is equally shared by the target and the bullet, half of this heat is available for raising the temperature of the bullet.

$$\text{Rise in temperature} = \frac{\text{Heat}}{\text{mass} \times \text{sp. heat}} = \frac{\frac{1}{2} \times \frac{1}{2} \times m \times 16 \times 10^8}{4.2 \times 10^7 \times m \times 0.03} = 317.5\ {}^\circ C.$$

(4) The Victoria Falls are 343 ft in height. Calculate the difference in temperature of the waters at the foot and at the top, assuming a value of 778 ft. lb per Btu for J.

Solution : Consider a mass m lb of water. The potential energy of this mass at the top of the fall is $343\,m$ ft. lb. When converted into heat this will give $343m/778$ Btu. If t is the temperature rise of water, then, since the sp. heat of water is unity, we have

$$mt = 343m\ /778 \text{ or } t = 343/778^\circ F = 0.44^\circ F.$$

10-3. Joule's experiment for the determination of J. The first exact determination of the mechanical equivalent of heat was made by Joule with the apparatus shown in Fig. 10.1. C is a cylindrical copper vessel containing water. Four radial partitions or vanes are fixed vertically inside the cylinder. A paddle can be rotated in the water about a vertical spindle (S). It carries eight sets of vanes which pass through spaces cut in the radial partitions as the paddle rotates. In the figure the radial partitions are represented by V_1 and the movable vanes by V_2. To turn the spindle (S) it is attached by a pin (P) to a rod (R). Two threads wound round the rod (R) pass over two pulleys (P_1, P_2) and may be loaded at the other ends with known weights.

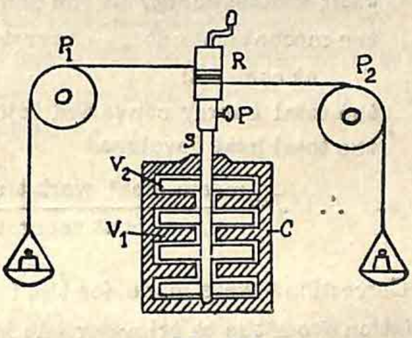

Fig. 10.1

When the weights are allowed to fall under gravity, the rod rotates and so does the paddle. As the water is churned its kinetic energy is destroyed at the radial vanes (V_1) and converted into heat. When the weights reach the ground, the pin (P) is detached and the weights are wound up again by turning a handle attached to the rod (R). As the churning continues water is gradually heated.

The elevated weights possess potential energy. When they reach the ground they have some kinetic energy.* The difference of the two energies gives the kinetic energy imparted to the water. This is converted into heat.

Let the mass of each load attached to a string $= m$ g,
distance through which each weight falls $= h$ cm,
acceleration due to gravity $= g$ cm/s^2,
total number of times each weight is made to fall $= n$,
speed of a weight when it reaches the ground $= v$ cm/s,
water-equivalent of the calorimeter with contents $= W$ g,
initial temperature of the water $= t_1$°C,
final temperature $= t_2$°C.
Then, the potential energy of the weights $= 2mgh$,
their kinetic energy at the end of the fall $= 2\times\frac{1}{2}mv^2$,
the mechanical energy converted into heat at each fall $= m(2gh - v^2)$,
the total energy converted into heat in n falls $= mn(2gh - v^2)$,
the total heat developed $= W(t_2 - t_1)$ cal.

$$\therefore\ J = \frac{\text{mechanical work transformed}}{\text{heat generated}} = \frac{mn(2gh - v^2)}{W(t_2 - t_1)}$$

Corrections were made for the frictional losses and the loss due to radiation from the calorimeter. As in all calorimetric work, conduction and convection were minimized.

In one of Joule's experiments the weights were 30 lb each and fell 20 times in all, each time through an effective height of $5\frac{1}{3}$ ft. The water equivalent of the calorimeter and its contents was 14 lb and the rise in temperature, 0.59°F.

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*Due to resistance offered by water to the motion of the paddles, they fall with a terminal (constant) velocity v. v can easily be measured during the fall.

$\therefore$ Heat generated $= 14 \times 0{\cdot}59 = 8{\cdot}26$ Btu.

Work done $= 2 \times 20 \times 30 \text{ lb} \times 5\frac{1}{3} \text{ ft} = 6400$ ft-lb.

whence $J = 6400 \text{ ft-lb} \div 8.26$ Btu.

$= 775$ ft-lb/Btu.

More accurate experiments to find the value of J were performed by Callendar, Rowland and others.

Since heat is energy, it can well be measured in terms of joules. In that case there would be no necessity of having any quantity named 'mechanical equivalent of heat' nor any separate heat unit such as the calorie (see last paragraph of Sec. 5-1). Many modern writers use 'joule' instead of 'calorie', 1 calorie being put equal to 4.2 joules (or more accurately, 4.1855 joules).

10-4. First law of thermodynamics. When heat is added to a body, the internal energy of the body increases. The body can also do some work if conditions are appropriate. Take for example a gas contained in a cylinder and fitted with a gas-tight piston. If a quantity of heat is supplied to the gas its temperature will increase. The pressure will also rise. The increased pressure will push the piston outward against the atmospheric pressure until the gas pressure equals the pressure outside. Thus, on addition of heat, the gas increases in internal energy and also does some work.

Let Q be the heat added to a body, U' the *increase* of internal energy of the body and W the work done *by* the body. The relation between these three quantities is given by

$$Q = U' + W \tag{10-4.1}$$

The quantities must all be expressed in the same units. They may be either heat units or work units.

The relation is known as the *first law of thermodynamics.* It covers all cases of conversion of heat into work and vice versa. Q is positive when heat is added, U' is positive when the internal energy increases and W is positive when work is done *by* the body.

Note that in the worked examples of Sec. 10·2 or in Sec. 10-3, there is no heat flow, that is, flow of energy due to temperature difference. Still we speak of heat. This can be easily reconciled with Eq. 10-4.1. Since there is no addition of heat $Q = 0$. W is negative as work has been done *on* the body. Therefore $U' - W = 0$ or $W = U'$. This means that *work done on the body increases its internal energy* by U'. We have equated U' to an amount of heat $Q=mst$. The temperature rise t corresponds to an increase U' of internal energy. The same increase could have been brought about by adding $Q=mst$ amount of heat.

We can represent the transformations of energy as $W=U'=Q$ $(=mst)$. W/Q is J, the mechanical equivalent of heat if W is expressed in work units and Q in heat units. If Q is also expressed in work units, the need for J does not arise.

(In the language of present day physics, the cases are those of transformation of work into internal energy.)

10-5. Isothermal and adiabatic expansion of gases. If the temperature of a gas remains constant during expansion, the expansion is said to be **isothermal**. Imagine a quantity of gas contained in a cylinder of uniform cross-section A (Fig. 10.2) fitted with a frictionless gas-tight piston. The gas presses on the piston with a pressure P and force PA. An equal pressure P is applied from outside on the piston to keep it in equilibrium.

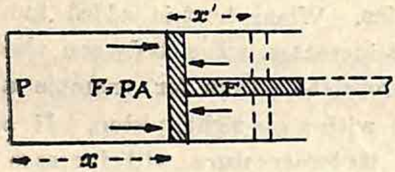

Fig. 10.2

If now the external pressure on the piston is decreased even slightly, the gas will expand. During expansion the gas does work as it pushes the piston outward. Let the difference of pressure on the two sides of the piston be infinitely small. If the piston moves outward by a very small distance x', the work done by the gas will be $PA \times x' = PV'$ where $V'(=Ax')$ is the volume swept out by the piston, that is, the increase in volume of the gas.

Unless energy is supplied to the gas, the energy for doing this work PV' must come from the internal energy of the gas. As a result the gas will be cooled. In order to keep the temperature of the gas constant during expansion, an amount of heat $Q = PV'$ must be supplied to it. (From the first law of thermodynamics it follows that if U' is to be 0, we must have $Q = W = PV'$.)

We therefore find that during an isothermal expansion of a gas, the gas has to be supplied with an amount of heat equivalent to the work it does. If W is the work done, we must supply to the gas an amount of heat Q equal to W/J (J=Joule's equivalent).

Conversely, if the external pressure on the piston is increased, work will be done on the gas and the gas will be compressed. The work done on the gas will increase its internal energy. The temperature of the gas will therefore rise. To keep the temperature constant an amount of heat equivalent to the work done on the gas must be withdrawn from the gas.

Adiabatic expansion. If during expansion of a gas no heat is allowed to enter or leave the gas, the expansion is said to be **adiabatic** A gas does work while expanding. So, in an adiabatic expansion a gas will be cooled (since no heat can enter it). For the same reason, a gas will be heated in an adiabatic compression. The rise in temperature will correspond to a supply of heat equivalent to the work done on the gas. During expansion the fall in temperature is similarly considered.

If we want to apply the first law of thermodynamics to an adiabatic expansion (or compression), we must put $Q = 0$ in Eq. 10-4.1. For expansion, W is positive. Hence $W = -U'$. For compression, W is negative. Therefore $W = U'$. Loss of internal energy $(-U')$ means cooling ; gain of internal energy $(+U')$ means heating.

An adiabatic expansion may be brought about in either of two ways. The gas may be contained in a cylinder made of thermally non-conducting material. (But no material is a perfect non-conductor of heat. So, in such a case, the expansion is very nearly adiabatic.)

Another way is to make the expansion so sudden that heat does not get time enough to enter the gas.

For an isothermal expansion the cylinder should be made of good conducting material. It will then be able to pass on heat to the gas from outside.

10-6. Specific heats of gases. When heat is added to a gas the resulting change of temperature depends on the manner in which the gas is allowed to expand. During expansion the gas does work against the external pressure. The energy required for this purpose is derived from the heat supplied to the gas. Thus, of the total heat supplied to a gas, a part may go to do external work and the rest raises its temperature.

Hence if we are to define the specific heat of a gas the conditions under which the heating takes place must be stated. To each possible mode of expansion there is a corresponding specific heat of the gas.

The two specific heats of a gas generally considered are the *specific heat at constant volume* (c_v) and that at *constant pressure* (c_p). If unit mass of a gas is heated through one degree under the condition that its volume remains constant, the heat it requires is called its **specific heat at constant volume**. For a mass m undergoing a change of temperature t under this condition the heat involved is mc_vt. When unit mass of a gas is heated through one degree under the condition that its pressure remains constant the heat required is called its **specific heat at constant pressure**.

To understand the difference between these two specific heats consider one gram of a gas contained in a vertical cylinder fitted with a movable frictionless piston (Fig. 10.3). Weights may be placed on the piston to alter the pressure on the gas. Supply sufficient heat to raise the temperature of the gas 1°C, at the same time keeping the volume constant by increasing the load on the piston (Fig.s 10 3*a* and 10.3*b*). The quantity of heat supplied is the specific heat at constant volume. The whole of this heat goes to increase the thermal energy of the molecules, while no work is done by the gas against the external pressure, as the piston does not move. Now start the operation afresh, with the same initial conditions. Supply sufficient heat to raise the temperature of the gas 1°C, but this time without altering the weights on the piston. The quantity of heat supplied is the specific heat at constant pressure.

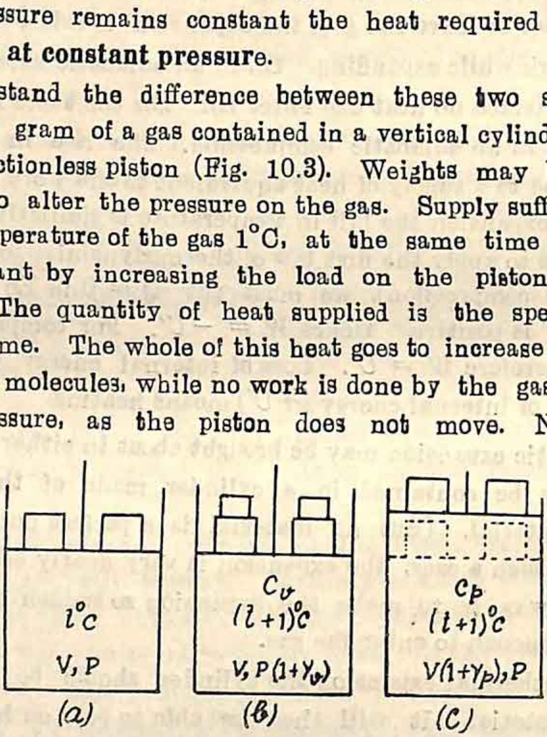

Fig. 10·3

In the second case, as the gas expands it raises the weights and so does work (Fig. 10.3*c*). Let V_1 and V_2 be the initial and final volumes of the gas, A the area of the piston, x the distance through which the piston moves and P the pressure on the piston. The force on the piston is PA and the work done against the external pressure is $PA\times x=P\ (V_2-V_1)$. The heat which is equivalent to this work is $P(V_2-V_1)/J$, where J is the mechanical equivalent of heat. Hence the specific heat at constant pressure

= heat required to increase the thermal energy corresponding to a rise of temperature of 1°C + heat equivalent of the work done

= specific heat at constant volume + heat equivalent of the work done.

In symbols $$c_p=c_v+P(V_2-V_1)/J \qquad (10\text{-}6.1)$$

It follows that c_p is greater than c_v.

A distinction is sometimes made in the symbols for the specific heat of 1 gram and that of 1 gram-molecule of a gas. The former is written with a small *c* as done above, and the latter with a capital *C*. Thus C_v stands for gram-molecular (or molar) specific heat at constant volume, and C_p for gram-molecular specific heat at constant pressure. But the distinction is not always observed.

Eq. 10-6.1 enables us to determine J when the other quantities are known. Note that $V_2-V_1=V_o\alpha_p$, where V_o is the volume of the gas at 0°C and α_p its coefficient of volume expansion at constant pressure.

Exercises

1. What arguments would you offer to show that heat is a form of energy? What is your idea of the nature of heat?

2. Explain the term *mechanical equivalent of heat.* Describe Joule's experiment for the determination of this quantity.

How much is a calorie equivalent to in ergs? In joules?

3. Comment on the statements (*i*) 'Heat is energy in transit due to temperature difference'; (*ii*) 'The joule can serve as the unit of heat; it is unnecessary to introduce the calorie'.

4. When heat is added to a body a part of it will increase the internal energy of the body and a part may do external work. Express this result in mathematical symbols, explaining what the positive or negative sign of a symbol would mean. Illustrate the equation with an example.

5. Explain what is meant by the first law of thermodynamics.

6. Explain what you understand by (*i*) *isothermal* expansion, (*ii*) *adiabatic* expansion of a gas. How can adiabatic expansion be practically realized?

7. Define the specific heat of a gas at constant pressure (C_p) and that at constant volume (C_v). Explain why they are different. Which is bigger and why?

8. In a determination of Joule's constant two 10 kg weights are allowed to fall 20 times through a height of 3 m. This increases the temperature of the calorimeter by 0·4°C, the water equivalent of the calorimeter and its contents being 7 kg. Calculate the value of J from these data.

[Ans. : $4{\cdot}2 \times 10^7$ ergs per calorie]

9. From what height must 100 g of copper (sp. gr. = 0·1) fall so that its temperature may rise by 1°C ? Assume that all the energy is retained by the copper. $J = 4{\cdot}2$ J/cal.

[Ans. : About 43 m]

For a greater mass of copper will the height be greater ?

10. A glass tube, 1 m long, is closed at both ends. It contains 250 g of lead shots and 1 litre of water. The tube is set up vertically and is then suddenly inverted. How many times must this be done so that the temperature of the water may rise by 1°C. Ignore effects due to buoyancy of lead shots and the heat that goes into the lead shots and the glass tube. [Ans. : 1723 times]

11. A lead shot hits a rigid target and increases in temperature by 200°C. If the entire energy is retained by the shot what was its velocity ? Sp. heat of lead = 0·03.

[Ans. : 224·5m/s]

12. A body weighing 10 kg drops from a height of 1 km. If its potential energy is fully converted into heat, how many calories will be generated ? $J =$ 4·2 joules/calorie.

[Ans. : $2{\cdot}33 \times 10^4$]

13. A lead shot moving with a speed of 336 m/s hits a target. If 75% of the kinetic energy is converted into heat in the shot, what will be its temperature rise ? $J = 4{\cdot}2 \times 10^7$ ergs/cal. Sp. heat of lead = 0·03. [Ans : 336 C°]

14. In an experiment for determining J, a card board tube, 1 m long and containing 800 g of lead shots, is inverted 50 times. The temperature of the shots is found to have increased by 28·84 $C°$. Sp. heat of lead is 0·031 and $g =$ 980 cm/s². Assuming that the heat generated has been fully retained by the shots, calculate the value of J. [Ans : 4.12×10^7 erg/cal]

15. A water-fall is 300 m high. If 50% of the potential energy is converted into heat and retained by the water, what is the temperature difference between the water at the top and the bottom. $J = 4{\cdot}2$ joule/cal. [Ans : 0·58 C°]

APPENDIX
TO
HIGHER SECONDARY PHYSICS

BY

D. P. RAY-CHAUDHURI

Containing

- **(i) Material on the TEACHING GUIDE-LINES laid down by the W. B. Council of Higher Secondary Education.**
- **(ii) Answers to EXAMINATION QUESTIONS of 1978.**
- **(iii) Answers to critical questions from the SPECIMEN QUESTIONS drawn up by the Council.**

PREFACE

With the holding of the first Higher Secondary Examination many doubts and uncertainties in the minds of teachers and students have been removed. Further, with the publication of TEACHING GUIDE-LINES and SPECIMEN QUESTIONS by the W.B. Council of Higher Secondary Education a positive direction has been given to teaching and examination.

The existing textbooks were all written long before the Guide-lines and Questions were available. Authors wrote as they thought fit. This led to much difference in contents and treatment.

Guide-lines from the Council show that the course has to be taught intensively and critically. The emphasis is on understanding the basic principles and acquiring an ability to apply them to the solution of problems.

Fortunately for the present author, the original textbook was written with this emphasis, with additional emphasis on modernization in the use of units. This small APPENDIX provides the additional material necessary for the student and the teacher.

Appendixes for the two parts of the book are attached separately to each part.

D. P. R.

Please note the following :

1. The actual wordings of the Teaching Guide-lines have been quoted in italics as TGL.
2. REFERENCES in this Appendix (such as Text, Mechanics, p. 40, etc.) are to the original text.
3. Specimen Questions of the Council have been marked Q(S).
4. Questions of the Higher Secondary Examination of 1978 have been marked Q(78).
5. The following items have been dropped from the original syllabus :

Part I. Simple surface tension phenomena. Motion in fluids—Viscosity—Streamline and turbulent flow. Isothermal and adiabatic expansion of gases. Specific heats of gases. Definitions of C_p, C_v.
Stefan's law—statement and applications.
Principles of recording and reproduction of sound.

Part II. Spectrometer. Rainbow. Fluorescence and phosphorescence. Principles of telegraphy, telephony and microphone.
Conduction of electricity through gases. Cathode ray tube. Television. Radar.
Measurement of atomic masses. Mass spectrograph. Cosmic rays and mesons.

GENERAL GUIDE-LINES

TGL 1. *Adequate arrangements should be made for usual demonstrations (with the aid of appropriate apparatus, films, slides, etc.). These may also be improvised by teachers.*

Remarks. This hardly needs emphasizing. Unfortunately, demonstration of apparatus and experiments is the most neglected part in our teaching, both in schools and colleges. Placing an oscillating pendulum before a class teaches the class much more about the nature of simple harmonic oscillations than a whole period of lecture.

Demonstration apparatus should be kept out of view of the students until the time for actual demonstration. Otherwise, the students' attention will be diverted by the exhibits.

TGL 2. *CGS units to be employed. Relation with SI units to be briefly mentioned.*

Remarks. SI units should get greater emphasis. They were introduced in 1960 by international agreement for use in all scientific measurements (see Text, Pt I, p. 3). All western countries have adopted them. The Central Board of Education at New Delhi has adopted the system. We in West Bengal are among the few who are still sticking to the older systems. The fps system, which has gone out of use even in England, still appears in our questions.

Science always tries to simplify. SI units do away with multiplicity of units, such as the fps, cgs e.s.u., cgs e.m.u., etc. If one system serves all purposes, why burden students with a multiplicity of systems ?

We may allow some time for transition from CGS to SI units. SI units are quite simple. Many of them are already known even to an oldtimer. The transition should be quick.

Even when we are using the cgs system we should stick to the international recommendations for the use of unit symbols (See Text, Pt. I, p.5.). It is unfortunate that the same question paper uses both g and gm for gram, s and sec for second, cm and cm. for centimetre. We expect consistency in the use of unit symbols in question papers. Let us not confuse the student. (Unit symbols have been made even more confusing in the Bengali rendering of the questions of the 1978 examination by using both English and Bengali. The reason which led us to accept international numerals (1, 2, 3, etc,) applies also to unit symbols.

TGL 3. *Units of physical quantities must be stressed.*

Remarks. This hardly needs emphasizing. It should be remembered that

$$\textit{physical quantity} = \textit{number} \times \textit{unit}.$$

Without mention of unit, the number carries no sense.

TGL 4. *Problems should be worked out wherever necessary and should be of such a nature as to illustrate the significance of the basic physical concepts and equations involved.*

Remarks. Ability to solve questions is a test of the student's grasp of basic principles. Numerical problems should be carefully selected. They should be realistic in nature and apply to life situations. A question on motion should not speak of 'a weightless elephant sliding down a perfectly smooth hill.'

PART—I

MECHANICS

TGL 1. *The two aspects of the equation $F=ma$ should be stressed:*

(*i*) *It gives a measure of force ;*

(*ii*) *It enables us to find the equation of motion.*

Material. a in the equation is the instantaneous acceleration and F is the *net* instantaneous force, that is, the *vector sum* of the forces acting at the moment. F and a are international symbols for force and linear acceleration respectively. The vector equation is written as $\mathbf{F}=m\mathbf{a}$.

Aspect (i) is stressed in all books (e.g., see Text, Mechanics, p. 31).

Equation of motion generally means a differential equation.

For motion in a straight line, the equation of motion is written as $F=m\dfrac{dv}{dt}=m\dfrac{d^2x}{dt^2}$. Here m is the mass of the body, v=instantaneous velocity, x=instantaneous displacement along the line of motion from some fixed origin. $d\,dt$ is the rate of change with time of the quantity (v or x) that follows it. When such an equation is solved, we know v and x at any time t.

Q(S). *Show that the first law of motion can be derived from the second.*

Ans. The second law states that the rate of change of momentum is proportional to the resultant force, and is in the direction of this force. So, if the momentum does not change, the force is zero. Hence if there is no force, the momentum of a body will not change. Now, momentum is the product of mass and velocity. Mass is a constant quantity. If therefore the momentum is not to change, it will either remain constant or be zero. This means that a body at rest will continue to be at rest ; or if it is moving, it will move with a constant velocity unless some force acts on it. This is the first law.

Note. We often say that the *third law is contained in the first.* The principle of conservation of linear momentum is derived from the third law. The third law emphasizes that internal forces cannot alter the momentum of a system as a whole. To change the momentum of a system some force from outside must act on it. Looking at it in this sense one may say that the third law is contained in the first.

It seems therefore that we could have done with the second law alone, deriving the first and third laws from it as corollaries.

TGL 2. *Applications of Newton's laws in daily life to be stressed. (For instance, the cases of a horse pulling a cart and walking may be dealt with in some detail.)*

Material. Plenty of examples of application of Newton's law are given in the text books. Students may be asked to give examples from daily life. This may start a very interesting and useful dialogue.

Horse pulling a cart. The thing to be strongly emphasized in dealing with the third law is that *action and reaction are exerted on different bodies.* So the question of their producing equilibrium does not arise. Equilibrium of a body is determined by the forces acting on the body. Any force the body itself exerts on any other body or bodies does not come in the picture.

In this connection it will be profitable to remember the following :

Procedure in Problem Work

In solving problems involving the action of more than one force on a body or a system of bodies, much difficulty may be avoided by following the procedure suggested below :

(*i*) Select one of the bodies for consideration, usually the one whose rest or motion is to be discussed. Consider this body to be isolated from the rest of the system by an imaginary closed surface.

(*ii*) Construct a 'force diagram' in which the selected body is represented by a point and represent *all* of the forces acting *on* this body by suitable vectors. Be careful not to omit any force, but **never include any force which this body exerts on other bodies**. If any force is unknown, represent it by a vector, but mark it as unknown.

(*iii*) From the force diagram find the resultant of the forces. If the resultant is zero, the body is at rest. If otherwise, it is this unbalanced force which gives the body an acceleration along its own line of action. The magnitude of the acceleration is obtained by dividing this resultant force by the mass of the body.

The horse exerts a force T on the cart. Friction with the ground brings into play a force R acting on the cart opposite to T. The unbalanced force $T-R$ acting on the cart accelerates it.

The reaction T which the cart exerts on the horse merely retards the motion of the horse. The latter gets his acceleration from the unbalanced force $H-T$, where H is the horizontal component of the reaction to the thrust he exerts on the earth.

The action exerted by the horse on the cart helps to accelerate the cart; the reaction of the cart on the horse retards the motion of the horse.

Walking. Before a man starts to walk he stands erect and lifts one leg. His weight is supported by the vertical reaction of the ground. As his leg muscles bend his other leg, he leans forward and at the same time pushes forward his other leg. A large number of leg and body muscles take part in this action (which is too complicated to discuss). But in this position the point of application of the reaction of the ground on him moves forward (and the heel may be raised). The reaction increases in magnitude than before and is directed up the inclined leg in touch with the ground. The vertical component of this reaction supports the man's weight. The horizontal component provides the force for forward motion.

TGL. 3. Conservation of momentum—*the fact that the system must be 'closed' should be stressed.*

Material. We often select for discussion two or more bodies which exert forces on one another. For convenience of analysis we imagine the bodies under discussion to be *isolated* from all other bodies by a *closed* imaginary surface which does not allow exchange of any energy with the outside. Such bodies are variously referred to as a (i) *system of bodies*, (ii) *isolated system* or (iii) **closed system.**

The principles of conservation of energy and linear momentum apply to closed systems.

TGL. 4. Elastic collisions—*Momentum and energy principles should be applied to prove that the velocity of approach of two particles is equal to the velocity of recession.*

See Text, Mechanics, p. 39, Eq. 1-12. 5.

Q(S). A ball rolling on a smooth table strikes head-on another identical ball, which is stationary. Explain why the first stops and the second begins to move.

Ans. See Text, Mechanics, p. 40, Eq. 1-12. 10. It can also be deduced from the energy and momentum principles, taking m_1 equal to m_2.

TGL. 5. Jets and rockets. *Acceleration due to loss of mass should be discussed briefly.*

Material. (Text, Mechanics, p. 38.) As the rocket rises the fuel burns and the mass of the rocket diminishes. If the fuel burns at a constant rate it provides a constant force acting on the rocket. Hence the acceleration of the rocket goes on increasing so long as the fuel lasts. The same applies to jets. Application of simple mathematics shows that

$$\text{acceleration} = \frac{\text{downward velocity of gas relative to rocket}}{\text{mass of rocket}} \times \text{rate of loss of mass } minus \text{ acceleration due to gravity.}$$

With increase of height above the earth g diminishes. If the fuel burns at a constant rate the acceleration goes on increasing due to loss of mass.

TGL 6. Friction *should be explained with examples and demonstrations. These will reveal the advantages and disadvantages of friction. Rolling friction should be briefly introduced and its importance in machinery should be pointed out.*

Distinction between conservative and dissipative forces should be pointed out.

Material. (Text, Mechanics, pages 41-43.) Students may be asked to suggest examples from daily life. Such dialogues will be interesting both to the student and the teacher. Simple demonstrations are obvious. Stoppage of a sliding body may start the dialogue.

Conservative and dissipative forces will be discussed with 'Work and Energy' in TGL 10.

Q(S). 100 lb of wooden craft is being pushed across a stone floor with a horizontal force of 45 lb. How long will it take the craft starting from rest to travel 200 ft? (Coeff. of friction = 0·4)

Ans. Resultant force on craft = applied force − frictional force.

Applied force = 45 lbf = 45×32 poundals. Frictional force = coefficient of friction × normal reaction = $0{\cdot}4 \times (100 \times 32)$ poundals = 40×32 pdl.

Acceleration of the craft = Resultant force ÷ mass = $(45-40) \times 32$ pdl ÷ 100 lb = $(5 \times 32) \div 100 = 1{\cdot}6$ ft s^2.

Time t required to travel 200 ft from rest is given by $200 = \frac{1}{2}$ acceleration $\times t^2$ or $200 = \frac{1}{2} \times 1{\cdot}6 \times t^2$ or $t^2 = 400/1{\cdot}6 = 250$ or $t = \sqrt{250} = 16$ s nearly.

Q(S). Why can we not walk on a frozen lake? Explain briefly.

Ans. In walking the horizontal component of the reaction between the foot and the ice provides the force for forward motion (TGL 2. Mechanics.).

Friction allows such a force to be exerted. The coefficient of friction between the foot and the ice is almost zero. Hence friction is unable to exert any horizontal force on the man. In attempting to walk the man will fall forward.

Q(S). How could a person at rest completely in frictionless ice crossing a pond reach the shore, rolling or kicking his feet? Explain briefly.

Ans. The man is practically an isolated system. No horizontal force can act on him in the absence of friction. In rolling or kicking the feet, only the internal forces in the closed system do work. According to the principle of conservation of momentum such action cannot impart momentum to the system. So he cannot move towards the shore.

If, however, he had a sufficient number of stones at his disposal he could, by throwing them in one direction, move in the opposite direction by recoil.

TGL 7. Statics : Centre of mass and centre of gravity. *Illustrative examples should be given to reveal the difference between the two.*

Material. (Text, Mechanics, pp. 70-71.) For astronomical objects the centre of mass is the only meaningful concept. It greatly simplifies problems. The motion of a star or galaxy may be treated as though it were a particle with the mass concentrated at the centre of mass. In general, whether on the earth or not, the centre of mass is a more universal concept than the centre of gravity. For example, in applying Newton's law $\mathbf{F}=m\mathbf{a}$, we have really been talking about the acceleration $\mathbf{a}$ of the centre of mass of a body.

TGL 8. *Idea of torque should be explained by citing common examples (e.g. the force required to turn a door about its hinges).*

Condition of equilibrium of a system of particles should be taught after the concept of moment of force has been introduced.

Material. Torque—See Text, Mechanics, p, 54.
Moment of force—See Text, Mechanics, p. 52.
Conditions of equilibrium—See Text, Mechanics, p. 63.

TGL 9. Dynamics of circular motion. *Expression for centripetal acceleration should be deduced with the help of vector diagram. The shape of the earth and other examples to be discussed.*

Material. To deduce the expression for centripetal acceleration with the help of vector diagram.

Let a particle be moving with uniform angular velocity ω in a circle of radius r (Fig. 1). The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ represent the instantaneous velocities at P and Q. The vectors have the same length but different directions. The average acceleration $\bar{\mathbf{a}}$ is the *vector change in velocity* between P and Q, divided by the time taken. The vector change in velocity means the *vector difference* $\mathbf{v}_2-\mathbf{v}_1$. Let us write it as $\delta\mathbf{v}$.

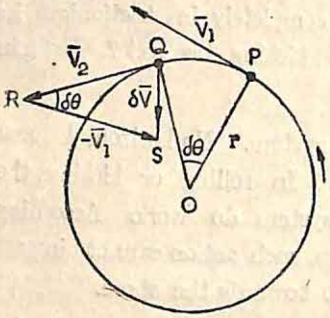

Fig. 1.

Now, $\mathbf{v}_2-\mathbf{v}_1$ is the vector *sum* of $\mathbf{v}_2$ and $-\mathbf{v}_1$. $-\mathbf{v}_1$ is a vector equal and opposite to $\mathbf{v}_1$. To get $\delta\mathbf{v}$, we lay down $-\mathbf{v}_1$ from the end point of $\mathbf{v}_2$, and join the initial point Q of $\mathbf{v}_2$ to the end point S of $-\mathbf{v}_1$. The line segment $\overrightarrow{QS}$ represents the vector difference $\delta\mathbf{v}$.

If the particle took a short time δt in moving from P to Q then the average vector acceleration between P and Q is

$$\bar{\mathbf{a}}=\delta\mathbf{v}/\delta t.$$

It points inwards in the same direction as $\delta\mathbf{v}$. Let v be the magnitude of either $\mathbf{v}_1$ or $\mathbf{v}_2$, and $\delta\theta$ the angle between OP and OQ, that is, between $\mathbf{v}_2$ and $-\mathbf{v}_1$. Then from the isosceles triangle QRS, $QS=2QR\sin\frac{1}{2}\delta\theta$. If $\delta\theta$ is very small we can write $\sin\frac{1}{2}\delta\theta=\frac{1}{2}\delta\theta$. So, when P and Q are extremely close together, the magnitude of $\delta\mathbf{v}$ is

$$\delta v=QS=2QR.\tfrac{1}{2}\delta\theta=v.\delta\theta.$$

When P and Q are infinitely close together, both $\delta\theta$ and δt are infinitesimal. We then get the instantaneous acceleration by dividing both sides by δt. This acceleration is directed towards the centre O of the circle. Its magnitude is

$$a=\frac{\delta v}{\delta t}=v.\frac{\delta\theta}{\delta t}=v\omega$$

or, centripetal acceleration = linear speed × angular velocity.

[Since $\omega=v/r$, we can also write $a=v^2/r$. The instantaneous acceleration points towards the centre O and has magnitude $a=v^2/r=v\omega$. The corresponding (centripetal) force $F=mv^2/r=mv\omega$, where m

is the mass of the moving particle. It has the direction of the centripetal acceleration, that is, the centripetal force is directed towards the centre of the circle.]

Shape of the earth. When a spherical body rotates about a diameter, particles on its equatorial plane experience the greatest centrifugal force. As a result, the body tends to flatten out into a disc. This tendency is opposed by the rigidity of the body. Depending on the rigidity, the body acquires a bulge in the middle and is flattened at the poles. This is the shape of most celestial bodies with a spin, including the earth.

Other examples in Text—Mechanics, pp. 59-61. A few *additional examples* are given below.

Centrifugal drying machine. In consists of a cylindrical vessel with perforated walls and can be spun rapidly round its axis. Damp cloths are placed in the cylinder, which is set into rapid rotation. As a result of the centrifugal force, the water is forced out through the perforations in the walls of the cylinder, and cloths are dried thereby. Alternatively, we may say that as there is not enough centripetal force, the water particles fly tangentially away through the holes.

Centrifuge. It is a device in which small tubes or vials, hinged near their tops, are spun at a high speed in a horizontal circle by air pressure or electrically driven gears. As the machine spins, the lower ends of the tubes rise (as the governors of a steam engine). The tubes ultimately become practically horizontal and move in a horizontal circle.

The tubes contain particles in suspension in a liquid of different density, such as fat particles in milk, the red corpuscles in blood, etc. The purpose of the centrifuge is to separate these particles from the liquid. In the centrifuge a particle of volume V, density σ and angular speed ω needs a force $V\sigma\omega^2r$ to keep it rotating in a circle of radius r. The surrounding liquid can exert on it a centripetal force $V\rho\omega^2r$ where ρ is the density of the liquid. If $\rho>\sigma$, the particle moves towards smaller values of r, i.e., towards the axis of rotation. If $\sigma>\rho$, the particle moves towards larger values of r, i.e., away from the axis of rotation.

The rate of sedimentation under gravity is principally determined by the difference in densities between the sediment and the liquid. When ω is large, the force $V(\sigma-\rho)\omega^2r$ tending to displace the particle is much greater than that due to gravity which is $V(\sigma-\rho)g$. Now ω^2r can be made much greater than g. With modern techniques it is possible to obtain values of $\omega^2 r$ approaching a million times that of gravity. Hence sedimentation is brought about much more quickly by a centrifuge than by gravity.

The centrifuge is of great value in various fields, particularly in medical and biological research, e.g., in separating proteins, hormones, viruses etc., from sera or other liquid media. It is also used for separating cream from milk, and has many other similar uses.

Reality of centrifugal force. A man stands on a rotating turn-table holding two weights on his outstretched hands. In pulling the weights towards his body his muscles will do work against the centripetal force. (The man is in the rotating reference frame.)

TGL 10. Work, Energy and Power. *That work is a scalar quantity should be emphasized.*

The fact that potential energy is undefined to the extent of an arbitrary additive constant should be stressed.

Material. Work $W = F.\ s \cos\theta$. Both F and s are vector quantities. But that does not make W a vector quantity. Whatever the direction of F and s, so long as the quantity $F.\ s \cos\theta$ remains the same, W remains the same. W (work) has no sense of direction associated with it. Work is a scalar quantity.

Absolute value of potential energy cannot be determined. Consider the case of gravitational potential energy. A body raised to a height h above the ground level has a potential energy mgh, if it can come back to the ground level. If the body was on a table of height h', its potential energy relative to the table top would have been $mg(h-h')$. So the value of the gravitational potential energy depends on which level we take as the level of zero energy. Now, the choice of this zero energy level is *arbitrary*, that is, we choose it according to our convenience. Any value of potential energy that we derive depends on our choice of the zero level of potential energy, which is arbitrary. If the zero level is altered, the value will alter by some constant amount.

Potential energy of all kinds is always measured with respect to some standard configuration. We never know what the potential energy is in the standard configuration. Hence we say *potential energy is undefined to the extent of an arbitrary additive constant.* This constant is the potential energy in the standard configuration. We never know this quantity.

Conservative and dissipative forces. (From TGL 6.) When an outside agent lifts a body vertically he does work. The work done is equal to the increase in gravitational potential energy of the body. Such an agent also does work when he pulls the body along a rough horizontal plane ; but in this case the potential energy of the body does not increase.

The reason for this difference can be understood if we consider what happens when we restore the body to its initial position. In being lowered, the body can do work at the expense of the gravitational potential energy it had acquired. The amount of this work is, in the ideal case, equal to the energy it had gained. So, in this case the work spent on the body in lifting it, is *recoverable.*

In the other case, when we reverse the motion of the body to restore it to its initial position on the rough surface, the force of friction is also reversed. So, the same work is done against friction during the return motion as in the forward motion. The work is *not recoverable.*

We may express the difference in another way also. In the first case (case of gravity), as much work will be done by the body in falling from a level A to a level B as will be done on it in lifting it from B to A. In still other words, *the work for a round trip is zero.* But not so in the case of friction ; work is done by the outside agent for both trips.

When a force is such that the work done against it is recoverable, it is called a **conservative force.** In such a case potential energy increases. When the work done against a force is not recoverable, as in the case of friction, the force is called a **dissipative force.** The work done in such a case appears as heat.

The mechanical energy of an isolated system is conserved only when all the forces acting on it are conservative. When dissipative forces are present, the total energy of the system is conserved, but not the mechanical energy. Some of the latter is generally converted into heat.

QUESTIONS. 1. Discuss the principle of conservation of energy when friction is present.

2. Distinguish between *conservative* and *dissipative* forces. How would you apply the energy principle when dissipative forces are present ?

Q(S). *A man rowing a boat up stream is at rest with respect to the shore. Is he doing any work ?*

Ans. Yes. A force is said to do work when it moves its point of application in its own direction. Here the oar is a lever ; its fulcrum ;

is in the water; the load is where it is attached to the boat and the effort is applied at the end where the oarsman pulls it. As the man pulls the oar, the point of application of the force moves in the direction of the force. So work is done.

Situation with respect to the shore is immaterial.

GENERAL PROPERTIES OF MATTER

TGL 11. Gravitation. *Variation of g on the surface of the earth should be discussed with reference to (a) Latitude effect, (b) Effect due to rotation of the earth (Detailed treatment should be avoided).*

For g inside the earth only physical explanation need be given. The mean density and mass of the earth are to be worked out.

Material. For variation of g, see Text, Properties of Matter, p. 4. g inside the earth is due only to the sphere which lies between the body and the centre of the earth. The earth shell outside this inner sphere does not exert any resultant pull on the body. Action of any part of this shell is equal and opposite to that of its other part.

Mean density and mass of earth (See Text, Properties of Matter, p. 2.)

TGL 12. *The* **laws of simple pendulum** *should be stated and explained.*

The principle of determination of g by a simple pendulum should be discussed in the theoretical class along with limitations.

Derivation and discussion on gravitational potential are not required.

Material. See Text, Vibrations and Waves, p. 93 for laws of simple pendulum.

Q(S). (*i*) *How would you determine the height of a mountain with the help of a simple pendulum?*

Ans. Find the periodic time $T(=2\pi\sqrt{l/g})$. g at height h above the surface of the earth is $g=g_o(1-2h/R)$, where g_o=value at the surface of the earth $(=GM/R^2)$, R=earth radius. [See Properties of Matter, p. 4, Eq. 1-4.1.]

Q(S). (*ii*) *Find the length of a seconds pendulum where $g=981$ cm/s^2.*

[See answer to Q. 4, Paper I, 1978 Examination.]

Q(S). (*iii*) *Would you ever construct a simple pendulum? Argue.*

[See Text, Vibrations and Waves, p. 92, Sec. 1-4.1 (i). Also see para (i) under "Limitations etc." that follows here.]

Q(S). (*iv*) *A hollow sphere is filled with water through a small hole in it. It is hung by a long thread, and as the water flows slowly out of the hole at the bottom, one finds that the period of oscillation at first increases and then decreases. Explain.*

Ans. With the sphere full of water the centre of gravity is at the centre of the sphere. The effective length of the pendulum is the distance between its point of suspension and c.g. of the bob. As the water runs out, c.g. is lowered. Hence l increases, increasing T. c.g. is lowered more and more as half the water runs out. Hence T goes on increasing until then. As more water runs out, c.g. slowly rises and reaches the centre when all water has run out. During this period effective length gradually diminishes, and so does the periodic time.

Limitations of the simple pendulum method of determining g.

The method cannot give an accurate result for the following reasons :

(*i*) The formula $T = 2\pi\sqrt{l/g}$ is derived for a simple pendulum. It is an ideal pendulum which we cannot get in practice. No real string is perfectly flexible nor inextensible or weightless. Since these conditions are not fulfilled, there is bound to be some unknown error in the value of g that we get.

(*ii*) The formula assumes that the amplitude is infinitely small. As the amplitude increases, T also increases. Keeping the linear amplitude within about $\frac{1}{10}$th the length of the pendulum confines the error in T to about 1 or 2 parts in 1000.

(*iii*) There is difficulty in accurately measuring the distance between the point of suspension and the centre of gravity of the bob. The measured value of l has therefore some error in it.

(*iv*) An error also occurs in the measurement of the time for a given number of oscillations. We cannot start or stop the stop-clock at the exact moments. So the measured value of T will have an error in it. Besides, the stop-clock may not be running correctly.

All these factors lead to an error in the calculated value of g. If the error could be confined to within 1%, the result should be considered good. Since the correct value is about 980 cm/s^2, a 1% error means values in the approximate range 980 ± 10, i. e., between 970 and 990 cm/s^2. Any value in this range should be considered equally acceptable, and of equal merit. Special emphasis should *not* be given to values near 980 cm/s^2.

TGL 13. *Kepler's laws should be stated with general discussion on the physical background and description of the trajectory. The orbital velocity of satellites with circular orbits should be calculated.*

Material. Kepler's laws. Tycho Brahe (1546—1601), a Danish astronomer, made fairly accurate records of the positions of planets in their motion across the sky. From a close study of these records, extended over several years, his assistant Johannes Kepler (1571-1630) derived three laws known as Kepler's laws. The laws are of basic importance in the study of the motion of planets round the sun, and of satellites (natural or artificial) round a planet.

Kepler's laws may be stated as follows :

Law (i). Each planet moves in an elliptic path around the sun with the sun at one focus of the ellipse.

Law (ii). An imaginary line joining the sun and the planet sweeps out equal areas in equal times.

Law (iii). The squares of the periods of revolution of the planets round the sun are proportional to the cubes of their mean distances from the sun.

For most planets the two foci of their elliptical paths are so close together that the orbits may be taken as *circular* for ordinary purposes.

Kepler's laws are of great importance and significance. Newton's law of gravitation can be derived from them. Besides, the orbit of a satellite can be determined from these laws.

At the initial stage, we shall take the orbits as circular.

For *orbital velocity,* see answer to Q. 4(c) of 1978 Exam. (Also see Text, Properties of Matter, p. 5, Sec. 1-6.)

TGL 14. Elastic Properties of Matter. *Definitions with physical interpretations should be given. Simple examples of elastic moduli in practical life are to be given .*

Relations between different elastic constants are not required.

Material. Elastic moduli and molecular structure. In *solids,* atoms and molecules are arranged in a regular geometrical pattern. There are both attractive and repulsive forces between them. The equilibrium position of atoms and molecules is determined by these forces. For closer approach than the equilibrium distance, there will be repulsion. Any force tending to increase this distance brings attractive forces into play. A solid will therefore resist any force tending to alter the relative positions of its constituent atoms or molecules. It will resist any change of length, volume, and also of twist or shear. It will therefore have all the three moduli of elasticity, namely, Young's modulus, bulk modulus and rigidity modulus.

In *liquids*, the molecules are free to move, but the mean distance between neighbouring molecules remains constant at a given temperature. So a liquid can resist only a change of volume, but not any change of shape. Due to mobility of molecules it takes the shape of the container. It thus has bulk modulus only, but no rigidity modulus or Young's modulus. Application of a horizontal force to a liquid molecule will cause it to move continuously under the force. But in a solid, such a molecule will be displaced by only a finite amount as restoring forces due to rigidity come into play. *The fact that a liquid cannot resist a shear is of basic importance*; it explains the hydrostatic behaviour of liquids.

Gases are like liquids. Their molecules are much farther apart than in liquids and are much more free to move than liquid molecules. The expansibility of gases is due to the very weak attracting forces between molecules. Gases have bulk modulus only ; but it is much smaller than that of liquids.

Some examples of elastic moduli in practical life. Young's modulus (E) and shear modulus (G) are of great importance in structural engineering. The ability of pillars and struts to support loads is determined by the Young's modulus of their material. Horizontal beams supporting loads are both elongated and compressed ; they are also sheared. Hence in their case, both E and G are of importance. Compression or extension of springs is determined mainly by E when the turns are very close. But when the turns are more open G also plays an important part.

Thermal expansion or contraction can exert a large force. The force is determined by the temperature change, area of cross-section and Young's modulus of the material.

The performance of rotating axles in machinery in transmitting power is determined by its modulus of rigidity (G).

The high value of the bulk modulus (K) of liquids makes them a suitable substance for absorbing shocks, such as recoil of guns.

Examples of this kind may be multiplied without limit.

[For *definitions* of the moduli, see Text, Properties of Matter, pages 11, 13 and 14.]

Q (S). (i) What force will be required to stretch a steel wire 1 sq. cm. in cross-section to double its length ? E for steel $=2\times10^{12}$ cgs units

Ans. By definition, Young's modulus

$E=\dfrac{\text{applied force }(F)\text{ area }(\alpha)}{\text{elongation }(e)/\text{original length }(l)}$. Here $\alpha=1$ cm^2 and $e=l$

$\therefore\ F=E\times(e/l)\times\alpha=2\times10^{12}\times1\times1$ in cgs units $=2\times10^{12}$ dynes.

(*Note.* A wire snaps long before its length has been doubled. Besides, with elongation, the cross-section diminishes.)

Q (S). (ii) Which is more elastic, rubber or steel ?

Ans. Strictly speaking, a material should be called more elastic if a stronger force is required to bring about the same deformation. In this sense steel is much more elastic than rubber.

Popularly, we call a substance elastic if it is easily stretched. It should then have a large elastic limit and a small Young's modulus. In this popular sense, we call rubber more elastic.

TGL 15. Specific gravity. *Principles of the methods of determination should be briefly discussed.*

Material. Principles of methods of determining specific gravity.

Specific gravity (s) is *relative density*, that is,

$$s=\frac{\text{density of the substance}}{\text{density of water at }4^\circ\text{C}}=\frac{\text{weight of a volume }V\text{ of the substance}}{\text{weight of the same volume of water at }4^\circ\text{C}}$$

So, to determine s, we have to weigh the same volume of the substance and water. In the cgs system of units the numerical value of the volume of water in cm^3 is equal to the mass in grams.

(*i*) We can use the balance and apply Archimedes' principle for measuring the masses of the same volume of the substance and water.

(*ii*) For liquids, we can use a density bottle (also called specific gravity bottle) to determine the masses of equal volumes of a liquid and water.

The density bottle can also be used to find the specific gravity of a solid powder by partially filling the bottle with powder.

(*iii*) For liquids, there are other methods.

(*a*) One depends on the hydrostatic pressure $h\rho g$ exerted by a liquid. If two liquid columns balance each other, then $h_1\rho_1g=h_2\rho_2g$, or $\rho_1/\rho_2=h_2/h_1$. If ρ_1 is the density of the experimental liquid and ρ_2 that of water, then $s=\rho_1/\rho_2=h_2/h_2$. This is known as the *principle of balancing columns*. Hare's method of determining s depends on this principle.

(*b*) We can also apply the *principle of flotation.* All hydrometers depend on this principle. In *common hydrometers* (known as *variable immersion hydrometers*) a loaded hollow glass tube, when floated in different liquids, sinks to different depths in different liquids. So, its stem can be directly graduated in terms of density or specific gravity.

In another type, known as the *constant immersion type,* a hollow cylinder is made to sink up to a given mark in different liquids. The additional weights to be put on it will be different for different liquids. Knowing the weight of the cylinder and the additional weights necessary, it is possible to calculate the specific gravity. *Nicholson's hydrometer* works on this principle.

The different methods for determining s are thus found to depend on the following principles :—

(1) Archimedes' principle, using the hydrostatic balance.

(2) Direct weighing by using density bottles.

(3) Principle of balancing columns.

(4) Principle of flotation :

(*a*) Variable immersion, (*b*) Constant immersion.

TGL 16. Archimedes' Principle. *Applications of the principle should be discussed.*

Material. In the laboratory, we use this principle for determining volumes of bodies and specific gravities of substances.

The ascent of balloons is an application of this principle.

All cases of flotation are determined by this principle.

Buoyancy forces are very important to many marine and freshwater animals. Many sea animals have gas-filled chambers and gas-filled spongy bones which make their density the same as that of sea-water. This makes quick movement very easy for them. Some have a gas-filled bag, called a swim bladder, whose volume they can vary. Squeezing the bag, the fish makes itself heavier and sinks. Some have a control valve to adjust the amount of air in the bladder. Ease of adjustment to various depths in water is a great boon to marine life. The principle on which the adjustment is made is Archimedes' principle.

Q(S). *Explain with theory how Archimedes' principle may be used to determine the proportion of two pure metals in a sample of alloy.*

Ans. Let m_1 grams of a metal of density ρ_1 form an alloy with m_2 grams of another metal of density ρ_2. Assuming that the volume remains unchanged on alloy formation, the volume V of the alloy is $V=m_1/\rho_1+m_2/\rho_2$. Its density is $\rho=(m_1+m_2)/V=(m_1+m_2)(m_1/\rho_1+m_2\rho_2)$. The proportion n of the first metal to the second in the alloy is $n=m_1/m_2$. Dividing both numerator and denominator of the last equation by m_2, we get

$$\rho=\frac{(m_1/m_2+1)}{(m_1/m_2)/\rho_1+1/\rho_2}=\frac{n+1}{n\rho_1+1/\rho_2}.$$

If ρ_1 and ρ_2 are known and we determine ρ of the alloy using Archimedes' principle, we can find n from this equation.

Q(S). A soft plastic bag weighs the same when empty as when filled with air at atmospheric pressure. Why? Would the weights be the same when measured in vacuum?

Ans. In air, the apparent weight = true weight − buoyant force due to air on it. When the bag is full of air, the buoyant force due to air increases by the weight of the additional air displaced. Hence whether empty or full we get only the weight of the bag (less the weight of the little air it displaces). But in vacuum, there is no force of buoyancy. Hence the bag containing air will weigh more.

TGL. 17. Pascal's law. *Basic experiments on the principle should be discussed. The principle of the hydraulic press should be given, without details of construction.*

Material. See Text, Properties of Matter, p. 34.

Principle of transmissibility of pressure: Hydrostatic bellows. A simple apparatus designed by Pascal, illustrates the *principle of transmissibility of pressure in a liquid.* A stout bladder or leather bellows filled with a liquid (water) has a vertical tube attached to it (Fig. 2). The liquid stands at the same height in both. When a load is placed on the platform attached to the bladder (or leather bellows), the liquid in the tube rises some distance and balances the load. The condition for balance is hydrostatic pressure in the tube = pressure in the bladder (or bellows). If α is the cross-sectional area of the bellows, W the weight on it, and h the height of the liquid in the tube above the liquid-level in the bellows, then $h\rho g=W/\alpha$.

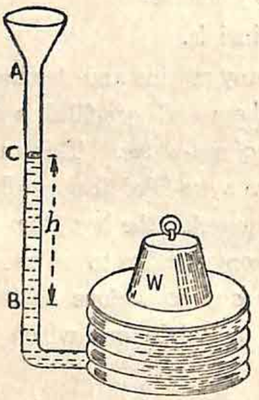

Fig. 2

If α is big enough, a small height of liquid in the tube will be enough to support a large weight. Suppose the platform has a diameter of 30 cm. A heavy boy, weighing 70 kg, can be balanced by a height of about only 1m of water in the tube.

Demonstration of Pascal's law. The accompanying figure shows a piece of apparatus to demonstrate Pascal's law. It consists of a glass barrel fitted with a plunger. The barrel ends in a bulb pierced with holes of the same size. It is filled with water by dipping the bulb in water and slowly raising the plunger. When the plunger is pushed in, water shoots out equally from all the holes. This shows that the pressure applied to the plunger has been transmitted uniformly throughout the water. A stronger force applied to the piston gives the water jets greater velocity.

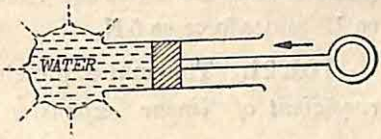

Fig. 3

TGL 19. Pumps. *Only principles of working with simple diagrams are required. For vacuum pumps, piston pump should be taught. Rotary pump should be mentioned. Idea of vacuum attainable with rotary and diffusion pumps should be given.*

Material. See Text, Properties of Matter, pages 44-48.

Well constructed rotary pumps can produce vacua of the order of 0·001 mmHg (1 mmHg is the unit of pressure used in vacuum practice). It is called a *torr* (in honour of Torricelli) and is equal to the hydrostatic pressure exerted by a column of mercury 1 mm tall. 1 mmHg$=1333$ dyn/cm^2.

High vacuum pumps called *diffusion pumps* can ordinarily attain vacua of the order of $10^{-5}-10^{-6}$ torr. With special care they can reach 10^{-8} torr.

HEAT

TGL 20. Recapitulation. *To start with, the concept of temperature may be introduced from analogy and the kinetic interpretation given later. CGS and SI units should be used generally, but B. Th. U. also should be mentioned.*

Material. See Text, Heat, pp. 58-59, Kinetic interpretation of temperature—Heat, p. 159, Sec. 9-5.

CGS unit of heat=Calorie (Heat, p. 97) ; B. Th. U. (Heat, p. 97)

SI unit of heat in the Joule (J). 1 cal=4·1855 J (=4·2 J nearly). (Heat, p. 98)

SI unit of temperature is the *Kelvin* (K, not °K). A difference of temperature of 1 K=a difference of temperature of 1°C. We take the temperature of ice=0°C=273·15 K (273 K for ordinary purposes).

Note. Temperature is a physical quantity. It should be expressed as the product of a number and a unit. The unit is Kelvin ; hence 273 K (there is to be no degree sign). Note that this is similar to writing a length as 10 m or a force as 5 N.

TGL 21. Thermal expansion. *Principle of measurement of the coefficient of linear expansion by Pullinger's apparatus should be described.*

Determination of the coefficient of apparent and real expansions of liquids are not required.

Verification of Boyle's law and Charles' law should be discussed briefly.

Determination of volume and pressure coefficients should be omitted.

Material. Pullinger's apparatus (Heat, p. 65).

Verification of Boyle's law is a prescribed laboratory experiment.

A method of verifying Charles' law is described.

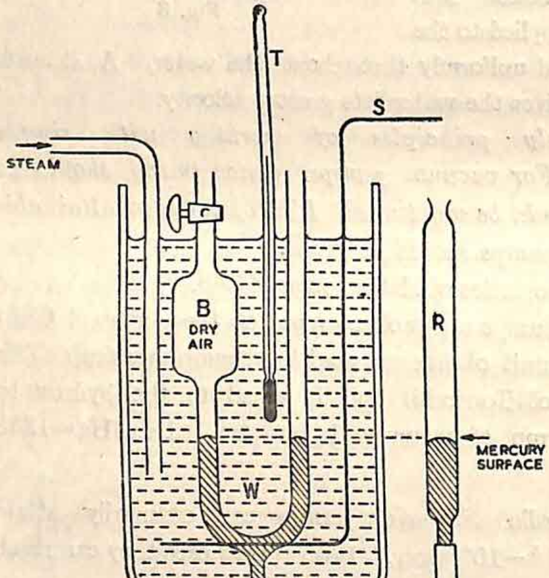

Fig. 4

Verification of Charles' law. The enclosed figure represents one form of apparatus for verifying Charles' law. The bulb B contains a fixed mass of the gas under consideration (say, air). B forms part of a U-tube, and is graduated in cm^3. The U-tube is connected to a mercury reservoir R as shown. R can be raised or lowered as desired. The temperature of the water bath can be raised by passing steam through it.

The temperature of the bath is held constant at some value. R is then raised or lowered until the mercury levels in R and both limbs of the U-tube are at the same height. The volume V of the gas in B is read off.

The observations are repeated at intervals of several degrees. As many readings as possible are taken of the volume (V) and corresponding temperature (θ). Adjustment of the mercury levels makes the pressure in B equal to the atmospheric pressure. We thus get a set of values of V, θ at constant pressure for the same mass of gas.

V is then plotted against θ. The graph is found to be a straight line. This confirms Charles' law that volume increases with temperature at a constant rate when the pressure and mass are constant.

TGL 22. Calorimetry. *Specific heat should be defined as the heat required to raise the temperature of unit mass through unit temperature. Definition of specific heat as a ratio* **should not be taught.**

Remarks. A very welcome directive : Old methods and concepts that have been dropped from scientific literature should no longer persist in our books. It would have been better to modernize calorimetric terms by dropping 'water-equivalent' and replacing it by 'heat capacity', 'specific heat' by 'specific heat capacity', 'latent heat' (for unit mass) by 'specific latent heat'. The use of the term 'specific' is now confined to quantities per unit mass or per unit volume.

[Ref. Heat, p. 98 (bottom half), p. 99 (top), p. 101 (centre), p. 108 (bottom)]

TGL 23. Change of state. *Franklin's and Bottomley's experiments should be demonstrated.*

Applications of Hygrometry in meteorology should be pointed out.

Remarks. It is easy to demonstrate Franklin's experiment (Heat, p. 120). Most schools will find it difficult to get a suitable block of ice in time to demonstrate Bottomley's experiment (Heat, p. 113). If ice is available, a thin, bare copper wire loaded as heavily as possible should be used.

Applications of hygrometry—See Heat, p. 132, Sec. 7-9.

TGL 24. Mechanical equivalent of heat. The equation PV^r=Const. should not be proved.

Remarks. In the original pamphlet of the Council 'PV=const.' is a misprint for PV^r=Const.

TGL 25. Transmission of heat. (i) Principle of Searle's method of determination of thermal conductivity of solids should be described avoiding details.

(ii) Thermometric conductivity should not be introduced.

(iii) That good thermal conductors are also good electrical conductors should be mentioned.

Material. (i) Searle's method of determining thermal conductivity.

The method is applicable only to good conductors. A diagram of the apparatus is given below.

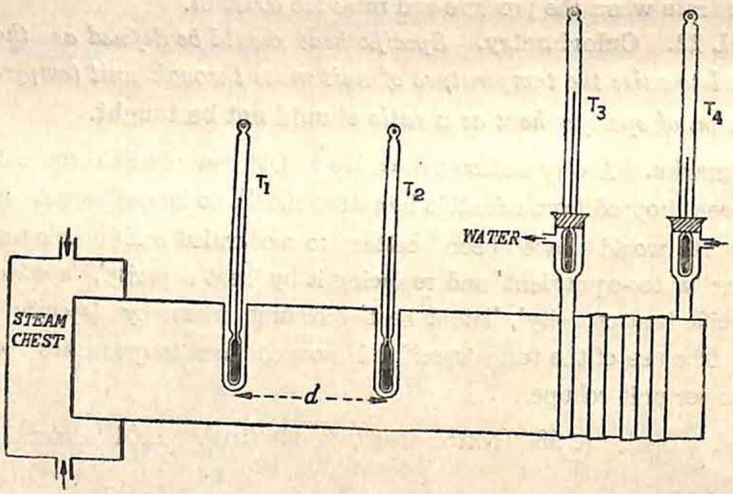

Fig. 5

A copper bar about 5 cm in diameter is polished, surrounded by dry felt and enclosed in a wooden box. Steam circulating through a chamber attached to one end of the bar maintains this end at 100°C. A steady flow of cold water is kept passing through a copper spiral soldered round the other end. It keeps this end at a steady lower temperature. This enables the rate of flow of heat through the bar to be measured. Thermometers T_1 and T_2 are placed in holes in the bar at a known distance d apart. The holes contain mercury to ensure good thermal contact.

Two more thermometers T_4 and T_3 record inlet and outlet temperatures of flowing water.

When all thermometer readings are steady, the number m of grams of water flowing through the spiral in a known time t seconds is noted. The temperatures are all recorded.

Let θ_1 and θ_2 be the steady temperatures indicated by the thermometers T_1 and T_2, and θ_4 and θ_3, the inlet and outlet temperatures respectively. Then the heat flowing through the bar in t seconds is $m(\theta_3-\theta_4)$. If r is the radius of the bar, its area of cross-section is πr^2. Therefore, we shall have (from the definition of thermal conductivity)

$$m(\theta_3-\theta_4)=K\pi r^2(\theta_1-\theta_2)t\ d.$$

Since all values except K are known, it can be calculated. Using cgs units with t in °C, we get K in cgs units.

(iii) It may be noted that *good thermal conductors, such as metals, are also good electrical conductors.* In fact, the ratio of the thermal conductivity to the electrical conductivity is the same for all metals at a given temperature. This ratio is proportional to the absolute temperature. (The result is known as Wiedemann-Franz law.) Most pure metals obey the law with reasonable accuracy at ordinary temperatures.

The sameness of the ratio shows that the same process must be responsible for conduction of heat and electricity in metals. In metals, free electrons conduct electricity. Therefore, they must also be responsible for thermal conduction in metals.

VIBRATIONS AND WAVES

TGL 26. (i) *Examples of oscillations from common experience are to be given and discussed. The mathematical expression for SHM is to be derived and quantities associated with the general expression should be discussed. Graphical representations are also necessary.*

Material. Some examples are motion of pendulum of a clock, children's swing, motion of the needle of a sewing machine, to-and-fro motion of the piston of a railway engine, motion of any part of a string under tension. Students may suggest more. Say which is longitudinal and which is transverse.

Expression for SHM—Ref. Vibrations and Waves, pp. 88-89, Eqs. 1-2.1 to 1-2.4. Consider Sec. 1-2.2 fully.

Graphical representation—Ref. Same text ; pp. 95, 96. (Fig.s 1-10, 1-11).

TGL. 26. (ii) *The energy of a simple harmonic oscillator should be calculated assuming the expression for potential energy.*

Material. Ref. Vibrations and Waves, p. 96, Sec. 1-7.

TGL 26. (iii) *Transverse and longitudinal vibrations are to be explained with the help of suitable examples. Resonance and forced vibration should also be explained (qualitatively). Demonstration should be given.*

Material. Distinguish between transverse and longitudinal vibrations in relation to examples in (i) above. A vibrating steel strip is transverse. Vibration of a body suspended by a rubber cord or long spring is longitudinal.

Forced vibration and resonance—Ref. Vibrations and Waves, p. 100, Sec. 1-10. Demonstration—Same ref.. p. 102, Fig. 1.16. Examples—p. 102, Sec. 1-11.

TGL. 26. (iv) (*a*) *Distinction between vibrations and waves should be clarified with suitable examples.*

(*b*) *The different types of waves should be explained physically and pictorially.*

(*c*) *The important features of transverse waves should be demonstrated with a ripple tank if available.*

Material. (a) **Difference between vibrations and waves.** A vibration spreading out through a medium is a wave. Both require *elasticity* and *inertia.* In a vibration, such as the vibration of a load supported by a spring, inertia is localized in the load and elasticity is localized in the spring. When such a vibrating body is placed in an extended medium (such as water or air) waves spread out into the medium. Vibration occurs at every point of the medium. So the medium must have elasticity and inertia distributed throughout it so as to be capable of carrying waves.

Particles along the line of advance of a wave are in different states (or phases) of vibration. Particles away from the source acquire at a later time the state (or phase) of vibration of a particle lying nearer to the source.

(b) Ref. Vibrations and Waves, p. 108, Sec. 2-2.2, p. 109, Sec. 2-2.3 ; Figures on pp. 112 and 113.

(c) Perhaps even 1% of our teaching institutions do not possess

'Ripple Tanks'. But it is a most useful apparatus, which can demonstrate before a class all properties of waves with the help of ripples of water. A single demonstration clarifies wave properties much better than any number of lectures even with demonstration slides. A 'film strip', if available, is an equally good substitute.

Teaching institutions should try to acquire a ripple tank, or a suitable film strip with projector. Perhaps the USIS or the British Council may help.

TGL 26.(v) *The essential similarities and dissimilarities in properties between sound and light waves should be explained with suitable examples. The elastic solid theory of light should* **not** *be taught.*

Essential similarities and dissimilarities in the properties of light and sound waves.

Similarities. Both show such wave properties as (a) reflection, (b) refraction, (c) interference, (d) diffraction and (e) scattering. Both move with different speeds in different media, though their speeds are widely different.

Dissimilarities. Light waves can be polarized, but not so the sound waves. This shows that light waves are transverse in nature. Sound waves are longitudinal.

Light waves can pass through vacuum, but not so the sound waves. Sound waves are longitudinal, elastic waves in a material medium. Light waves are known to be electromagnetic waves. They consist of vibrations of an electric intensity vector **E** and a magnetic intensity vector **H**. **E** and **H** are perpendicular to each other. Both are perpendicular to the direction of propagation of light.

Frequency of audible sound lies in the approximate range 20 to 20,000 Hz. Frequency of visible light lies in the approximate range 4.3×10^{14} to 7.5×10^{14} Hz.

[See also Text, Vibrations and Waves, p. 176, Sec. 9-1.]

TGL 27. Doppler effect *in relation to sound should be dealt with. The fact that a similar effect exists in light and its importance—the red shift—should also be mentioned.*

Material. For Doppler effect in Sound, see Vibrations and Waves p. 119, Sec. 2-6.

Doppler effect in light. Light waves also show Doppler effect. When the spectra of light from the eastern and western edges of the sun are separately examined, it is found that Fraunhofer lines are displaced towards the red end in one case and towards the violet in the other. A shift towards the red means an increase in wavelength and hence a lowering of frequency. Lowering of frequency is explained if we assume that the source is moving away (receding) from us. Violet shift means that the source is moving towards the observer. The red and violet shifts of spectral lines from the two edges of the sun indicate that the sun is rotating about a north-south axis. (The period of rotation has been determined in this way.)

Spectral lines of light from very distant galaxies show a *red shift.* From the amount of shift one can calculate the speed with which such a galaxy is moving away from us. Accurate measurements gave the very important and astonishing result that *distant galaxies are all moving away from us with a speed proportional to the distance from us.* This observation gave rise to the theory of the *expanding universe* in cosmology. (Cosmology is the science of the nature, origin and history of the *universe.*)

TGL 28. (i) Light as a wave phenomenon. (*i*) *Method of determination of velocity of light should not be taught, but its importance should be stressed.*

Material. See Text, Vibrations and Waves, Chap. 9, pp. 176-182.

Importance of determination of velocity of light. The velocity of light *in vacuum* is Nature's speed limit. No material particle can exceed, or even reach, this limit. This is a law of Nature, and is not due to any inability on our part to apply a large enough force.

The velocity of light in vacuum is the same under all circumstances. It is not affected by the motion of the sources or of the observer. This fact is the foundation stone of Einstein's theory of Special Relativity.

An accurate determination is also of importance in measuring distances. We can get a beam of light (in fact, microwaves) reflected from the object whose distance we want to find. When the velocity of light is known, a knowledge of the time interval between emission and reception back (of the microwaves) will give us the distance. It should be noted that the velocity of light in vacuum does not depend on wavelength. So, long radio waves, shorter microwaves or even shorter visible light travels with the same speed in vacuum. Light beams are used for measuring distances of artificial satellites and of the moon. Time intervals of the order of 10^{-8} s are no longer difficult to measure.

TGL 29. (ii) *Electromagnetic nature of light waves should be pointed* *out and qualitative discussion on the electromagnetic spectrum should be given.*

Material. Nature of light waves (See Text, Vibrations and Waves, pp. 176-177).

Electromagnetic spectrum. An electromagnetic wave is an oscillating electric force travelling through space, accompanied by a similarly oscillating magnetic force in a plane at right angles to it. Such waves travel in vacuum with the same velocity as light, irrespective of the wavelength (or frequency) of the waves.

The wavelength range extends from about 10^6m to 10^{-14}m. The frequency range extends from about 10^2 to 10^{23} Hz (hertz=cycle per second).

Waves in the different ranges have different names, though waves of different names have no sharp dividing lines with respect to wavelength or frequency. Some names and approximate ranges are given below.

Name	Range (in m)	Range (in Hz)
Power	10^7-10^5	10^1-10^3
Radio	10^5-10^{-2}	10^5-10^{10}
Microwaves	10^1-10^{-3}	10^9-10^{11}
Infrared	$10^{-3}-7\times10^{-7}$	$10^{11}-10^{14}$
Visible	$4\times10^{-7}-7\times10^{-7}$	$4\cdot3\times10^{14}-7\cdot5\times10^{14}$
Ultraviolet	$4\times10^{-7}-2\times10^{-8}$	$\sim10^{15}-10^{17}$
X-rays	$10^{-8}-10^{-12}$	$10^{16}-10^{20}$
γ-rays	$10^{-10}-10^{-14}$	$10^{18}-10^{22}$

Standard broadcasting frequency is around 10^6 Hz.

TV broadcasting is around 10^8 Hz and radar, 10^{10} Hz.

H. S. EXAMINATION QUESTIONS, 1978

Paper I

GROUP A

Q. 1. *State Newton's second law of motion. Show how his law will give us a unit of force when the unit of mass is 1 kg and the unit of acceleration is 1 m/s^2. What is the name of this unit of force ?*

(*b*) *Express the weight of 1 kg in the MKS unit of force.*

(*c*) *A train is moving along a straight track. Explain what would be the nature of velocity-time graph in the following cases :—*

(*i*) *It is moving with a uniform acceleration.*

(*ii*) *Its acceleration is increasing.* [6+2+2]

Ans. (*a*) and (*b*) [See Text, Pt I, Mechanics, p. 29, p. 32, p. 33.]

(*c*) A velocity-time graph is one in which the instantaneous velocity is plotted against time. Time is plotted along the x-axis and velocity along the y-axis.

(i) Since the acceleration is uniform, velocity will increase at a constant rate. Hence the velocity-time graph will be a straight line sloping upward. (See Text, Mechanics, p. 10, Fig. 1.3).

(ii) When acceleration is increasing, velocity will increase at a faster rate than when acceleration is constant. The graph will therefore be a curve rising faster than the straight line mentioned in (i). The curve will be concave upward.

Q. 2. (*a*) *Define kinetic energy. Obtain an expression for it in the case of a particle of mass m moving with a velocity v.*

(*b*) *A body of mass 100 g is let loose from a tower 100 m high. Calculate the kinetic energy of the body* (i) *one second after release, and* (ii) *when at the bottom of the tower.*

(*c*) *A man rises in a lift carrying a bucket of water. Explain :—*

(i) *If any work is done by the man on the bucket of water, and*

(ii) *If the energy of the bucket of water would remain unaltered.*

[4+4+2]

Ans. (*a*) [See Text, Mechanics, p. 81.]

(*b*) (i) The body falls from rest and acquires a velocity of 9·8 m/s in 1 second. Using SI units, its mass = 100 g = 0·1 kg and velocity = 9·8 m/s. Hence

K.E. $=\frac{1}{2}\times 0{\cdot}1\times(9{\cdot}8)^2$ SI units $=4{\cdot}9$ J (= joule, the SI unit of energy).

In cgs units, mass = 100 g, velocity = 980 cm/s.

$\therefore$ K.E. $=\frac{1}{2}\times 100\times(980)^2$ cgs units $=4{\cdot}9\times 10^7$ ergs (erg = cgs unit of energy).

(ii) At the bottom of the tower, the potential energy is fully converted into K.E.

$\therefore$ K.E. $=mgh=0{\cdot}1$ kg $\times 9{\cdot}8$ m/s$^2\times 100$ m $=98$ SI (or MKS) units $=98$ J.

In cgs units K.E. $=100$ g $\times 980$ m/s$^2\times 100\times 100$ cm $=98\times 10^7$ ergs.

(*c*) (i) To do work, an applied force must move its point of application through a distance. The man merely holds the bucket. The force he applies on the bucket does not move its own point of application. Hence the man does not do any work.

(ii) So far as the man in the lift is concerned there is no change in the energy of the bucket. To a man at rest outside the lift, the potential energy of the bucket increases as the lift goes up.

Q. 3. (*a*) *Explain what is meant by centripetal and centrifugal forces.*

(*b*) *Find an expression for the centripetal force.*

(*c*) *A cyclist is describing a circle of 20 m radius at a speed of 18 km per hour. What is his inclination to the vertical ? (Assume the rider and the cycle to be in one plane.)* [2+4+4]

Ans. (a) and (b) [See Text. Mechanics, pages 56, 57, 58.]

(c) The cyclist inclines himself towards the centre of curvature of his track. The horizontal component of the reaction of the earth on the cycle and the cyclist supplies the necessary centripetal force. Let θ be the inclination of the cycle to the vertical. The reaction P of the earth acts in the plane of the cycle (See Mechanics, p. 60, Fig. 2.10). The vertical component $P\cos\theta$ balances the weight mg of the cycle and the cyclist. The horizontal component $P\sin\theta$ supplies the necessary centripetal force.

$\therefore$ $P\sin\theta=mv^2/r$ and $P\cos\theta=mg$.

Or $\tan\theta=v^2/gr$.

Here $v=18$ km/hr $=18000$ m/$(60\times 60$ s $=5$m/s ; $g=9{\cdot}8$m/s^2 ; $r=20$m.

$\therefore$ $\tan\theta=(5\text{m/s})^2/(9{\cdot}8\text{m/s}^2\times 20\text{m})=25/196$.

(From the table of tangents θ comes out to be about 7°18′.)

[Note that reaction of the ground is greater when the cyclist takes a turn than when he rides straight. Why ?]

Q. 3, (Alternative). (*a*) *What is meant by elasticity of matter and elastic limit ? Define and explain the different types of modulus of elasticity.*

(*b*) *A mass of 5 kg is suspended from a vertical wire 1 m long and of 1 mm radius. If Young's modulus of the material of the wire be $2{\cdot}0\times10^{12}$ dynes/cm^2, find the length of the wire without load.* [6+4]

Ans. (*a*) [See Text, Properties of Matter, pages 8, 10, 11, 13.]

(b) Removal of load shortens the wire. So, effect of removal is the same as compression by an equal load. Let e be this contraction. Then, according to definition,

$$\text{Young's modulus} = \frac{\text{Force per unit area}}{\text{Contraction/original length}} = \frac{W/\alpha}{e/l}$$

where W = weight of the load,

α = area of cross-section and l, the original length.

Now, $W = 5$ kg-wt $= 5000\times980$ dynes (We use cgs units as E is given in that unit) ;

$\alpha = \pi(1\ \text{mm})^2 = 3{\cdot}14\times(0{\cdot}1\ \text{cm})^2 = 3{\cdot}14\times0{\cdot}01\ \text{cm}^2$;

$l = 1\ \text{m} = 100\ \text{cm}$;

$E = 2{\cdot}0\times10^{12}\ \text{dynes/cm}^2$.

Substituting these values in the equation for E, we get

$$2{\cdot}0\times10^{12} = \frac{5000\times980}{3{\cdot}14\times0{\cdot}01}\times\frac{100}{e}$$

$$\text{or,}\quad e = \frac{49\times10^7}{2{\cdot}0\times3{\cdot}14\times0{\cdot}01\times10^{12}}\ \text{in cgs units}$$

$$= 0{\cdot}0078\ \text{cm (on simplification).}$$

Hence length of wire without load $= 100 - 0{\cdot}0078 = 99{\cdot}9922$ cm.

Q. 4. (*a*) *What do you mean by a seconds pendulum ? Calculate its length in cm at a place where $g = 980$ cm/sec^2.*

(*b*) *State and explain whether the time period of the pendulum will change in the following cases :—*

(*i*) *If a hollow bob is taken instead of a solid bob.*

(*ii*) *If the hollow bob is partially filled with water.*

(*iii*) *If the pendulum is taken to the top of a mountain.*

(*c*) *Artificial satellite revolves around earth in a circular orbit at a height of 400 km above earth's surface. Assuming radius of earth to be*

6000 km and value of g at earth's surface to be 980 cm/sec^2, calculate the velocity of the satellite. [3+3+4]

Ans. *(a)* A pendulum which takes one second in moving from one end of its swing to the other is called a *seconds pendulum*. Its time period $T=2$ seconds.

For a simple pendulum, $T=2\pi\sqrt{l/g}$. In the case of a seconds pendulum $T=2$ s. Given, $g=980$ cm/s^2. Taking $\pi=3{\cdot}14$, we have

$$l=gT^2/4\pi^2=980\times4/(4\times9{\cdot}86) \text{ in cgs units}=99{\cdot}4 \text{ cm}.$$

(b) The periodic time of a simple pendulum is given by $T=2\pi\sqrt{l/g}$. Any change in l (length) or g (acceleration due to gravity) will affect T accordingly. *l is the distance between the point of suspension and the centre of gravity of the bob.*

(i) If the position of the centre of gravity of the hollow bob is the same as that of the solid bob, l will not change. Hence T will not change.

(ii) Filling the hollow bob partially with water lowers the centre of gravity of the bob. Hence l increases, and so does T.

(iii) At the top of a mountain g is smaller than at the surface of the earth. Hence at the top T will increase.

(c) For motion in a circular orbit, gravitational attraction = centripetal force. Let M = mass of earth, m = mass of satellite, v = orbital velocity of satellite, r = radius of orbit, G = universal gravitational constant, g = acceleration due to gravity on earth's surface.

Then $GMm/r^2=mv^2/r$ or, $v^2=GM/r$ or $v=\sqrt{GM/r}$;

$r=R+h$ where R = earth radius and h = height of orbit above earth surface.

Besides, $g=GM/R^2$ or $GM=gR^2$.

$$\therefore \quad v=\sqrt{gR^2/r}=R\sqrt{g/(R+h)}.$$

In mks (or SI) units, $R=6000$ km $=6\times10^6$ m, $g=9{\cdot}8$ m/s^2, $r=(6000+400)$ km $=6400\times10^3$ m. Substituting these values in the last equation, we get

$$v=6\times10^6\sqrt{9{\cdot}8/(6400\times10^3)}=6\times10^6\sqrt{98/(64\times10^6)}$$
$$=(6/8)\times10^3\times\sqrt{98}=0{\cdot}75\times9{\cdot}9\times10^3=7{\cdot}4\times10^3 \text{ in mks units}$$
$$=7{\cdot}4 \text{ km/s}.$$

Q. 5. (a) *Explain why a lump of iron (sp.gr. 7·8) sinks in water but floats in mercury (sp. gr. 13·6) ? State the principle you will need for your explanation.*

(b) *Calculate the fraction of the total volume of the iron lump which will be under the mercury surface when floating in the latter.*

(c) *State Pascal's law in relation to transmission of pressure in a liquid.* [4+4+2]

Ans. (a) and (b). A body will float in a liquid if it can displace its own *weight* of the liquid. According to Archimedes' principle a partly immersed body displaces its own *volume* of the liquid and feels an upthrust equal to the weight of the displaced liquid.

Let us consider 1 cm^3 of iron. Its weight is 7·8 gf. When a fraction 7·8/13·6 of it, that is, 78/136 cm^3 of iron is immersed in mercury the upthrust on the iron is $(78/136)\times 13·6 = 7·8$ gf = weight of the immersed iron. Hence the iron will float in mercury. A fraction 78/136 of its volume will be immersed.

The case is one of application of Archimedes' principle (State it).

In water 1 cm^3 of iron will feel an upthrust of only 1 gf. This cannot balance the weight of the iron. Hence the iron will sink in water.

[See Text, Properties of Matter, p. 25.]

(c) [See Text, Properties of Matter, p. 34.]

Q. 6. (a) *Show with the help of an experiment that the atmosphere exerts pressure.*

(b) *"At a certain place the atmospheric pressure is equal to 760 mm of mercury." What do you understand by the above statement ? Calculate the atmospheric pressure in cgs units.* (*Given g=980 cgs units and density of mercury=13·6 gm/c. c.*)

(c) *Explain briefly the working of a siphon.* [3+4+3]

Ans. (a) [See Text, Properties of Matter, p. 36, Torricelli's experiment.]

(b) It means that the atmospheric pressure at the place is equal to the hydrostatic pressure exerted by a column of mecury 760 mm tall.

[See Properties of Matter, bottom of p. 31 and top of p. 32.

Ans. 1·013 dyn/cm^2.]

(c) [See Properties of Matter, p. 43.]

GROUP—B

Q. 7. (a) *Define coefficient of expansion of a solid.*

(b) *Obtain the relation between the coefficient of linear expansion and volume expansion of a solid.*

(*c*) *A steel scale is correct at 68°F. The length of a brass rod measured by it at 50° C is found to be 1·5 m. Find out the true length of the rod at 50° C (Linear expansion coefficient of steel=$11{\cdot}2\times10^{-6}$ per °C).* [2+4+4]

Ans. (*a*) [See Text, Heat, p. 64.]

(*b*) [See Text, Heat, p. 68]

(*c*) $68°\text{F}=(68-32)\times\frac{5}{9}=20°\text{C}$.

The rise in temp. of the scale$=50°-20°=30°\text{C}$.

For this temp. rise 1m of the scale expands to $1(1+11{\cdot}2\times10^{-6}\times30)$ m $=1{\cdot}000336$ m. But it still reads 1m. Therefore the true length for a reading of 1·5 m on this scale at 50°C is $1{\cdot}5\times1{\cdot}000336=1{\cdot}500504$ m.

Q. 8. (*a*) *Explain what is meant by apparent and real expansion of a liquid. Which coefficient of a liquid is determined by a weight thermometer ?*

(*b*) *Describe the method.*

(*c*) *How does fish live in a frozen lake ?* [4+5+1]

Ans. (*a*) (See Text, Heat, p. 76, Sec. 3-1.) Apparent expansion coefficient.

(*b*) (See Text, Heat, p. 79, Sec. 3-3.)

(*c*) (See Text, Heat, p. 84.)

Q. 8. (Alternative). (*a*) *What do you mean by 'natural vibration' and 'forced vibration' ?*

(*b*) *Distinguish between 'forced vibration' and 'resonance'.*

(*c*) *Why orders are given to soldiers to break steps while crossing a suspension bridge ?*

(*d*) *What is the utility of the hollow box of a violin ?* [2+4+2+2]

Ans. (*a*) (See Text, Vibrations and Waves, pp. 99-100. 'Natural vibration' and 'Free vibration' mean the same thing.)

(*b*) (See Text, Vibrations and Waves, p. 100.)

(*c*) (—Do. —p. 103, Example 5.)

(*d*) The vibrating string communicates its vibration to the thin board of the box through the bridge. The board is thrown into forced vibration. This vibration is communicated to the air inside the hollow box. A large mass of air thus undergoes forced vibration. This increases the loudness of the sound. The string alone emits a feeble sound. (See also p. 150, para. 4 of Vibrations and Waves.)

Q. 9. (*a*) *What do you understand by the thermal capacity of a body ? How does it differ from the water equivalent of the body ?*

(*b*) *An alloy contains 60% copper and 40% nickel. A piece of the alloy weighing 50 g is heated to 80°C and is dropped into a calorimeter of water equivalent 10 g. The calorimeter contained 90 g of water at 10°C. Find the final temperature of the mixture.*

(*Sp. heat of copper=0·09 ; sp. heat of nickel=0·11*)

(*c*) *What part does the specific heat of water play in causing the sea-breeze ?* [3+5+2]

Ans. (*a*) (See Text, Heat, p. 100, Sec. 5-3).

(*b*) 50 g of the alloy contains $50 \times (60/100) = 30$ g of copper and $50 \times (40/100) = 20$ g of nickel. Instead of the single piece of alloy, we may assume that there are a piece of copper weighing 30 g and a piece of nickel weighing 20 g.

Here, the calorimeter and water gain heat. The alloy loses heat. Let $t°$ C be the final temperature.

Heat lost by the alloy = Mass × sp. heat × temp. fall

(in calories) $= (30 \times 0{\cdot}09 + 20 \times 0{\cdot}11)(80 - t)$

Heat gained by the calorimeter = mass × sp. heat × temp. rise

(in calories) $= (90 + 10) \times 1 \times (t - 10)$,

since the specific heat of water = 1(cal/g °C).

$$\therefore \quad (30 \times 0{\cdot}09 + 20 \times 0{\cdot}11)(80 - t) = 100(t - 10)$$

Solving for t, we get $t = 13{\cdot}27$°C.

(*c*) Specific heat of water being much higher than that of land masses, land masses become warmer while sea-water remains cooler when the sun shines on them. This causes the air above the land to rise. Its place is taken by the cooler air from the sea, giving rise to sea-breeze.

The high value of the sp. heat of water is responsible for sea (and land) breezes.

(See Text, Heat, p. 147, Example vii.)

Q. 10. (*a*) *Explain why the sp. heat of a gas at constant pressure is greater than that at constant volume.*

(*b*) *A tube of heat insulating material, closed at both ends, contains 800 g of lead shots. The tube is 1 m long and is held vertically. It is then suddenly inverted so that the lead shots fall to the other end. After 50 such inversions the temperature of the lead shots is found* *to rise by* $3{\cdot}89°C$. *Assuming that the whole of the heat generated remains within the lead shots, calculate the mechanical equivalent of heat.* (*Sp. heat of lead*$=0{\cdot}03$).

(*c*) *What is the first law of thermodynamics ?* [3+5+3]

Ans. (*a*) (See Text, Heat, p. 170)

(*b*) Mechanical work done = Energy of fall × number of times the shots fell $= mgh \times n = 800\ \text{g} \times 980\ \text{cm/s}^2 \times 100\ \text{cm} \times 50 = 40 \times 98 \times 10^6$ cgs units (ergs).

Heat developed = mass × sp. heat × temp. rise

$$= 800\ \text{g.} \times 0{\cdot}03\ \text{cal/g}^\circ\text{C} \times 3{\cdot}89^\circ\text{C} = 24 \times 3{\cdot}89\ \text{cal}$$

$$\therefore\ J = \frac{\text{Work}}{\text{Heat}} = \frac{392}{93{\cdot}36} \times 10^7 = 4{\cdot}2 \times 10^7\ \text{ergs/calorie.}$$

(*c*) (See Text, Heat. p. 167, Sec. 10-4, Eq. 10-4.1.)

Q. 11. (*a*) *State the fundamental assumptions in kinetic theory of gases.*

(*b*) *Discuss the pressure of a gas and the concept of temperature according to kinetic theory.* [5+5]

Ans. (*a*) (See Text, Heat p. 158, Sec. 9-3.)

(*b*) (Do. Sec. 9-4 and p. 159, Sec. 9-5.)

Q. 11. (**Alternative**). (*a*) *What is Laplace's correction of Newton's formula for the velocity of sound in a gas ? Why was the correction necessary ?*

(*b*) *Discuss characteristics of a musical sound. On what factors does the quality of a musical sound depend ?* [5+5]

Ans. (*a*) (See Text, Vibrations and Waves, p. 127, Sec. 3-5.1.)

(*b*) (Do. p. 164, Sec. 7-2 ; p. 166, Sec. iii)

Q. 12. (*a*) *Define the following terms in connection with wave motion—*(*i*) *Wave length,* (*ii*) *Frequency,* (*iii*) *Amplitude.*

(*b*) *What is a stationary wave ? Describe a simple experiment to demonstrate the formation of a stationary wave.*

(*c*) *Discuss the formation of beats.* [3+4+3

Ans. (*a*) (See Text, Vibrations and Waves, p. 114, Sec. 2-4,)

(*b*) (Do, p. 132, Sec. 4-5 ; p. 139, Sec. 4-5.2.)

(*c*) (Do. p. 136 ; use Sec.s 4-2 and 4-3.)

SELECTED SPECIMEN QUESTIONS

(Framed by the H. S. Council)

Paper—I

[Here we have considered answers to some selected critical questions framed by the Council. They are such that a student broadly familiar with the text may find it difficult to answer.]

Q. 1. *Distinguish between average and instantaneous acceleration. Which of them is important in the laws of mechanics ? Show that the instantaneous acceleration at any point in the velocity-time graph equals the slope of the tangent to the graph at that point.*

If v_1 is the velocity at any moment t_1 and v_2, that at t_2, then $(v_2-v_1)/(t_2-t_1)$ is the average acceleration between the moments t_1 and t_2. If t_2-t_1 is extremely short, then the value of the ratio is called the instantaneous acceleration at moment t_1.

Instantaneous acceleration is important in the laws of mechanics. Newton's second law $F=ma$ is the fundamental law of motion in mechanics. In this equation a is the instantaneous acceleration.

In the figure, let P and Q be two points on the velocity-time graph. P represents a velocity v_1 at time t_1, and Q represents a velocity v_2 at time t_2. The vertical distance QR between P and Q is v_2-v_1 ; the horizontal distance PR is t_2-t_1. QR/PR= $\tan\theta'$ is the average acceleration between t_1 and t_2.

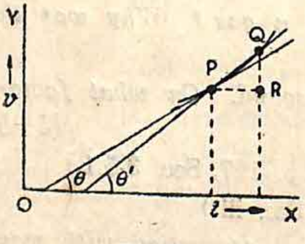

Fig. 6

If we make Q approach closer and closer to P, t_2-t_1 gradually diminishes. When Q is *indefinitely close* to P, the ratio QR/PR becomes equal to $\tan\theta$ where θ, is the angle between the tangent line at P and the time axis. By definition, this ratio is the instantaneous acceleration at P.

Hence the instantaneous acceleration at any point in the velocity-time graph equals the slope of the tangent to the graph at that point.

[Students familiar with the calculus will recognize that the differential coefficient of a variable v with respect to a variable t is defined in this way. Acceleration is the time rate of change of velocity. In the language of the calculus $a=dv/dt$.]

Q. 2. *(Some questions involving inertia and reference frame.)*

(i) *A boy sitting in a railway carriage moving with constant velocity throws a ball straight up into the air. Will the ball fall (a) behind him, (b) in front of him, (c) into his hand ?*

(ii) *What happens when the carriage (a) is accelerated forward, (b) goes round a curve when the ball is in the air ?*

(iii) *Explain briefly (a) why you fall forward when a moving train decelerates to a stop and (b) fall backward when the train accelerates. (c) What would happen if the train rounded a curve at constant speed ?*

Ans. (i) The ball has the same velocity as the train and will return into the boy's hand. It is an example of inertia of motion.

(Consider a reference frame fixed in the carriage. The boy is at rest in this frame. The ball is thrown vertically up. Pull of gravity will bring down the ball to its starting point. In this reference frame the motion of the carriage is immaterial so long as it is *not accelerated.*)

(ii) The boy gets the forward acceleration, but not the ball. Hence the ball falls behind the boy.

When the train takes the curve, the ball continues in a straight line with the velocity it has. It does not get the train's centripetal acceleration, and continues to move tangentially to the train's curved path.

(iii) The part of the body in contact with the train comes to a stop ; but the rest of the body tends to continue in its motion. Hence the forward fall. On acceleration, part of the body in contact with the train moves forward. But the rest of the body tends to continue in its state of rest. Hence the backward fall.

When the train rounds a curve he experiences the centrifugal force. This is directed away from the centre of curvature of the path.

Q. 3. *Can two vectors of different magnitudes be combined to give zero resultant ? Can three vectors ? Explain in brief.*

Ans. In addition of two or more vectors by the geometrical method we lay down the vectors successively with the tail of one to the head of the previous one. The resultant is obtained by joining the initial point (tail) of the first vector to the end point (head) of the last vector.

Two vectors of unequal magnitude will always leave a resultant, whatever the angle between them. There cannot be a zero resultant.

If three vectors, added in the manner stated, form a *closed triangle*, the resultant is zero.

(This leads us to the rules of triangle of forces and triangle of velocities.)

Q. 4. *Show that the first law of motion can be derived from the second.* [See Q(S) in TGL 1.]

Q. 5. *Briefly criticise the statement that the mass of a body is the measure of "quantity of matter" in it.*

Present-day physics recognizes 'operational definitions' only. An 'operational' definition means one which also indicates a method for measuring the physical quantity so defined.

Seen in this light, the given statement cannot be taken as the definition of *mass* ; no method of measurement is suggested by the statement.

(In elementary physics, we have, however, to introduce fundamental *concepts*, such as of matter, energy, temperature, etc. What we say there, relates to the concepts. The corresponding physical quantities are defined in some other way which makes measurement possible.)

Q. 6. *When a horse pulls a wagon the pull of the horse on the wagon is equal and opposite to the pull of the wagon on the horse. How can then the wagon be set moving ?*

(See TGL 2—The two paragraphs before "Walking".)

Q. 7. *A boy sitting on a railroad car moving with constant velocity drops a coin while leaning over the car. Describe the path of the coin as seen (a) by the man on the train, (b) by a person standing on the ground near the rail.*

With respect to the man on the ground, the coin has two motions at the same time—(i) a horizontal velocity, the same as that of the train, (ii) an accelerated vertical motion under gravity. (We ignore the resistance of the air to the motion of the coin.) The combination of these two motions makes the coin move along a parabolic path. (This is the kind of path described by a horizontal jet of water.)

To the boy in the car, the coin falls vertically under gravity. He has the same horizontal velocity as that of the coin. The common velocity does not produce any relative displacement between them.

(In reality, air resistance retards the motion of the coin.)

Q. 8. *Explain how conservation of momentum applies to a handball bouncing off a wall.*

The principle of conservation of momentum applies only to closed systems.

(See TGL 3). The ball alone does not form the closed system in this case. The wall and the ball, which interact between them, is the closed system. Momentum of this closed system does not change.

[See Text, Mechanics, p. 40, item (d).]

Q. 9. *(i) Elucidate why a space vehicle in orbit is a freely falling body with an acceleration towards the earth equal to the value of g at that point.*

(ii) Hence explain weightlessness.

(iii) Does Archimedes' principle hold in a satellite moving in a circular orbit ? Explain. Does Archimedes' principle hold for a free fall ?

(*i*) To make a space vehicle move in a circle around the earth th necessary centripetal force mv^2/r is supplied by the earth's pull mg on it g being the acceleration due to gravity at the distance r. If mg did not ac the vehicle would continue to move in a straight line with velocity v. m is *fully spent* in constantly pulling the vehicle from the straight path an keeping it on the circle. No part of mg is spent otherwise. Its direction i towards the centre of the earth. So the vehicle is a body falling freel towards the centre of the earth with an acceleration g appropriate to th place where the vehicle is.

(*ii*) Our sense of weight arises from the reaction of the floor on whic we stand. If this reaction is zero we shall feel weightless. In an eart satellite, the force mg on any piece of matter (of mass m) is fully spent i keeping it on the orbital path. There is no mechanical force which it ca exert on anything in contact with it so long as it is at rest relative to i surroundings. So a person in a space vehicle does not exert any force the floor. Hence there is no reaction of the floor on the person. The perso therefore feels weightless.

(*iii*) As stated above (in ii) a body in an artificial satellite does n exert any mechanical force on bodies in contact with it so long as it is rest with respect to them. A lump of iron kept on a beaker of water in satellite will remain where it is. If pushed someway down into the wa it will stay where it is. Archimedes' principle, which is based on the acti of g, does not operate here. It is like having no g at all.

Archimedes' principle will not hold when bodies are falling freely.

Q. 10. *Explain why the radian measure of angle is satisfactory for all systems of units.*

An angle in radian measure is the ratio of two lengths and is a pure number. A pure number does not depend for its value on any system of units. An angle in radians has the same value in all systems of units.

Q. 11. *A man rowing a boat up-stream is at rest with respect to the shore. Is he doing any work ?*

(See Q(S) at the end of TGL 10.)

Q. 12. *A bucket of water is suspended from a spring balance . Does the balance reading change when a piece of iron held by a string is immersed in water ?*

The water exerts an upthrust on the iron equal to the weight of the displaced water. The iron exerts an equal and opposite reaction on the water. This is directed downward. Hence the reading of the spring balance increases by this amount.

Q. 13. *Estimate the force required to pull apart two evacuated hemispheres of radius r.*

Atmospheric pressure acts equally on both hemispheres whichever way their plane of separation lies. Assume this plane to be vertical. Then the total vertical thrust on each will be zero. The total horizontal thrusts are equal and opposite. Horizontal thrust on any hemisphere$=\pi r^2 P$ where P is the atmospheric pressure.

To separate the hemispheres we shall have to apply on each a force of $\pi r^2 P$ directed outward.

Q. 14. *What happens to the molecules of a mass of gas when the temperature is doubled on the (i) absolute scale, (ii) centigrade scale ?*

According to the kinetic theory of gases the kinetic energy of a molecule is proportional to the absolute temperature. When the absolute temperature is double the kinetic energy of the molecules of the mass of gas is also doubled.

If the initial celsius temperature was θ°C, the final temperature is 2θ°C. The corresponding absolute temperatures are $273+\theta$ and $273+2\theta$. So

$$\frac{\text{initial kinetic energy of gas molecules}}{\text{final kinetic energy}}=\frac{273+\theta}{273+2\theta}.$$

The kinetic energy of the molecules increases in the above manner.

Q. 15. *(i) A certain mass of oxygen gas occupies a volume 400 c.c. at 27°C and 76 cm pressure. What would be its volume at N.T.P.?*

(ii) By how much will the product PV change? (R=2 calories)

(i) This part is clear enough and needs no help. The answer is

$$V=\frac{70}{76}\times\frac{273}{300}\times 400=335{\cdot}3\ \text{cm}^3.$$

(ii) This part has been made confusing by the strange datum $R=2$ calories. The molar gas constant is nearly 2 calories per kelvin per mole. Does R mean this? It should have been clearly stated. Otherwise one may take

$P_1V_1=RT_1$ and $P_2V_2=RT_2$, whence $P_1V_1-P_2V_2=R(T_1-T_2)$.

Putting $R=2$ cal in this equation gives the two sides *unequal dimensions.*

Q. 16. *Why is it difficult to walk on the surface of a frozen lake?*

In walking we push back on the ground with one foot.

The ground must be capable of exerting a force equal and opposite to the horizontal component of this thrust. This is ordinarily supplied by friction. The coefficient of friction between the ice and the foot (not clad in spiked shoes) is very small. The ice cannot support a horizontal back push of any appreciable extent. This makes taking a stride on ice very difficult.

(See the portion marked 'Walking' in TGL 2.)

Q. 17. *Why is it that at a given temperature we feel comfortable only if the relative humidity is within a definite range?*

When R.H. is below a certain value there is too much evaporation from the skin, giving us a feeling of dryness and hence discomfort. When R.H. is above a certain value evaporation of moisture from the skin is retarded. The body cannot get rid of the excess heat and we feel uncomfortable.

This range of R.H. values depends on the temperature. Besides, the range is not sharply defined at the boundaries.

Q. 18. *What is simple harmonic motion? Point out its correlation with uniform circular motion. Obtain a relation for the velocity of a body executing SHM.*

(*Remark.* The first two sentences of the question, taken together, is an unhappy combination. It is perfectly legitimate to say in answer to both that an SHM is the projection of a uniform circular motion on any diameter. If that was meant, the question should have been differently worded. In old textbooks, SHM was generally introduced by presenting the characteristic features of its motion. Then it was shown that the projection of a uniform circular motion has that character. This approach should no longer continue. SHM should be defined kinematically or dynamically and the characteristics determined from the definition.)

(See Text, Vibrations and Waves, p. 87. Sec. 1-2.)

Q. 19. *Give an instance of total reflection of sound waves.*

(See Text, Vibration and Waves, p. 132, Sec. 3-8.1.)

Q. 20. *What do you mean by coherent sources of light.*

Two sources of light are said to be *coherent* if the waves emitted by them are always in the same phase or always have a constant phase difference.

Q. 21. *State the basic points of similarity and dissimilarity between light waves and radio waves.*

Both are electromagnetic waves (See TGL 30 ii) and therefore have all the wave properties in common. The most prominent difference is that in wavelength. While the average wavelength of light is about 5×10^{-7}m, the range of wavelength of radio waves is about 10^5 to 10^{-2}m.

Another point of difference is in the mode of generation. Light waves are generated by transition of electrons from one energy level to another. The process occurs inside atoms and molecules. Radio waves are generated by oscillating electrons in the radio antenna.

An accelerated charged particle can produce either radio waves or light under appropriate conditions.

Since energy in electromagnetic waves is proportional to frequency, light waves have much more energy than radio waves.

No electromagnetic radiation other than light can create the sensation of sight.

Q. 22. *Explain why rectilinear propagation of light as assured in geometrical optics is only approximately true.*

Light waves (like all waves) have the property of bending round corners. This phenomenon is known as *diffraction*. In passing through an aperture of breadth a, the extent of bending is given by θ where $\sin\theta = \lambda/a$. λ is the wavelength of light, the mean value of which may be taken as about 6×10^{-5}cm. So an aperture of only 1 cm diameter will cause an angular spreading of 6×10^{-5} radian=about $\frac{1}{5}$ of a minute of arc. Such small spreading cannot be detected with the eyes. The beam appears, for all practical purposes, to move in a straight line.

In geometrical optics, apertures of beams that we use are of this order of magnitude. Hence light beams appear to travel in straight lines.

(See Text, Vibrations and Waves, pp. 180-181, Sec.s 9-4 and 9-5.)